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JOINT OPTIMIZATION OF PRODUCTION AND DISTRIBUTION

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To my mother and father,
To my brother and sisters,
To my nephews and nieces,
To my dear friends...

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CONTENTS

Acknowledgments . . . v

List of Figures . . . x

List of Tables . . . xii

List of Algorithms . . . xiv

Publications . . . xv

Acronyms . . . xvii

General Introduction . . . xix

1 INTRODUCTION TO SUPPLY CHAIN MANAGEMENT AND OPTIMIZATION 1
- 1.1 Introduction . . . 1
- 1.2 Introduction to Supply Chain and Supply Chain Management . . . 1
  - 1.2.1 Definition of Supply chain . . . 1
  - 1.2.2 Supply Chain Management (SCM) . . . 5
  - 1.2.3 Decisions in Supply Chain Management . . . 6
- 1.3 Supply Chain Coordination . . . 7
  - 1.3.1 Mechanisms for Supply Chain Coordination . . . 7
- 1.4 Supply Chain Planning . . . 9
  - 1.4.1 Sequential Planning . . . 9
  - 1.4.2 Integrated Planning . . . 10
  - 1.4.3 Vehicle Routing Problem . . . 11
  - 1.4.4 Lot Sizing Problem . . . 12
  - 1.4.5 Integrated Lot Sizing with Direct Shipment Problem . . . 15
  - 1.4.6 Inventory Routing Problem . . . 16
  - 1.4.7 Production Routing Problem . . . 16
- 1.5 Solution Approaches . . . 20
  - 1.5.1 Exact Methods . . . 20
  - 1.5.2 Heuristics . . . 22
  - 1.5.3 Metaheuristic . . . 22
- 1.6 Conclusion . . . 25

2 A DECISION-MAKING FRAMEWORK FOR VMI AND RMI CONTRACTS . . . 27
- 2.1 Introduction . . . 27
- 2.2 Literature Review . . . 28
  - 2.2.1 Supply Chain Coordination . . . 28
  - 2.2.2 Vendor-Managed Inventory Contracts . . . 29
  - 2.2.3 Analytical Methods for Selecting Between VMI and RMI . . . 31
- 2.3 Problem Description and formulations . . . 33
  - 2.3.1 All or nothing model . . . 34

2.3.2 The Best-VMI model . . . 39
2.4 Numerical Experiments . . . 40
2.4.1 Data Generation . . . 40
2.4.2 Results and Discussion . . . 42
2.5 Conclusion and Future Works . . . 56

3 RANKING TOOL FOR VMI AND RMI CONTRACTS . . . 59
3.1 Introduction . . . 59
3.2 Literature Review . . . 60
3.3 Problem Description and formulations . . . 60
3.3.1 Ranking Process . . . 63
3.3.2 Two-Stage Stochastic Programming Models . . . 63
3.3.3 Risk-Averse Two-Stage Stochastic Models . . . 66
3.4 Numerical Experiments . . . 68
3.4.1 Data Generation . . . 68
3.4.2 Two-stage Stochastic Models Results . . . 71
3.5 Conclusion and Future Research . . . 76

4 INTEGRATED PRODUCTION AND DISTRIBUTION PROBLEM . . . 79
4.1 Introduction . . . 79
4.2 Literature Review . . . 80
4.2.1 Lot Sizing Problem . . . 80
4.2.2 Integrated Lot Sizing with Direct Shipment . . . 81
4.2.3 Multi-Trip Vehicle Routing problem . . . 82
4.2.4 Solution Approaches . . . 82
4.2.5 Contribution . . . 83
4.3 Problem Description . . . 83
4.4 Solution Approach . . . 87
4.4.1 The hybrid simulated annealing algorithm . . . 87
4.4.2 Pseudo-code of the heuristic . . . 90
4.5 Numerical experiments . . . 91
4.5.1 Archetti et al. benchmark instances . . . 92
4.5.2 Generated data-sets . . . 94
4.5.3 Analysis of the performance of HSA heuristic . . . 94
4.5.4 The case study: Comparing Company's Strategy with our proposed approach . . . 99
4.6 Conclusion and Future Works . . . 104

5 TWO-ECHELON PRODUCTION ROUTING PROBLEM . . . 105
5.1 Introduction . . . 105
5.2 Literature Review . . . 106
5.2.1 The single echelon and the two-echelon PRP . . . 106
5.2.2 Solution approaches . . . 107
5.3 Problem Description and Formulation . . . 109
5.3.1 Numerical Example . . . 114
5.4 Solution Approach . . . 115
5.4.1 Three-Phase Iterative (3PI) Heuristic Approach . . . 115
5.4.2 Simulated Annealing with Path Relinking Procedure (SA-PR) 121
5.5 Numerical Experiments . . . 126

5.5.1 Generated data-sets 126
5.5.2 Performance Analysis 128
5.5.3 Computational Performance of Heuristics on Medium and Large Sized Instances 131
5.5.4 Results Analysis 133
5.5.5 Managerial Insights 134
5.5.6 Case Study 135
5.6 Conclusion 140

6 CONCLUSIONS AND FUTURE WORK 143

A APPENDIX 149

BIBLIOGRAPHY 149

LIST OF FIGURES

Figure 1.1 Supply chain structures (SENOUSSI 2016) 4
Figure 1.2 House of Supply Chain Management (Stadtler 2005) 6
Figure 1.3 Lot Sizing with Direct Shipment Problem (Adulyasak et al. 2015b) 15
Figure 1.4 Inventory Routing Problem (Adulyasak et al. 2015b) 17
Figure 1.5 Production Routing Problem (Adulyasak et al. 2015b) 17
Figure 2.1 VMI adoption rate under high transportation cost and for different demand patterns. 44
Figure 2.2 VMI adoption rate under low transportation cost and for different demand patterns. 45
Figure 2.3 VMI adoption rate under high inventory cost and for different demand patterns. 47
Figure 2.4 VMI adoption rate under low inventory cost and for different demand patterns. 48
Figure 2.5 Comparison between models when changing transportation costs. 50
Figure 2.6 Comparison between models when changing inventory costs. 51
Figure 2.7 Selected VMI contracts from both models with transportation costs **Model1**: All-or-Nothing, **Model2**:Best-VMI models 52
Figure 2.8 Selected VMI contracts from both models with inventory costs **Model1**: All-or-Nothing model, **Model2**: Best-VMI model 53
Figure 2.9 Distribution of selected VMI contract types across both models. Labels indicate which party among vendor (V) and retailer (R) bears inventory (I) and transportation (T) costs. . 53
Figure 2.10 VMI rate with changes of demand volume 54
Figure 2.11 VMI rate with changes of demand varibility 55
Figure 2.12 VMI rate with intermittent demands 55
Figure 3.1 The Expected Value of Perfect Information and the Value of Stochastic Solution for the contracts 72
Figure 3.2 Expected Cost and Expected Deviation evolution vs varying λ 74
Figure 3.3 Average Expected Cost 74
Figure 4.1 Representation of lot-sizing with Direct shipment 81
Figure 4.2 A general description of the investigated network 84
Figure 4.3 Representation of the multi-trip concept (Benfedel et al. 2025a) 84
Figure 4.4 Comparing Gaps of different acceptance probability in Simulated Annealing for $(T, N, P) = (6, 4, 4)$ 97
Figure 4.5 Comparing CPU time of different acceptance probability in Simulated Annealing 97
Figure 4.6 comparison of integrated and sequential approaches 102
Figure 4.7 Total cost comparison of two VMI policy options in Integrated and Sequential Approaches 103
Figure 5.1 Graphical Illustration of Supply Network (Benfedel et al. 2025b) 111

Figure 5.2 Flowchart of the proposed Three-Phase Iterative (3PI) Heuristic Approach. 117
Figure 5.3 Mixed Path Relinking. 123
Figure 5.4 Comparison of Average Gap Between 3PI and SA-PR Heuristics for all Cases 134
Figure 5.5 Comparison of Average CPU Time Between 3PI and SA-PR Heuristics for all Cases 134
Figure 5.6 Comparing total cost results of the three situations 136
Figure 5.7 Transportation cost results of the three situations 136
Figure 5.8 Inventory cost results of the three situations 137
Figure 5.9 Comparing total costs 138
Figure 5.10 Comparing setup costs 138
Figure 5.11 Comparing inventory costs 139
Figure 5.12 Comparing transportation costs 139
Figure 5.13 Total cost results of the three strategies 139
Figure 5.14 setup cost and customers inventory cost results of the three strategies 140
Figure 5.15 Inventory cost results of the three strategies 140
Figure 5.16 Transportation cost results of the three strategies 140

LIST OF TABLES

Table 3.1 Contracts' costs of Example 1 70
Table 3.2 Contracts' costs of Example 2 70
Table 3.3 Ranking Results for the Numerical Example 76
Table 4.1 Performance of the proposed metaheuristic against heuristic of Archetti et al. (2011) 93
Table 4.2 Instance generation. 95
Table 4.3 Performance of the proposed metaheuristic for small instances against CPLEX 99
Table 4.4 Performance of the proposed metaheuristic for large instances against CPLEX 100
Table 5.1 Production quantity and Setup times 115
Table 5.2 Inventory quantity 115
Table 5.3 Delivery quantity of first-level 115
Table 5.4 Delivery quantity of second-level 115
Table 5.5 Visiting Cost SC_{wit} 120
Table 5.6 Visiting periods H_{wit} 120
Table 5.7 New Visiting Cost SC_{wit} 120
Table 5.8 Solution 1 120
Table 5.9 Solution 2 120
Table 5.10 Performance of the proposed heuristics against CPLEX for small instances 129
Table 5.11 Performance of the proposed heuristics against CPLEX for medium-sized instances 130
Table 5.12 Performance of the proposed heuristics against CPLEX for large-sized instances 131
Table 5.13 Performance Comparison of Proposed Heuristics and CPLEX with 45 and 75 Second Time Limits for Medium-Sized Instances 132
Table 5.14 Performance Comparison of Proposed Heuristics and CPLEX with 200 and 450 Second Time Limits for Large-Sized Instances 133
Table A.1 Impact of high transportation costs on VMI contract rates. . 149
Table A.2 Impact of low transportation costs on VMI contract rates. . 149
Table A.3 Impact of high inventory costs on VMI contract rates. 150
Table A.4 Impact of low inventory costs on VMI contract rates. 150
Table A.5 Detailed Average Cost Data and Results for selected VMI contracts under High Transportation costs. 150
Table A.6 Detailed Average Cost Data and Results for selected VMI contracts under Low Transportation costs. 151
Table A.7 Detailed Average Cost Data and Results for selected VMI contracts under High Inventory costs. 151
Table A.8 Detailed Average Cost Data and Results for selected VMI contracts under Low Inventory costs. 151

Table A.9 Distances between warehouses of the Soft Drink company and customers 152

LIST OF ALGORITHMS

Algorithm 1.1 Simulated Annealing Algorithm 23
Algorithm 1.2 Tabu Search Algorithm 24
Algorithm 1.3 Genetic Algorithm 25
Algorithm 1.4 Variable Neighborhood Search 26

Algorithm 2.1 The “All-or-Nothing" Selection process 38

Algorithm 3.1 Ranking process 64

Algorithm 4.1 Simulated annealing algorithm 91

Algorithm 5.1 Path Relinking 124
Algorithm 5.2 Simulated Annealing with Path Relinking 127

PUBLICATIONS

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ACRONYMS

2E-PRP	Two-Echelon Production Routing Problem
2E-PRP-MDM	Two-Echelon Production Routing Problem With Mixed Delivery Modes
3PI	Three-Phase Iterative
3PL	Third-party Logistics
APS	Advanced Planning Systems
CIM	Customer Inventory Management
CLSP	Capacitated Lot Sizing Problem
CPFR	Collaborative Planning Forecasting And Replenishment
CRP	Continuous Replenishment Programs
DLSP	Discrete Lot Sizing And Scheduling Problems
EDI	Electronic Data Interchange
ELSP	Economic Lot Scheduling Problem
EOQ	Economic Order Quantity
ERP	Enterprise Resource Planning
EVPI	Expected Value Of Perfect Information
HSA	Hybrid Simulated Annealing
HSAGA	Hybrid Simulated Annealing And Genetic Algorithm
IIM	Integrated Inventory Management
ILP	Integer Linear Programming
ILS	Iterated Local Search
IRP	Inventory Routing Problem
IT	Information Technologies
LTL	Less-than-truckload
MILP	Mixed Integer Linear Programming
OWMR	One-Warehouse Multi-Retailer Problem
POS	Point-of-sales
PPE	Personal Protective Equipment
PRP	Production Routing Problem
RFID	Radio Frequency Identification
RMI	Retailer Managed Inventory
RP	Recourse Problem
SA	Simulated Annealing
SA-PR	Simulated Annealing Algorithm With Path Relinking
SC	Supply Chain

SCM	Supply Chain Management
TL	Truckload
TS	Tabu Search
TSP	Traveling Salesman Problem
ULSP	Uncapacitated Lot Sizing Problem
VICS	Voluntary Interindustry Commerce Standards Association
VMCI	Vendor-Managed Consignment Inventory
VMI	Vendor Managed Inventory
WS	Wait-and-See

GENERAL INTRODUCTION

In today's competitive global marketplace, organizations continuously face pressure to enhance efficiency, reduce costs, and elevate customer satisfaction. These challenges are intensified by globalization, digital transformation, and evolving consumer preferences, compelling businesses to adopt innovative strategies to sustain their market positions. To effectively address these pressures, companies must continuously improve their performance by efficiently delivering products and services that promptly and cost-effectively meet customer expectations.

This efficiency requires optimizing every stage of the value creation process, from raw materials sourcing and production to distribution and final delivery to customers. Such comprehensive optimization drives organizations to collaborate closely within structured supply chain networks. Consequently, effective Supply Chain Management (SCM), characterized by robust coordination, agility, and responsiveness, is essential to achieving and sustaining competitive advantages in the marketplace.

Despite advancements in SCM, firms encounter persistent systemic inefficiencies, most notably the bullwhip effect, in which fluctuations in orders increase as they move up the supply chain from retailers to wholesalers to manufacturers and ultimately to suppliers. (Chopra and Meindl 2013). This amplification of demand fluctuations generates significant operational inefficiencies, such as leading to elevated operational costs across the supply chain and a decline in customer service levels. This phenomenon drives all partners away from the efficient frontier, ultimately diminishing both customer satisfaction and overall supply chain profitability. Consequently, addressing the bullwhip effect through enhanced supply chain coordination has become a central focus in both academic research and industry practice, highlighting the need for integrated strategies and effective collaborative frameworks.

Among recent coordination mechanisms, Vendor Managed Inventory (VMI) has gained empirical validation as an effective strategy to mitigate demand distortion. VMI shifts inventory control and replenishment decisions from the retailer to the manufacturer or supplier, who often retains ownership of the inventory until it is sold. This system requires retailers to share demand data with manufacturers, enabling improved production planning and forecasting. When inventory decisions account for both retailer and manufacturer margins, VMI can enhance profitability for individual firms and the entire supply chain.

This operational implementation of VMI represents a broader class of integrated supply chain planning models that aim to coordinate multiple supply chain functionssuch as production, storage, and distributionin order to minimize the total cost generated by these interdependent activities. The literature offers numerous models that support such integration, emphasizing the value of aligning decision-making across at least two functional areas.

This thesis is positioned within the framework of VMI and integrated production-distribution planning. It aims to address practical and strategic challenges faced by supply chain partners by developing advanced decision-support tools. The primary objective is to enable more effective management of resources and operations at minimal cost while integrating production, inventory, and distribution planning.

The research is structured around three key objectives: The first project of this thesis focuses on assisting the vendor in deciding whether to adopt the VMI policy or retain the traditional Retailer Managed Inventory (RMI) approach. We develop novel decision-support tools that help evaluate and select the most appropriate contract based on the optimization of both total and individual costs. To the best of our knowledge, this is the first study to propose such a comprehensive contract evaluation framework.

We have introduced a set of enhanced VMI contracts based on shared responsibilities between the vendor and the retailer. The selection of the optimal contract is achieved through two Mixed Integer Linear Programming (MILP) models, each incorporating a distinct selection process. Additionally, a dedicated ranking tool has been developed to assist the vendor in contract selection based on predefined preferences and constraints.

The second project investigates the integrated production and distribution problem in a setting where a manufacturing plant directly delivers to a set of customers and warehouses. The model allows vehicles to perform multiple trips per period, which is particularly relevant for industries such as Personal Protective Equipment (PPE) manufacturers that operate in urban environments with small trucks and constrained delivery capacities.

This problem, called the Lot Sizing Problem with Direct Shipment and Multi-Trips, is formulated using MILP and solved through a Hybrid Simulated Annealing (HSA) approach. The goal is to determine the production and delivery quantities for each period, assign deliveries to vehicles, and optimize the trips requiredall while satisfying customer demand and respecting production, storage, and transportation capacity constraints. The objective is to minimize total costs, including production, inventory, and transportation.

The final project extends the previous work by considering a two-echelon supply chain, where products are first delivered to customers and warehouses, which in turn supply a set of retailers in the second echelon. In the literature, this problem is known as the Two-Echelon Production Routing Problem (2E-PRP). Despite its practical relevance, especially for PPE and soft drink industries in countries like Algeria, it remains underexplored due to its high complexity. Motivated by this gap, we propose a solution framework that includes a MILP formulation, a Simulated Annealing algorithm with Path Relinking (SA-PR), and a Three-Phase Iterative (3PI) Heuristic. The remainder of this thesis is organized as follows.

Chapter 1 is designed to familiarize the reader with the concept of the supply chain, supply chain management (SCM), and the main functions within a supply chain. Particular attention is given to the integration of these functions, highlighting the various coordination mechanisms and dimensions involved. The chapter then explores different optimization methods relevant to integrated planning problems.

Chapter 2 presents decision-making tools designed to assist in selecting between VMI contracts and the traditional RMI policy. The positioning and originality of this study are highlighted through a comparison with existing literature.

The chapter introduces the VMI contract and MILP formulations underlying the proposed tools and concludes with a discussion of the numerical experiments conducted, along with the managerial insights derived from the results

Chapter 3 builds upon the previous chapter by introducing a comprehensive ranking tool designed to assist vendors in systematically evaluating multiple contract options. This tool not only facilitates the selection of the most suitable contract but also highlights alternative choicessuch as second- or third-ranked optionsthat may better align with the vendor's specific objectives or constraints. The proposed framework accommodates both deterministic and stochastic demand scenarios and integrates varying levels of vendor risk aversion. The chapter concludes with a detailed discussion of the numerical examples and the experiments conducted under stochastic programming, with particular emphasis on the impact of risk-aversion parameters.

Chapter 4 presents the integration of production and distribution planning within a VMI framework, focusing on a multi-item, multi-trip direct shipment setting. The chapter begins with a literature review that highlights the originality and research gap addressed by this study. It then introduces a MILP formulation of the problem, followed by the development of an efficient HSA algorithm to solve it. The chapter concludes with a comprehensive discussion of the results obtained from the numerical experiments, along with managerial insights derived from the analysis.

Chapter 5 builds upon the previous chapter by extending the integrated production and distribution planning problem to a Two-Echelon Production Routing Problem with Mixed Delivery Modes (2E-PRP-MDM). The chapter begins with a literature review that situates our work within existing research, highlighting both relevant studies for Production Routing Problem (PRP) and the notable lack of work addressing the 2E-PRP. This demonstrates the originality and contribution of our study. A MILP formulation of the problem is then proposed, followed by the development of two solution approaches: a SA-PR, and a Three-Phase Iterative Heuristic (3PI). The chapter concludes with a detailed discussion of the numerical experiments and the managerial insights derived from the results.

Chapter 6 concludes this thesis. It begins by summarizing the main contributions and key findings. Next, it highlights notable aspects that were not fully explored within the scope of this work. Finally, it outlines potential future research directions related to integrated production and distribution planning in a VMI context.

Note that we chose not to include a chapter for the literature review because the thesis consists of three distinct contributions, all within the same context of integration. Each contribution addresses a different aspect of the problem, which allows us to present a dedicated literature review section within each chapter. This approach avoids repetition and helps the reader better understand the originality and motivation behind each contribution.

1 INTRODUCTION TO SUPPLY CHAIN MANAGEMENT AND OPTIMIZATION

This chapter covers the basics of supply chain management (SCM), defining key concepts, flows, and decision types. It explores supply chain functions, integration mechanisms (e.g., VMI), and the importance of joint optimization. Finally, it reviews integrated optimization models and solution methods, including exact, heuristic, and metaheuristic approaches.

1.1 introduction

The integration of the various functions of the supply chain represents a major current challenge. The research problem we have presented consists of integrating production, storage, and distribution decisions into a single model while minimizing the total cost associated with these functions. To address this problem, it is appropriate to position it within a well-established research area: supply chain management. We will therefore begin by examining in more detail what constitutes a supply chain and what supply chain management entails. We will then focus on the concept of integration, its close relationship with supply chain management, and its various mechanisms and dimensions.

1.2 introduction to supply chain and supply chain management

In this section, we will present a clear definition of the concepts of supply chain and supply chain management and highlighting their key components.

1.2.1 *Definition of Supply chain*

There is no universal definition of the Supply Chain (SC). The literature offers a wide range of definitions, with some adopting a product-oriented perspective, others focusing on the enterprise perspective, and others using the process perspective to identify the actors within the supply chain.

We present three of the existing definitions.

H. L. Lee and Billington (1995) define the supply chain as a system of facilities that acquire raw materials, convert them into components, and subsequently into completed goods, then transfer the completed goods to the customer.

According to Beamon (1998), the supply chain is an integrated process wherein several entitiessuppliers, manufacturers, distributors, and retailerscooperatively obtain raw materials, transform them into certain completed goods, and then provide these products to consumers.

M. Christopher (2022) define the supply chain as a network of companies involved, both upstream and downstream, in various processes and activities that create value in the form of products and services delivered to the end consumer. In other words, a supply chain consists of multiple companies, including upstream (providing raw materials and components), downstream (distribution), and the final customer.

1.2.1.1 *Classification of Supply Chain Entities*

The entities within the same supply chain can be classified according to physical, organizational, and functional criteria.

1. Physical Classification: Three types of physical entities are present in a supply chain:
   - Sites: These can be production or storage sites.
   - Goods: These can include raw materials, finished products, or semi-finished products that are exchanged between sites via means of transportation.
   - Means of Transportation: These include various types of carriers (e.g., truck fleets, vehicles, etc.) that ensure the movement of goods between the different sites of the supply chain.
2. Functional Classification: The entities of a supply chain can be identified based on the function they perform within the chain. The major activities within a supply chain are: transportation, storage, and production.
3. Organizational Classification: This classification is generally used when the supply chain is defined in relation to a specific company. It involves identifying each actor in the chain based on their relationship with that company. Three essential links are distinguished:
   - Procurement and Supply: This link involves supplying an operating system, such as a production line or a warehouse, with raw materials. It includes all entities that are upstream of the company.

- Production: This link consists of entities involved in the various stages of manufacturing a given product. These are typically the different departments of the main company.
- Distribution: This includes all entities located downstream of the company, responsible for transporting products that no longer require further transformation to the customers.

Companies belonging to the same supply chain are interconnected by various flows that pass through them.

1.2.1.2 *Flows in a Supply Chain*

Three types of flows are exchanged between members of the same supply chain: information flow, financial flow, and physical flow.

- Information Flow: This flow consists of a data flow and a decision flow, which are essential for the proper functioning of a supply chain. Indeed, by understanding how other links in the chain operate, a manager can make the best decisions for the functioning of their own company or department. Information systems such as ERP (Enterprise Resource Planning) or EDI (Electronic Data Interchange) have been developed to provide technical support that ensures the exchange of information between companies (Desgrippes 2005).
- Financial Flow: Financial flows represent the exchange of monetary values. These flows are generated by the various activities associated with physical flows, such as production, transportation, storage, recycling, etc. They are also used as a performance indicator for the functioning of these activities (Akbalik 2006).
- Physical Flow: Also called product flow, physical flows describe the materials that move between the different links in the chain. These materials can include components, semi-finished products, finished products, or spare parts. These flows are the heart of a supply chain, without which the other flows would not exist. They can be grouped into three stages: produce (or transform), store, and transport. These activities are typically carried out by different actors specialized in each of these areas (Akbalik 2006 and Desgrippes 2005).

The concept of a supply chain implies that companies take into account their environment through the three flows highlighted above. This environment can change depending on the objectives and alliances that the actors establish with each other. Thus, based on these alliances, several supply chain structures can be identified.

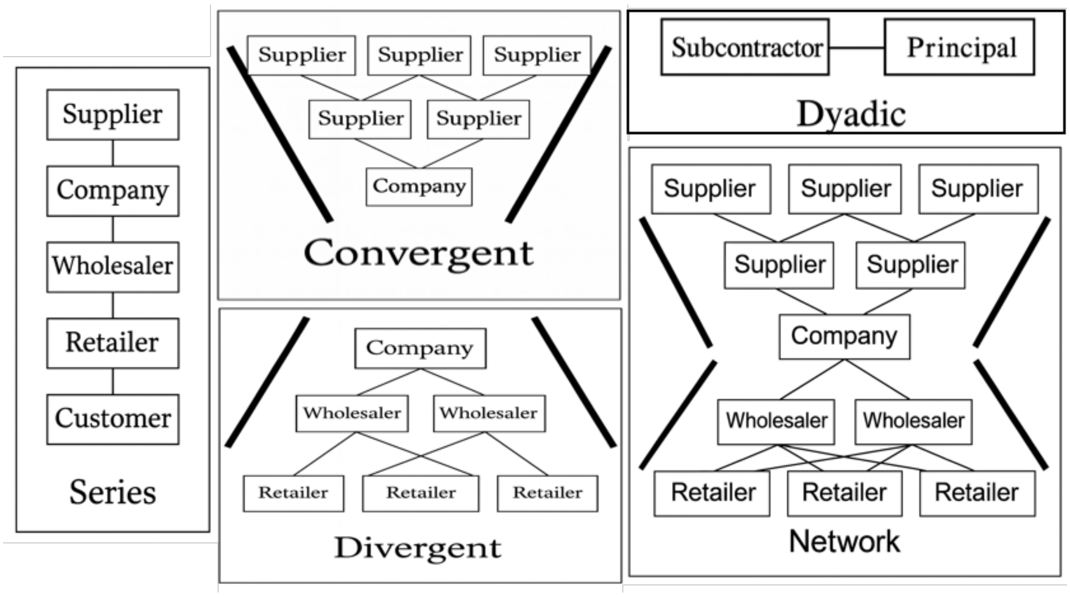


Figure 1.1: Supply chain structures (SENOUSSI 2016)

1.2.1.3 *Structures of the Supply Chain*

The structure of a supply chain depends on its nature and the objectives set during its design. The typical structures of supply chains commonly found in the literature are categorized by Huang et al. (2003) into: Serial, Divergent, Convergent, Network, and Dyadic. These structures are illustrated in Figure 1.1.

- Serial Structure: The serial structure corresponds to a linear manufacturing process where each entity in the chain supplies only one downstream entity.
- Divergent Structure: A structure is said to be divergent if one supplier supplies multiple customers. It is used to model a distribution network.
- Convergent Structure: A structure is said to be convergent if multiple suppliers supply one customer. It also represents an assembly process.
- Network Structure: The network structure is a combination of convergent and divergent structures, allowing for the modeling of more complex supply chains.
- Dyadic Structure: The dyadic structure can be seen as a special case of a serial chain limited to two stages. It can serve as a basis for studying client/supplier or contractor/subcontractor relationships.

To improve the overall performance of a supply chain, it is necessary to make a number of decisions. The goal is to achieve better fluidity in the circulation of the three flows (material, information, and financial) while reducing the costs of the entire system.

The literature is quite rich in terms of models and methods for flow management, which the scientific community groups under a common field called "supply chain management." The following section summarizes some generalities about supply chain management

1.2.2 *Supply Chain Management (SCM)*

The term Supply Chain Management (SCM) emerged during the 1980s. This period was characterized by a significant internationalization of businesses and a concentration of their activities. This led to the fragmentation of processes and the specialization of actors, resulting in the globalization of exchanges. These exchanges increased the complexity of flows within the supply chain, making it more difficult to coordinate the various actors involved. To address the coordination problem, new challenges arose to integrate independent companies and coordinate the flow of materials, information, and finances. Since then, the concept of SCM has found its place in both academic and professional spheres. Similar to the supply chain definition, SCM also has multiple definitions.

1.2.2.1 *Definition*

Simchi-Levi et al. (2003) define SCM as a comprehensive set of methodologies designed to integrate suppliers, manufacturers, warehouses, and distribution centers. The objective is to ensure that finished products are manufactured and distributed with the requisite quality, within specified timeframes, while minimizing total costs and meeting the desired service levels.

Thomas and Griffin (1996) describe SCM as the systematic management of material and information flows, both within and across the entities of the supply chain, including suppliers, manufacturing and assembly facilities, and distribution centers.

Mentzer et al. (2001) propose the following definition: “It is the systemic and strategic coordination of traditional operational functions and their respective tactics within a single enterprise, as well as among partners across the supply chain. The objective is to enhance the long-term performance of each member enterprise and the supply chain as a whole".

Stadtler (2005) characterize SCM as the process of integrating the various organizations that constitute the supply chain and coordinating the flows of materials, information, and financial resources. This coordination aims to fulfill end-customer demands and enhance the overall competitiveness of the supply chain.

The multifaceted aspects of SCM, as outlined in the definition by Stadtler (2005), are illustrated through their conceptual framework referred to as the House of SCM, as depicted in Figure 1.2. The roof of this framework represents the over-

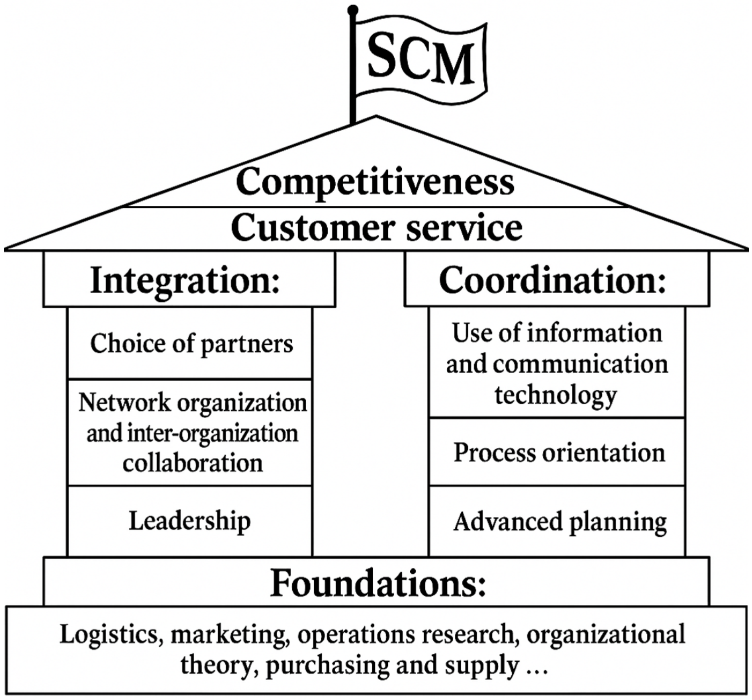


Figure 1.2: House of Supply Chain Management (Stadtler 2005)

arching objectives of SCM, which include addressing customer requirements and improving the competitive positioning of the supply chain. These objectives are supported by two foundational pillars:

1. Integration of the network comprising the various supply chain partners.
2. Coordination among the diverse actors within the network.

The integration pillar focuses on establishing the supply chain structure and fostering collaborative partnerships among its participants. The coordination pillar pertains to the effective management of the three critical flows: material, information, and financial. The base of the framework encompasses the essential elements of industrial management that facilitate the advancement and implementation of SCM (Galasso 2007).

1.2.3 *Decisions in Supply Chain Management*

Decisions in supply chain management are typically structured into three hierarchical levels: the strategic level, the tactical level, and the operational level. Each of these levels is associated with a different time horizon.

- **Strategic Level:** The impact of strategic decisions is long-term. These decisions concern the design of the supply chain structure, including the selection of suppliers, the choice of production or storage sites (capacity, dimensions), the selection of transportation modes, the design of new products, and expansion into new markets or business sectors. They also involve determining the terms of relationships between supply chain partners, such as contracts, profit-sharing, and information-sharing agreements.

- **Tactical Level:** This level generally involves medium-term decisions (ranging from a few weeks to a few months). These decisions focus on issues related to the planning of the company's resources. Examples include allocating supply sources to factories, determining production batch sizes, managing inventory levels, and selecting carriers.
- **Operational Level:** This level deals with short-term decisions that affect how products move through the supply chain and ensure its daily functioning. These decisions must take into account existing strategic and tactical decisions. Examples include optimizing flow, scheduling and controlling production systems, and planning vehicle routes.

1.3 SUPPLY CHAIN COORDINATION

The performance of a supply chain is intricately linked to the actions of all stages; a single weak link can disrupt the efficiency of the entire chain. While the basic aim of optimizing supply chain performance is widely supported in principle, individual firms often prioritize their objectives. This self-serving behavior can undermine the overall efficiency of the supply chain. The bullwhip effect is one of the consequences of such behaviors, resulting in larger costs and less efficiency (Krajewski and Malhotra 2022). It has been underlined that coordination among the different stages of the supply chain increases efficiency and reduces the costs occurring as a consequence of the bullwhip effect (Chopra and Meindl 2013).

Effective coordination relies extensively on information sharing and clear communication between supply chain parties enabling them to make informed choices. This includes the use of more accurate forecasts and the consideration of the wider implications for the supply network (Zheng and Zipkin 1990). Research has shown the benefits of information sharing, which can result in substantial reductions in inventory and cost savings for vendors while simultaneously decreasing overhead, processing costs, and the inventory costs of retailers (Bourland et al. 1996).

1.3.1 *Mechanisms for Supply Chain Coordination*

One effective approach to mitigating information distortion is to centralize replenishment responsibilities within the supply chain under a single entity. A single point of decision-making ensures visibility and a common forecast that drives orders across the supply chain. We present the most common industry practices that assign a single point of responsibility.

- **Continuous Replenishment Programs (CRP):**
  Continuous Replenishment Programs (CRP) involves regular replenishment of a retailer by the wholesaler or manufacturer, typically based on warehouse withdrawals rather than Point-of-sales (POS) data. CRP can be managed by the supplier, distributor, or a third party. Retailers prefer sharing warehouse withdrawal data as it is easier to implement. Effective CRP relies on Information Technologies (IT) systems linking the supply chain (Chopra and Meindl 2013).

- **Collaborative Planning, Forecasting, and Replenishment (CPFR)** :
  CPFR was established in the United States in 1996 through the Collaborative Planning Forecasting and Replenishment (CPFR) subcommittee of Voluntary Interindustry Commerce Standards Association (VICS). It is grounded in a collaborative approach between clients and suppliers that facilitates the development of sales forecasts, replenishment schedules, production plans, and distribution strategies to ensure optimal stock replenishment. In this framework, key functions such as procurement, marketing, sales, and customer service actively collaborate with partners. Successful implementation relies on effective resource organization and a foundation of mutual trust. Information sharing encompasses orders, inventory levels, sales data, forecasts, promotions, and commercial strategies.
  The objectives of CPFR include reducing inventory levels, enhancing customer service rates, increasing sales, shortening response times to demand, improving cycle times, minimizing production capacity requirements, boosting forecast accuracy, and lowering overall costs (SENOUSSI 2016).

- **Vendor Managed Inventory (VMI):**
  Before the implementation of advanced supply chain management practices, Retailer-Managed Inventory (RMI) was the basis of transactions between vendors and retailers. In this case, retailers managed their stock levels and placed orders when needed to minimize inventory holding costs. This method required suppliers to manage production according to schedules set by retailers.
  Among the different contracts and coordination strategies, VMI, has gained significant popularity in recent years. In a VMI system, the vendor holds ownership of the inventory located at the retailers site. The earliest implementations of VMI were in the grocery sector. Due to the success of major retailer Walmart (Andel 1996), it has been adopted by many companies such

as Kmart, Dell, Intel, and Fred Meyer (Chopra and Meindl 2013, Bookbinder et al. 2010). Magee (1985) seems to be the first to address the idea of VMI within a conceptual framework for the design of production control systems.

1.4 SUPPLY CHAIN PLANNING

Supply chain planning serves as a cornerstone of modern operational management, focusing on key principles that optimize the coordination of activities from production to distribution. These principles stem from the critical need to synchronize the flow of products, information, and finances effectively, ensuring adaptability to market dynamics. Logistics planning adopts a systemic approach, integrating concepts such as inventory management, production planning, and transportation optimization to enhance operational efficiency (Zeddam 2024).

The core principles of logistics planning encompass cost minimization, maximizing operational efficiency, and lead time reduction. Cost minimization entails strategic inventory management, route optimization, and production planning to eliminate inefficiencies. To maximize operational efficiency, optimal resource utilization, alignment of production capacities with demand, and the integration of advanced technologies like real-time tracking are essential. A key challenge is lead time reduction, requiring agile planning to adapt to demand fluctuations, supply chain disruptions, and changing operational conditions.

The strategic challenges of logistics planning involve balancing often conflicting objectives. These include managing costs while maintaining operational flexibility, optimizing routes while minimizing environmental impact, and ensuring effective inventory management while responding swiftly to demand shifts. Tackling these complexities necessitates a holistic approach that incorporates advanced mathematical models, sophisticated information systems, and close collaboration with supply chain partners.

Whether sequential or integrated, supply chain planning is guided by principles of efficiency and flexibility while navigating intricate strategic challenges. Technological advancements, sophisticated mathematical modeling, and agile decision-making act as critical levers for overcoming these challenges and achieving optimal supply chain management in an operational environment.

1.4.1 *Sequential Planning*

For many years, supply chain functions have been managed independently, with each function operating as a separate entity responsible for optimizing its activities. In this traditional approach, each function develops its planning autonomously. For example, once production is scheduled, the corresponding plan is used to determine raw material and finished product inventory levels. Subsequently, a transportation schedule is created based on these produced and stored quantities, without the ability to modify the production schedule. This approach is known as sequential planning (Akbalik 2006). However, it presents several disadvantages, which are outlined below:

- **High Costs**
  The lack of coordination between functions leads to inefficiencies such as lost sales due to stockouts or excessive holding costs from surplus inventory. These issues arise when demand and production are not synchronized, and manufacturing relies solely on inaccurate forecasts.

- **Lack of Data Sharing**
  Although necessary data exists across various points in the supply chain, it is often not utilized effectively at the right time and place, creating an illusion of data scarcity. This results in flawed assumptions and suboptimal decision-making.

- **Internal Conflicts**
  Different functions within an organization may compete for the same resources simultaneously, leading to unnecessary internal conflict and operational inefficiencies.

- **Duplication of Efforts**
  A lack of coordination can result in redundant tasks being performed at multiple points along the supply chain, increasing costs. For example, independently maintaining and updating finished product inventory databases at both production sites and distribution centers results in duplication. This inefficiency can be avoided by implementing a real-time, centralized database.

- **Lack of Strategic Vision**
  Addressing individual functional issues in isolation may provide short-term solutions, but it does not contribute to a cohesive, long-term strategy. Over time, the negative consequences of this fragmented approach become apparent.

1.4.2 *Integrated Planning*

Integrated planning is a vital strategic method aimed at enhancing coordination and synchronization throughout different stages of the supply chain. It aims to maximize operational efficiency while minimizing costs and delays. D. Christopher (2000) emphasize that "the integration of logistics planning across the entire chain provides a means to achieve greater efficiency."

One of the core aspects of integrated planning is synchronizing supply and demand. This involves accurately anticipating demand fluctuations and adjusting production accordingly. As Simchi-Levi et al. (2009) highlighted in their study on supply chain management, effective synchronization helps reduce storage costs while preventing stockouts.

Close collaboration with supply chain partners is another key element of integrated planning. Sharing information and data among stakeholders optimizes the flow of goods and improves production capacity planning. As Chopra and Meindl (2013) note, "integrated planning requires effective communication and collaboration between suppliers, manufacturers, distributors, and retailers."

A critical enabler of integration is the advancement of information and communication technologies, including Electronic Data Interchange (EDI), the Internet, Radio Frequency Identification (RFID), Enterprise Resource Planning (ERP) systems, and Advanced Planning Systems (APS). These tools facilitate real-time information exchange and enhance visibility across the supply chain, enabling more comprehensive and data-driven decision-making. Before diving into integrated problems in the supply chain, it is important to first introduce delivery planning, specifically the Vehicle Routing Problem (VRP)

1.4.3 *Vehicle Routing Problem*

The Vehicle Routing Problem (VRP) is a combinatorial optimization challenge frequently encountered in logistics and distribution. It involves a set of customers, each with specific demands, and one or more vehicles with limited capacity. The goal is to determine the optimal routes for the vehicles to meet all customer demands while minimizing operational costs, such as total distance traveled and travel time (Toth and Vigo 2014).

CLASSIFICATION OF VEHICLE ROUTING PROBLEM The Vehicle Routing Problem (VRP) has numerous variants; we present the following:

- **Capacitated Vehicle Routing Problem (CVRP)**: Each vehicle has a maximum load capacity, and the goal is to deliver goods to all customers while ensuring that the vehicle's capacity is not exceeded.
- **Vehicle Routing Problem with Time Windows (VRPTW):** Customers have predefined time windows during which they must be served. The objective is to plan vehicle routes that adhere to these time constraints.

- **Simultaneous Pickup and Delivery VRP (SPDVRP):** Vehicles must deliver goods while simultaneously picking up other goods during the same trip, all while respecting capacity constraints.
- **Heterogeneous Fleet VRP (HFVRP):** A fleet consists of vehicles with varying capacities and operating costs. The objective is to minimize total costs by efficiently utilizing this diverse fleet.
- **Multiple Depot VRP (MDVRP):** Vehicles are dispatched from multiple depots, with each depot having its own fleet. Vehicles can only serve customers assigned to their respective depots.
- **VRP with Pickup and Delivery (VRPPD):** Vehicles transport goods from specific pickup locations to designated delivery points while adhering to capacity constraints.
- **Time-Constrained VRP (TCVRP):** Customers must be served within strict time constraints, requiring optimized scheduling.
- **Stochastic VRP (SVRP):** Customer demands or travel times are uncertain and follow probability distributions, necessitating robust planning to account for variability.
- **VRP with Variable Service Durations (VRPVSD):** Service times at customer locations vary based on the number of goods being delivered, influencing route scheduling.
- **Online VRP (OVRP):** Customer orders arrive dynamically, and routing decisions must be made in real-time without prior knowledge of future requests.

1.4.4 *Lot Sizing Problem*

The Lot Sizing Problem (LSP) in the literature involves determining the optimal production lot size for a product, aiming to balance production costs and storage costs. Frequent production runs can be costly due to setup costs while producing in large quantities at less frequent intervals leads to high inventory holding costs.

1.4.4.1 *Classification of Lot Sizing Problem*

Several classifications of lot-sizing problems have been proposed in the literature, including the works of Drexl and Kimms (1997), Staggemeier and Clark (2001), B. Karimi et al. (2003), Wolsey (2002), and Brahimi, Dauzere-Peres et al. (2006). For instance, whereas Brahimi, Dauzere-Peres et al. (2006) evaluate the research on the single-product capacitated lot sizing problem (LSP), B. Karimi et al. (2003) offer a thorough synthesis of the literature on single-level lot size problems and their several extensions. Additionally, a classification scheme for lot sizing models has been proposed by C. (2008).

Lot sizing problems can be categorized according to many factors, including:

- **Constant and Variable Demand:** Demand is classified as constant if it remains uniform across the planning horizon; otherwise, it is variable. This distinction leads to different solution approaches:
  - Constant demand models typically generate a repeatable production cycle over the horizon.
  - Variable demand models determine production quantities for predefined periods within the planning horizon.
- **Product Structure:** The product structure defines the relationship between parent and component products. If no such relationship exists, the problem is single-level; otherwise, it is multi-level.
- **Costs:** Costs in lot sizing models typically include:
  - Unit production costs
  - Setup costs, associated with preparing production resources
  - Holding costs, incurred for storing inventory
- **Setup Times:** This refers to the time required to reconfigure a resource when switching between different products. Some models explicitly account for these times, particularly when frequent changeovers are required.
- **Resource Capacities:** If production capacity is unlimited or not considered, the problem is uncapacitated; otherwise, it is incapacitated.
- **Objectives:** The most common objective is minimizing total costs. However, other objectives may include maximizing service levels or optimizing other performance measures.
- **Modeling of the Planning Horizon:** The planning horizon can be:
  - Finite: Most discrete models use a finite horizon, which is further categorized into:
    1. Big bucket models (e.g., Dynamic Lot Sizing Problems), where production can occur for multiple products in the same period. The Wagner and Whitin (1958) is a foundational example.
    2. Small bucket models (e.g., Discrete Discrete Lot Sizing and Scheduling Problems (DLSP)), where each period is short and typically allows for only one production setup.

  - Infinite: Continuous models often assume an infinite horizon. The Economic Order Quantity (EOQ) model is one of the first such models; further developments included the (Economic Lot Scheduling Problem (ELSP).

- **Type of information:**
  - Deterministic models assume that all parameters have known, fixed values.
  - Stochastic models incorporate uncertainty; some parameters are expressed as random variables.

1.4.4.2 *Single-Product Lot Sizing Problem*

The single-product lot sizing problem has received significant attention due to its simplicity and importance as a subproblem of more complex lot sizing problems.

Uncapacitated Lot Sizing Problem (ULSP)

This problem was first formulated by Wagner and Whitin (1958), as a polynomial problem solvable in $O(T^2)$ (T represents time periods) time using dynamic programming, with subsequent improvements reducing the complexity to $O(TlogT)$ (Aggarwal and Park 1993; Wagelmans et al. 1992). Some extensions to the Uncapacitated Lot Sizing Problem (ULSP) are models that include backlogging (Zangwill 1969), lost sales (Aksen et al. 2003), and time windows (Brahimi, Dauzère-Pérès and Najid 2006), and the algorithms can reach polynomial complexities like $O(T^4)$.

Capacitated Lot Sizing Problem (CLSP)

Bitran and Yanasse (1982) proved that this problem, widely encountered in practice, is NP-hard. However, specific cases allow for polynomial-time solutions. For instance, when capacity remains constant, Florian and Klein (1971) proposed an $O(T^4)$ algorithm, later improved to $O(T^3)$ by Van Hoesel and Wagelmans (1996). Pochet (1988) introduced a compact extended formulation with $O(T^3)$ variables and constraints. Additionally, when capacity is non-decreasing over time, with a non-speculative cost function and non-decreasing setup costs, the problem remains polynomially solvable, as demonstrated by Pochet and Wolsey (2010). In more general cases, Capacitated Lot Sizing Problem (CLSP) becomes significantly more complex, requiring exact solution methods such as dynamic programming (Shaw and Wagelmans 1998), Branch & Bound (B&B) (Lotfi and Yoon 1994), and cutting-plane methods (Loparic et al. 2003). These techniques are often used for small problem instances to generate benchmarks for heuristic approaches. Furthermore, advanced formulations and valid inequalities enhance the performance of commercial solvers like CPLEX and XPRESS-MP, enabling them to tackle real-world problem instances more effectively (Brahimi 2004).

1.4.4.3 *Multi-Product Lot Sizing Problem*

The multi-product lot-sizing problem can be divided into N single-product for uncapacitated lot-sizing problems. Here, N is the number of products. However, the problem has been extensively studied in the literature (Lemoine 2008), demonstrating that capacity constraints increase complexity. Researchers have developed exact and heuristic methods to tackle this problem.

1.4.5 *Integrated Lot Sizing with Direct Shipment Problem*

Integrated Lot-Sizing with Direct Shipment refers to a problem that combines both production and distribution decisions. It includes production setup costs, setup time, and customer-specific delivery charges, which consist of fixed costs and per-unit delivery prices. The goal is to minimize production, setup, inventory, and shipping costs throughout a planning period. Products can be produced and directly transported from the manufacturing plant to customers, with the option to store them either at the plant or at customers locations, incurring inventory-holding costs (Adulyasak et al. 2015b), as depicted in Figure 1.3.

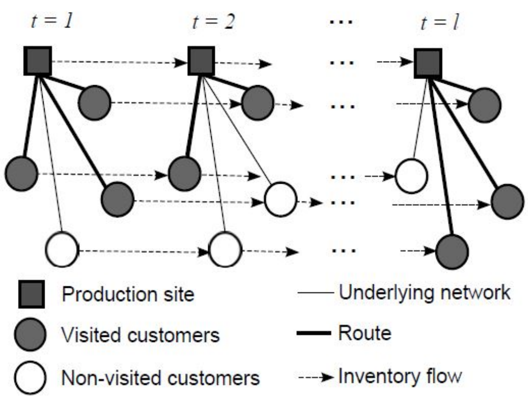


Figure 1.3: Lot Sizing with Direct Shipment Problem (Adulyasak et al. 2015b)

Classification of the Lot-Sizing with Direct Shipment Problem

The Integrated Lot-Sizing with Direct Shipment problem can be classified based on its specific situations, cost structures, and problem variations. A notable scenario is the One-Warehouse Multi-Retailer Problem (OWMR), which involves uncapacitated production and uncapacitated vehicles (Federgruen and Tzur 1999). The Truckload Distribution Problem involves a single product and a single retailer

(Alp et al. 2003). Additionally, Lot-Sizing with Production Substitution models scenarios where one product can satisfy the demand for another product (Hsu et al. 2005). The issue also showcases various cost structures, including fixed transportation expenses per container or shipment (Alp et al. 2003), piecewise linear transportation costs for Truckload (TL) and Less-than-truckload (LTL) shipments (C.-L. Li et al. 2004; Jaruphongsa et al. 2007), and cost discounts from transportation capacity reservations (van Norden and van de Velde 2005). Furthermore, variations in problem characteristics include multi-item versus single-item settings (Federgruen and Tzur 1999; Rizk et al. 2006), single-customer versus multiple-customer scenarios with allowing of backlogging (Chand et al. 2007), and split delivery under time-window constraints (Jaruphongsa and Lee 2008).

1.4.6 *Inventory Routing Problem*

When considering routing while disregarding production, the problem becomes an integrated inventory distribution and routing challenge, commonly referred to as the Inventory Routing Problem (IRP) (Andersson et al. 2010). Over the past decade, the IRP has been extensively studied, particularly in land transportation and maritime logistics, where inventory management plays a critical role. This problem is especially relevant in VMI systems, where suppliers monitor their customers' inventory levels and make replenishment decisions. By shifting inventory and ordering responsibilities from customers to vendors, VMI systems reduce workload, lower costs, and enhance supply chain efficiency through collaborative planning.

The IRP network structure is illustrated in Figure 1.4, beginning at a central warehouse without production decisions. Unlike traditional routing problems, a single vehicle can serve multiple customers along its route. As a generalized extension of the VRPwhich focuses on delivery quantities and routingthe IRP also incorporates timing decisions for fulfilling customer demand. This added complexity, involving periodic routing and inventory coordination, makes the IRP significantly more challenging than the classical VRP. Given that the VRP is a special case of the IRP, the latter is inherently NP-hard (Coelho et al. 2012).

1.4.7 *Production Routing Problem*

The two integrated problems discussed in the previous sections each overlook a critical dimension of operational planning in the supply chain. Specifically, the integrated lot-sizing problem with direct shipment does not account for routing decisions, while the IRP omits production planning. To address this gap, recent research has focused on a more comprehensive model known as the Production Rout-

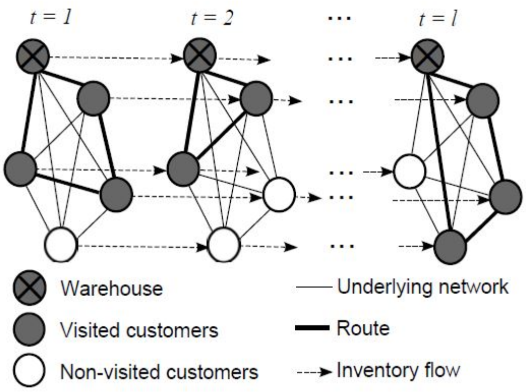


Figure 1.4: Inventory Routing Problem (Adulyasak et al. 2015b)

ing Problem (PRP) (Ruokokoski et al. 2010), which integrates lot-sizing, inventory management, distribution, and routing decisions into a unified framework.

The structure of the PRP is illustrated in Figure 1.5. The supply chain configuration includes a central production facility and several retailers, who act as customers of the production plant. Both the plant and the retailers are equipped with storage facilities (e.g., warehouses) to hold finished goods. Retailers face time-dependent demand that must be satisfied in each period over the planning horizon. In each period, the plant must decide whether or not to produce, determine the

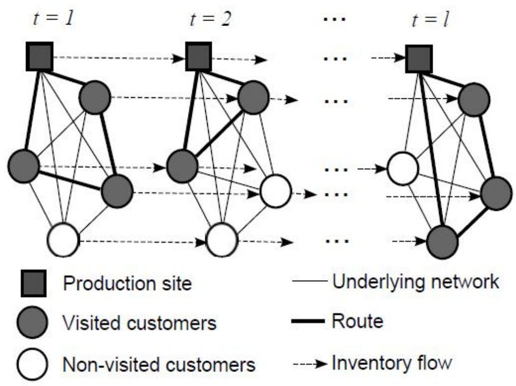


Figure 1.5: Production Routing Problem (Adulyasak et al. 2015b)

production quantity (lot size), and ensure that it remains within the available production capacity. Production activities incur a fixed setup cost as well as variable unit production costs. The finished goods are then delivered to the retailers using a limited fleet of capacitated vehicles, generating routing costs. As in typical supply chain settings, inventory holding costs are incurred for products stored either at the plant or at the retailers' warehouses.

Due to the combined complexity of production scheduling, inventory management, and vehicle routing, the PRP is inherently more challenging than its individual components. It is classified as an NP-hard problem, as it generalizes the classical VRP (Boudia et al. 2007; Archetti et al. 2011).

Basic Formulation for PRP

The PRP, as an integrated version of the LSP and VRP, supports various formulation schemes for effectively modeling the problem. We present the formulation according to Bard and Nananukul (2010).

We analyze a single-product PRP network that includes a production facility and several customers. The distribution network is represented by an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where N is the set of the manufacturing plant and retailers, while the edges in the graph are denoted as $\mathcal{A}$. The plant is denoted as node 1. Additionally, we define the set of periods $\mathcal{T}$ as the range $1, ..., T$. Let p represent the unit production cost, f indicate the setup cost, h_i represent the unit inventory holding cost at node i, and c_{ij} denote the transportation cost from node i to node j. We define C as the production capacity and Q as the vehicle capacity. Let K represent the maximum number of vehicles that can be dispatched during each period. The demand of customer i in period t is denoted as d_{it}, while the maximum inventory level at node i is indicated by U_i.
We introduce the following decision variables:.

- Production:
  x_t: Production level at period t.
  y_t: Binary setup variable equal to 1 if there is a production in period t and 0 otherwise.

- Inventory:
  I_{it}: Inventory level of the product at location i in period t while I_{i0} represent the initial inventory.

- Transport:
  q_{it}: Amount of product shipped to customer i in period t
  H_{it}: binary variable, equal to 1 if customer i is served in period t, 0 otherwise.
  Y_{ijt}: binary variable, equal to 1 if arc (i, j) is traversed in period t, 0 otherwise.

$$[\mathbf{PRP}] \quad \min \sum_{t \in \mathcal{T}} p x_t + \sum_{t \in \mathcal{T}} f y_t + \sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} h_{it} I_{it} + \sum_{i \in \mathcal{N} - \{1\}} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{N}} c_{ij} Y_{ijt} \tag{1.1}$$

$$s.t.$$

$$I_{1t} = I_{1,t-1} + x_t - \sum_{i \in \mathcal{N} - \{1\}} q_{it} \forall t \in \mathcal{T} \tag{1.2}$$

$$I_{it} = I_{i,t-1} + q_{it} - d_{it} \quad \forall i \in \mathcal{N} - \{1\}, \forall t \in \mathcal{T} \tag{1.3}$$

$$I_{it} \leq U_i \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \tag{1.4}$$

$$x_t \leq M_t y_t \quad \forall t \in \mathcal{T} \tag{1.5}$$

$$q_{it} \leq B \times H_{it} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N} - \{1\} \tag{1.6}$$

$$\sum_{j \in N} Y_{ijt} = H_{it} \quad \forall i \in \mathcal{N} - \{1\}, \forall t \in \mathcal{T} \tag{1.7}$$

$$\sum_{j \in \mathcal{N}} Y_{ijt} = \sum_{j \in \mathcal{N}} Y_{jit} \quad \forall i \in \mathcal{N} - \{1\}, \forall t \in \mathcal{T} \tag{1.8}$$

$$\sum_{j \in \mathcal{N}} Y_{1jt} \leq K \quad \forall t \in \mathcal{T} \tag{1.9}$$

$$\sum_{i \in N} \sum_{j \in \mathcal{N} - \{1\}} Y_{ijt} \leq \sum_{j \in \mathcal{N} - \{1\}} q_{it}/Q \quad \forall t \in T \tag{1.10}$$

$$y_t \in \{0, 1\} \quad \forall t \in \mathcal{T} \tag{1.11}$$

$$x_t, I_{it}, q_{it}, H_{it} \geq 0 \quad \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{1.12}$$

$$Y_{ijt} \in \{0, 1\} \quad \forall i, j \in \mathcal{N}, t \in \mathcal{T} \tag{1.13}$$

The objective function (1.1) aims to minimize total costs, which include production and setup costs, inventory costs at the manufacturing facility, customer inventory costs, and delivery transportation costs. Constraints (1.2), and (1.3) represent the inventory balance equations at the manufacturing plant and for customers, respectively. The constraints (1.4) respect the limits of the inventory capacity. Constraint (1.5) has a dual role: it ensures that production levels do not exceed capacity and establishes a link between the continuous production variables x_t and the binary setup variables y_t, indicating that a setup cost is incurred only when production occurs in period t. Constraints (1.6) ensure that delivery can only occur if customer i is visited during period t. Furthermore, constraints (1.7) ensure that each customer is served by no more than one vehicle per period. Constraints (1.8) ensure that a vehicle visits a customer and then leaves after serving them. Constraints (1.9) ensure that the number of trucks utilized does not exceed the size of the available fleet. Constraints (1.10) is the elimination of sub-tours proposed by Chandra and Fisher (1994). Finally, constraints (1.11) (1.13) are integrality and non-negativity constraints.

1.5 SOLUTION APPROACHES

The complexity of integrated supply chain planning problemswhich often involve simultaneously coordinating production, inventory, and distribution decisionsmakes exact solutions computationally challenging, particularly for large-scale instances. Consequently, a wide variety of solution approaches have been proposed in the literature, ranging from exact optimization methods to heuristic and metaheuristic techniques.

1.5.1 *Exact Methods*

Exact methods are a widely used approach for solving polynomial problems. These methods guarantee solution optimality, provided sufficient computational time is available for the optimization process to complete.

1.5.1.1 *Integer Linear Programming*

Integer Linear Programming (ILP) is a specific class of mathematical programming in which all decision variables must take integer values, and both the objective function and the constraints are linear. ILP is particularly well-suited for problems requiring discrete decisions, such as determining production quantities or scheduling tasks. These models are typically implemented using optimization solvers such as CPLEX or Gurobi, which search for feasible solutions that satisfy all constraints while optimizing the objective function. In addition to pure ILP models, related formulations include Mixed-Integer Linear Programming (MILP), where some decision variables are continuous while others are integers, and 0-1 Linear Programming (0-1 LP), where variables are restricted to binary values (0 or 1). These methods are widely applied in production planning, scheduling, and process optimization.

1.5.1.2 *Dynamic Programming*

Dynamic programming (DP) is generally utilized for problems involving a single item, with or without capacity constraints, and is categorized as a partial enumeration technique. The method, initially formalized by Bellman (1957), operates by dividing complex problems into smaller, manageable subproblems, thus simplifying the computational process. DP solves problems sequentially, with each step addressing a specific decision variable. Solutions derived from these smaller subproblems are systematically combined through recursive calculations, ultimately yielding an optimal solution for the original problem.

The term "dynamic programming" originates from its early applications in optimizing dynamic systems, where decisions occur sequentially over timea characteristic notably present in production planning scenarios. However, the approach is not restricted to time-dependent contexts, prompting the alternative, albeit less frequently used, term "multistage programming."

Dynamic programming is founded upon three core principles:

Principle of Optimality : This principle states that an optimal solution to the complete problem can be constructed efficiently from optimal solutions of its decomposed subproblems. It leverages recursive relationships and decomposable functions.

Stages : A stage represents a segment of the overall problem where multiple exclusive alternatives exist, among which an optimal choice must be identified.

States : A state typically describes the current condition or status of constraints linking the various stages within the optimization process.

To enhance computational efficiency, DP often employs strategies aimed at reducing the number of states.

1.5.1.3 *Branch and Bound*

Branch and Bound (B&B)is a systematic search method used to explore the entire solution space of combinatorial optimization problems. This technique operates by partitioning the overall search space into smaller subsets (branches) and systematically eliminating certain branches using calculated lower and upper bounds for each subset. This procedure continues iteratively until the optimal solution is identified or all feasible branches have been exhaustively examined. The branch-and-bound approach has been effectively applied across various optimization problems, including supply chain scheduling problem (N. Karimi and Davoudpour 2015), flowshop scheduling problems (Fattahi et al. 2014), and the Vehicle Routing Problem (Theurich et al. 2021).

1.5.1.4 *Branch and Cut*

Branch and Cut (B&C) is an exact optimization technique based on the branch-and-bound approach, integrating linear programming (LP) relaxations with the addition of valid inequalities, known as cuts, to tighten these relaxations. Ideally, generating all possible cuts would precisely characterize the convex hull of feasible solutions; however, in practice, identifying the complete set of such cuts is computationally intractable.

Typically, valid inequalities are dynamically generated at each node within the branch-and-bound tree to strengthen the current bounds and accelerate convergence toward optimality. Common examples of general-purpose cuts include Gomory cuts (Gomory 1958) and knapsack cuts (Crowder et al. 1983). Despite its theoretical rigor, the branch-and-cut method has not been extensively explored due to challenges associated with efficiently identifying effective valid inequalities, alongside significant demands on computational resources and memory.

1.5.2 *Heuristics*

Heuristic methods are problem-specific algorithms tailored to effectively address particular optimization problems or problem instances, which often limits their applicability to other contexts. Generally, heuristics can be classified into two main categories: construction heuristics and improvement heuristics. Construction heuristics build feasible solutions step-by-step from scratch, without relying on an initial solution. In contrast, improvement heuristics start from an existing solution and iteratively attempt to enhance its quality, progressively refining it over successive iterations (Brahimi 2004).

1.5.3 *Metaheuristic*

While heuristics are tailored to specific problems, metaheuristics are general-purpose algorithms designed to address a wide range of optimization problems. They serve as high-level frameworks that guide the construction and adaptation of heuristic methods for finding solutions. In this section, we present only the most used metaheuristics in the integrated planning.

1.5.3.1 *Simulated Annealing*

Simulated Annealing (SA) is a metaheuristic inspired by the annealing process in metallurgy, where controlled cooling of a material leads to a stable, low-energy configuration. Initially proposed by Kirkpatrick et al. (1983), SA was developed to efficiently escape local optima by probabilistically accepting non-improving solutions during the search process.

At each iteration, the algorithm explores the neighborhood of the current solution by generating a random neighbor. If the new solution improves the objective function, it is immediately accepted. Otherwise, the solution is accepted with a probability that depends on two factors: the degradation of the objective function value (ΔE) and a control parameter called the temperature (T). The acceptance probability follows a Boltzmann distribution:

$$\Gamma(\Delta E, T) = e^{-\Delta E/T}$$

The temperature progressively decreases according to a predefined cooling schedule, gradually reducing the likelihood of accepting worse solutions as the search advances. This mechanism enables SA to perform an extensive exploration of the solution space at the beginning of the search and to focus on exploitation near the end, increasing the chances of finding a high-quality solution. We present the pseudo code of the SA in algorithm 1.1.

Algorithm 1.1 Simulated Annealing Algorithm

```
Input: Initial solution s_0, initial temperature T_max
Output: Final solution s
s ← s_0
T ← T_max
repeat
    repeat
        Generate a random neighbor s' of s
        Compute ΔE = f(s') − f(s)
        if ΔE ≤ 0 then
            s ← s'                                  ▷ Accept better solution
        else
            Accept s' with probability e^{−ΔE/T}
        end if
    until equilibrium condition is met
    Update T according to cooling schedule
until stopping criterion is met
```

1.5.3.2 *Tabu Search*

Tabu Search (TS), introduced by Glover (1989), is a metaheuristic optimization method designed to overcome the limitations of local search (LS) techniques, specifically their tendency to get trapped in local optima. Unlike simulated annealing (SA), which uses probabilistic acceptance criteria to escape local optima, TS relies on deterministic rules combined with memory-based strategies to guide its search process.

At its core, Tabu Search systematically explores the neighborhood of the current solution and typically selects the best neighbor, even if this neighbor does not improve upon the current solution. This strategy allows TS to move beyond local optima by temporarily accepting worse solutions. To avoid cycling back to recently explored solutions, TS employs a distinctive memory structure known as a tabu list, which maintains records (attributes or moves) that have been previously visited or applied. By marking these moves or solutions as "tabu," the algorithm restricts immediate revisits.

However, the tabu restriction can sometimes be overly strict, potentially forbidding beneficial moves. To address this, TS introduces aspiration criteria, conditions under which a move listed as tabu may still be allowedtypically if the move results in a solution superior to the best-known solution or if it meets certain predefined favorable conditions. The algorithm 1.2 present the pseudo code of TS.

Algorithm 1.2 Tabu Search Algorithm

Input: Initial solution s_0
Output: Best solution found
$s \leftarrow s_0$ ▷ Initial solution
Initialize the tabu list, medium-term and long-term memories
repeat
    Find the best admissible neighbor s' ▷ non-tabu or satisfying aspiration criteria
    $s \leftarrow s'$
    Update tabu list, aspiration conditions, medium-term and long-term memories
    if intensification criterion holds **then**
        Perform intensification
    end if
    if diversification criterion holds **then**
        Perform diversification
    end if
until stopping criterion is satisfied

1.5.3.3 *Genetic Algorithms*

Genetic Algorithms (GAs) are evolutionary optimization techniques inspired by the principles of natural selection and genetics. Originally proposed by Holland (1992) and later popularized by Goldberg (1989), GAs simulate the evolution of a population of candidate solutions to progressively discover high-quality solutions.

Starting from an initial population generated randomly, the algorithm evaluates each individual according to a fitness function. The best-performing individuals are selected to undergo genetic operations: **crossover**, which combines parts of two parents to form offspring, and **mutation**, which introduces random changes to preserve diversity. **Selection** favors individuals with higher fitness scores, guiding the search towards more promising regions of the solution space.

The process of selection, crossover, and mutation is iteratively applied across multiple generations. By mimicking natural evolution, GAs balance exploration and exploitation, enabling the discovery of optimal or near-optimal solutions in complex, multi-dimensional spaces. Termination criteria typically involve a maximum number of generations or convergence to a sufficiently good solution. The following algorithm 1.3 presents the pseudo code of the GAs.

Algorithm 1.3 Genetic Algorithm

```
Input: Problem parameters
Output: Best solution found
Generate the initial population randomly
repeat
    for each individual in the population do
        Evaluate fitness
    end for
    Select individuals based on fitness
    Apply crossover to generate offspring
    Apply mutation to offspring
    Form the new population
until stopping criterion is satisfied
return best individual from the population
```

1.5.3.4 *Variable Neighborhood Search*

Variable Neighborhood Search (VNS) is a metaheuristic optimization technique introduced by Mladenovi and Hansen (1997). The fundamental idea of VNS is to systematically or randomly explore a set of predefined neighborhood structures to identify better solutions and escape local optima.

VNS capitalizes on the observation that different neighborhoods may lead to different local optima, and that a global optimum can be seen as a local optimum within at least one appropriately chosen neighborhood. By dynamically changing the neighborhood during the search process, VNS enhances both diversification and intensification, allowing the algorithm to explore the solution space more thoroughly and avoid premature convergence to suboptimal solutions.

Each iteration of VNS involves three main phases: shaking (randomly perturbing the current solution), local search (finding a local optimum starting from the perturbed solution), and neighborhood change (deciding whether to continue with the current neighborhood or move to the next one based on improvement). The pseudo code of the VNS in presented in Algorithm 1.4

1.6 CONCLUSION

In this chapter, we introduced key concepts and notions related to the supply chain and supply chain management (SCM). One of the fundamental pillars of SCM is integration, and among the mechanisms supporting integration is VMI. VMI is widely adopted in various vendorretailer relationships. Its advantages motivated

Algorithm 1.4 Variable Neighborhood Search

```
Input: A set of neighborhood structures {N_k} for k = 1, ..., k_max
Output: Best solution found
Generate initial solution x ← x_0
repeat
    k ← 1
    repeat
        Shaking: Generate a random solution x' in neighborhood N_k(x)
        Local Search: Apply local search starting from x' to obtain x''
        if f(x'') < f(x) then
            x ← x''
            k ← 1                    ▷ Return to the first neighborhood
        else
            k ← k + 1
        end if
    until k > k_max
until stopping criterion is satisfied
return best solution x
```

us, in this work, to study a supply chain across different structures: a single vendor and a single retailer, a single vendor and multiple retailers, and a two-echelon system comprising a single vendor, multiple retailers, and warehouses, all coordinated through the VMI mechanism.

We also presented the most commonly used solution approaches for addressing integrated planning problems. The objective of this thesis is to apply some of the solution approaches presented to solve various integrated production and distribution planning problems. In the following chapters, we will present these problems in detail along with the proposed solution methodologies.

2

A DECISION-MAKING FRAMEWORK FOR VMI AND RMI CONTRACTS

This chapter introduces a decision-making framework designed to assist partners in selecting between RMI and four VMI contracts, which are inspired by inventory ownership and transportation cost-sharing principles derived from INCOTERMS. We developed two MILP formulations for this purpose. In the VMI scenario, the formulations aim to minimize the overall supply chain cost while also determining the individual costs for the vendor and retailer. Conversely, in the RMI scenario, the models directly minimize the individual costs for each partner, with the total SC cost subsequently calculated as the sum of these individual costs. The effectiveness of these models has been evaluated through various scenarios, encompassing different demand patterns, demand variability, demand volumes, and intermittent demand.

The remainder of this chapter is structured as follows: Section 2.1 provides an introduction to the chapter. Section 2.2 offers a detailed review of relevant literature. Section 2.3 defines the problem context and presents the mathematical models employed. Section 2.4 discusses the outcomes of numerical experiments and computational analyses. Lastly, Section 2.5 summarizes the chapter's conclusions and outlines potential future research directions.

2.1 INTRODUCTION

In the modern competitive market, companies consistently face the dual challenge of controlling operational costs while responding to rapidly changing customer expectations (Ru and Wang 2010). Meeting these demands necessitates innovative approaches and advanced planning strategies to enhance operational efficiency without sacrificing service quality. Integrated planning of production and distribution, particularly when combined with inventory management approaches like VMI, has become an increasingly attractive alternative to traditional RMI policies (G. Cachon and Terwiesch 2008). Nevertheless, the adoption of VMI does not always guarantee improved performance, as illustrated in the findings by Dong and Xu (2002). Consequently, organizations frequently encounter difficulties when determining which inventory management strategy, whether VMI or RMI, is more cost-effective for their unique operational contexts. This uncertainty highlights the importance of developing robust decision-support tools to assist vendors and retailers in accurately assessing the appropriateness of these inventory management policies.

This chapter addresses this critical gap by proposing an integrated decision-making framework designed to assist both suppliers (vendors or manufacturers) and customers (retailers) in selecting between RMI and various VMI contract options under deterministic scenarios. The primary goal of the framework is to identify the optimal inventory management contract that minimizes aggregate supply chain costs while also considering the individual financial impacts on each participating organization. Specifically, this chapter aims to: (1) develop two distinct MILP models to evaluate VMI adoption scenarios comprehensively; (2) introduce new VMI contracts, detailed further in the subsequent Section 2.2; and (3) incorporate considerations for deterministic demand variability.

2.2 LITERATURE REVIEW

This section situates our research within the broader field of supply chain coordination and highlights its key contributions. First, it explores existing VMI contracts, both those implemented in practice and those proposed in the literature. Next, it reviews studies that advocate for VMI and its various contract structures, often emphasizing its benefits while providing little critical analysis of its effectiveness. Finally, this section examines analytical approaches developed to compare VMI and RMI, focusing on decision-support models that help determine the most suitable inventory management strategy.

2.2.1 *Supply Chain Coordination*

The performance of a supply chain is heavily dependent on the seamless coordination between its various stages. When supply chain members make independent decisions without considering their impacts on others, inefficiencies arise, significantly increasing overall costs. One well-known consequence is the bullwhip effect, characterized by amplified demand fluctuations that lead to higher inventory costs, inefficiencies, and reduced responsiveness (Krajewski and Malhotra 2022). Research has consistently demonstrated that improved coordination among supply chain partners can mitigate these inefficiencies, enhancing overall efficiency and reducing costs (Chopra and Meindl 2013).

A key aspect of supply chain coordination is the integration of production, inventory, and distribution planning. Early studies, such as that of Chandra and

Fisher (1994), revealed that aligning production and distribution decisions could result in cost reductions ranging from 3% to 20%. These findings laid the foundation for subsequent research on integrated decision-making models (Adulyasak et al. 2015b; Berghman et al. 2023).

To facilitate coordination, various contractual mechanisms have been developed. These include buy-back contracts (Pasternack 1985), sales-rebate agreements (Taylor 2002), revenue-sharing models (G. P. Cachon and Lariviere 2005), quantity flexibility contracts (Tsay 1999), and wholesale-price agreements (Gerchak and Wang 2004). These mechanisms aim to distribute risks and rewards among supply chain partners, ensuring higher product availability while aligning incentives across different tiers.

Effective coordination also relies on information sharing and transparent communication between supply chain partners. Access to accurate data allows firms to improve forecasting, make informed decisions, and optimize supply chain performance (Zheng and Zipkin 1990). Studies have shown that enhanced information sharing leads to lower inventory costs, reduced overhead, and better alignment of supply and demand (Bourland et al. 1996; G. P. Cachon and Fisher 2000). The benefits of such transparency have been observed in diverse settings, including stochastic demand environments (Gavirneni et al. 1999), correlated demand patterns (H. L. Lee et al. 2000), capacity-constrained supply chains (Gavirneni 2002), and global supply networks (Özer et al. 2014).

This chapter contributes to ongoing research on supply chain coordination by proposing a quantitative framework to evaluate different contractual agreements between vendors and retailers. By providing a structured analysis of these contracts, we aim to offer insights that support better decision-making in integrated supply chain management.

2.2.2 *Vendor-Managed Inventory Contracts*

Historically, inventory management between vendors and retailers relied on RMI, where retailers independently controlled their stock levels, placing orders based on demand forecasts to minimize holding costs. Suppliers, in turn, adjusted production schedules according to retailer-driven orders. However, this traditional approach often led to inefficiencies, prompting the adoption of more advanced strategies.

Among these, VMI has emerged as a widely accepted alternative. Under VMI, the vendor assumes responsibility for managing the retailer's inventory, often retaining ownership until the stock is sold. Initially popularized in the grocery sectormost notably by Walmart (Tyan and Wee 2003)VMI has since been adopted by major companies such as Dell, Intel, Kmart, and Fred Meyer (Chopra and Meindl

2013; Bookbinder et al. 2010). The conceptual foundations of VMI can be traced back to Magee (1985), who introduced early ideas on production control systems incorporating vendor-managed stock.

VMI has proven highly effective, particularly in single-vendor, single-retailer settings. Key benefits arise when the vendor retains ownership of inventory at the retailers location (Ru and Wang 2010; J.-Y. Lee and Cho 2014), when vendors incur penalties for stockouts at the retailer level (J.-Y. Lee and Johnson 2020), and when lot-sizing decisions are optimized to minimize overall costs (Rad et al. 2014). Beyond these simple arrangements, VMI has also been successfully applied to inventory routing problems (Archetti and Speranza 2016; Malicki and Minner 2021) and production routing problems (Neves-Moreira et al. 2019), where it enables better synchronization of inventory and transportation activities.

The success of VMI depends heavily on well-structured contractual agreements that define responsibilities and risk-sharing mechanisms between vendors and retailers (Dorling et al. 2006; Stapleton et al. 2006). For instance, the (z, Z) VMI contract introduced by Fry et al. (2001) establishes lower and upper inventory limits (z, Z) and applies penalties for deviations beyond these thresholds. This framework was later extended to multiple retailers by Darwish and Odah (2010). Similarly, J.-Y. Lee and Johnson (2020) analyzed VMI contracts for continuous-review (Q, r) inventory systems, considering the retailers perspective in contract negotiations. Further studies have examined VMI agreements in newsvendor settings, where contracts incorporating revenue-sharing, sales rebates, and quantity discounts optimize vendor-retailer interactions (Sainathan and Groenevelt 2019; Gerchak et al. 2007).

Building on these contractual frameworks, inventory ownership models provide additional flexibility in VMI arrangements. In Vendor-Managed Consignment Inventory (VMCI), the vendor retains ownership of inventory stored at the retailers site, invoicing only when stock is withdrawn. This approach has been shown to enhance sales and improve market penetration (Bichescu and Fry 2009; Gümü et al. 2008). Other VMI models transfer ownership to the retailer upon receipt of goods but delay payment until withdrawal, reducing financial risks for retailers while potentially affecting supplier cash flow (Ståhl Elvander et al. 2007; Wallin et al. 2006). In traditional VMI, the retailer assumes full ownership upon delivery, bearing all risks and costs but gaining protection against price fluctuations (Radzuan et al. 2018).

This chapter introduces four novel VMI contracts, integrating inventory ownership principles and transportation cost-sharing frameworks derived from INCOTERMS, which define vendor and retailer responsibilities in goods delivery. While inventory ownership contracts have been previously explored (Radzuan et al. 2018), the application of INCOTERMS within VMI contracts remains underexamined. The proposed contracts include:

- VMI-IVTV: The vendor retains ownership of inventory and covers transportation costs.
- VMI-IVTR: The vendor owns the inventory, but the retailer is responsible for transportation costs.
- VMI-IRTR: The retailer owns the inventory and also bears transportation costs.
- VMI-IRTV: The retailer owns the inventory, while the vendor pays for transportation.

Most existing studies on VMI focus on reducing overall supply chain costs rather than examining the distribution of costs among individual partners over time, especially in multi-partner environments. While much of the literature emphasizes the potential efficiency gains of VMI, research has also shown that VMI does not always lead to cost savings compared to RMI. Additionally, there is a lack of detailed economic analysis assessing the impact of different VMI contracts on individual supply chain entities, an aspect that this study aims to address (Dong and Xu 2002).

2.2.3 *Analytical Methods for Selecting Between VMI and RMI*

The effectiveness of VMI compared to RMI varies significantly depending on the specific supply chain context. Although VMI is commonly associated with improved coordination and potential cost reductions, evidence indicates that RMI can outperform VMI under particular conditions. For example, while certain contracts, such as the (z, Z) VMI framework (see Section 2.2.2), enhance coordination between production and distribution activities in newsvendor scenarios, RMI might be more effective when optimal information sharing is implemented (Fry et al. 2001). Additionally, transferring inventory management responsibilities to vendors does not always lead to improved outcomes unless suitable risk-sharing agreements are in place (C. C. Lee and Chu 2005). Furthermore, the cost benefits of VMI depend on variables such as IT system investment and cost structure distribution, situations in which RMI may present greater advantages (Ru et al. 2018). Nevertheless, previous studies are limited due to their reliance on simplified, single-period models and assume independent, identically distributed demand. In contrast, our research investigates dynamic, multiperiod scenarios under both deterministic and stochastic demand conditions, comparing four distinct VMI contracts against RMI.

In EOQ scenarios with backordering, Pasandideh et al. (2010) analyze the impact of VMI within a simplified two-tier supply chain involving one supplier and one retailer. Their findings suggest that VMI does not consistently achieve lower overall costs than RMI. However, their analysis is limited by a deterministic, single-period demand focus, thereby overlooking realistic multiperiod and uncertain demand characteristics. Our study extends their work by evaluating VMI and RMI performance across multiple periods with varied demand patterns.

The competitive interactions between manufacturers and retailers also shape the comparative benefits of VMI. Research has shown that while VMI may increase retailer profits by exploiting competition between manufacturers offering substitutable products, manufacturers might experience reduced profitability unless inventory holding costs favor them asymmetrically (Mishra and Raghunathan 2004). External factors such as exchange rate volatility also impact VMI effectiveness, with associated cost savings emerging only after initial inefficiencies are overcome (Yu et al. 2015). Additionally, Wei et al. (2020) indicate that under conditions involving stochastic learning effects, RMI can outperform VMI, particularly with high variability in learning rates. These findings collectively suggest that VMI is not universally superior to RMI, although existing studies frequently neglect critical multiperiod considerations, including setup, production, and transportation costs. Our research addresses these limitations by explicitly incorporating these cost factors and clarifying the responsibilities assigned to vendors and retailers.

Alternative inventory management approaches also offer pathways to efficiency. For example, supply chains driven by dominant retailers, with vendors and other retailers adapting accordingly (Almehdawe and Mantin 2010), and Customer-Managed Inventory (CMI), where customers control vendor inventory (Chen et al. 2024), represent practical alternatives to VMI.

Previous studies do not offer clear recommendations regarding the comparative cost-effectiveness of VMI and RMI for supply chain partners. To address this knowledge gap, we introduce a decision-making tool designed to help evaluate four different VMI contracts against the traditional RMI option. We develop two linear programming models: (1) the All-or-Nothing model, selecting the lowest-cost contract, and (2) the Best-VMI model, which favors the adoption of VMI unless RMI proves to be less costly. These models are validated through comprehensive testing across diverse demand scenarios, including variability, varying demand volumes, and intermittent demand patterns, to ensure their robustness and practical applicability.

2.3 PROBLEM DESCRIPTION AND FORMULATIONS

This section presents the mathematical models for the All-or-Nothing and Best-VMI approaches under deterministic demand. We present these models in both the traditional manner (RMI) and the integrated manner (VMI).

The supply chain structure comprises two key entities: a vendor and a retailer. The vendor is responsible for supplying a single type of product to the retailer over a predefined planning horizon spanning $\mathcal{T}$ periods. Before the product reaches the retailer, it incurs an inventory holding cost denoted as h_t^v, representing the vendor's storage expense. Once transferred to the retailer, the inventory holding cost is denoted as h_t^r. To specify cost allocation, we distinguish between cases where the vendor is responsible for inventory holding at the retailer level ($h_t^r = h_t^{r(v)}$) and those where the retailer assumes this cost ($h_t^r = h_t^{r(r)}$). Additionally, transportation costs differ based on which party covers them. If the vendor pays for transportation, the cost is represented as C_t^v, whereas if the retailer covers transportation, it is denoted as C_t^r.

Although the retailer incurs a purchasing cost for acquiring the product, this expense remains constant across all scenarios. It does not influence cost-sharing or decision-making within the optimization model. Therefore, the purchasing cost is excluded from the mathematical formulation.

The parameters and decision variables are as follows:

Sets:

$\mathcal{T} = \{1, \dots, T\}$: Set of time periods

Parameters:

P_t : Production cost in period t.

S_t : Setup cost in period t.

h_t^v : Inventory holding cost at the vendor in period t.

$h_t^{r(v)}$, $h_t^{r(r)}$: Inventory holding cost at the retailer, incurred by the vendor or retailer, respectively, in period t.

U : Storage capacity at the retailer.

I_0^v : Initial inventory level at the vendor

$I_0^{r(v)}$, $I_0^{r(r)}$: Initial inventory level at the retailer, owned by the vendor or retailer, respectively.

D_t : Retailers demand in period t.

C_t^r : Transportation cost paid by the retailer in period t.

C_t^v : Transportation cost paid by the vendor in period t.

Decision variables:

x_t : Production quantity in period t.

y_t : Binary variable, equal to 1 if production occurs in period t, otherwise 0 .

I_t^v : Inventory level at the vendor in period t.

$I_t^{r(v)}$: Inventory level at the retailer in period t, owned by the vendor.
$I_t^{r(r)}$: Inventory level at the retailer in period t, owned by the retailer .
v_t^r : Binary variable, equal to 1 if the retailer is supplied in period t and bears the transportation cost.
v_t^v : Binary variable, equal to 1 if the retailer is supplied in period t and the vendor bears the transportation cost .
q_t : Quantity of product shipped to the retailer in period t

Unless explicitly mentioned otherwise, we assume that the initial inventory levels at both the vendor and retailer are zero, i.e., $I_0^v = I_0^{r(r)} = I_0^{r(v)} = 0$.

2.3.1 *All or nothing model*

This section presents the structure of the All-or-Nothing model, which is composed of three key components: the RMI model, the VMI model, and the selection mechanism that determines the most advantageous contract for both the vendor and the retailer.

i) TRADITIONAL SUPPLY CHAIN (RMI) In the RMI framework, the vendor and the retailer operate independently, each seeking to optimize its own costs. The retailer determines the optimal order quantities to procure from the vendor, while the vendor treats these orders as external demand and optimizes its production schedule accordingly to minimize costs. As a result, the total supply chain cost is simply the aggregation of the costs incurred by both parties.

The retailer decision model: The retailer's objective is to determine the optimal order quantity q_t in each period t while minimizing the total costs associated with transportation and inventory holding.

$$\text{Retailer decision model: } [\mathbf{RMI} - \mathbf{R}] \quad \min R_{RMI} = \sum_{t \in \mathcal{T}} h_t^{r(r)} I_t^{r(r)} + \sum_{t \in \mathcal{T}} C_t^r v_t^r \tag{2.1}$$

$$I_t^{r(r)} = I_{t-1}^{r(r)} + q_t - D_t, \quad \forall t \in \mathcal{T} \tag{2.2}$$

$$I_t^{r(r)} \leq U, \quad \forall t \in \mathcal{T} \tag{2.3}$$

$$q_t \leq M \times v_t^r, \quad \forall t \in \mathcal{T} \tag{2.4}$$

$$v_t^r \in \{0,1\}, \quad \forall t \in \mathcal{T} \tag{2.5}$$

$$q_t, I_t^{r(r)} \geq 0, \quad \forall t \in \mathcal{T} \tag{2.6}$$

The objective function in equation (2.1) aims to minimize the retailer's total costs associated with inventory holding and transportation. The inventory balance constraints, represented in equation (2.2), ensure that stock levels at the retailer are updated appropriately over time. The notation $I_t^{r(r)}$ is used to emphasize that inventory management is the retailers responsibility. It is also assumed that the initial inventory level is zero($I_0^{r(r)} = 0$). Equation (2.3) enforces the storage capacity limitations at the retailer, ensuring that inventory levels do not exceed the available space. The constraints in equation (2.4) establish a link between the order quantity variables q_t and the binary decision variables v_t^r, indicating that transportation is required only when an order is placed in period t. The parameter M in this equation is a sufficiently large constant, typically defined as the total demand over the planning horizon T. Lastly, equations (2.5) and (2.6) define the integrality and non-negativity constraints.

Vendor Decision Model:
The quantity of products delivered to the retailer in period t, denoted as q_t, is determined based on the retailer's order decisions. This quantity is then used as an input parameter in the vendors optimization model, represented as $\hat{q}_t$. The vendor's objective is to minimize the overall costs associated with production, setup, and inventory holding while ensuring that the retailer's demand ($\hat{q}_t$) is met efficiently.

Here is the reformatted set of equations using the 'equation' environment:

$$\text{Vendor decision model: } [\mathbf{RMI-V}] \quad \min V_{RMI} = \sum_{t\in\mathcal{T}} P_t x_t + \sum_{t\in\mathcal{T}} S_t y_t + \sum_{t\in\mathcal{T}} h_t^v I_t^v \tag{2.7}$$

$$I_t^v = I_{t-1}^v + x_t - \hat{q}_t, \quad \forall t \in \mathcal{T} \tag{2.8}$$

$$x_t \leq y_t \sum_{t'\in\mathcal{T}} \hat{q}_{t'}, \quad \forall t \in \mathcal{T} \tag{2.9}$$

$$y_t \in \{0,1\}, \quad \forall t \in \mathcal{T} \tag{2.10}$$

$$x_t, I_t^v \geq 0, \quad \forall t \in \mathcal{T} \tag{2.11}$$

The objective function in equation (2.7) aims to minimize the vendor's overall costs, encompassing production, setup, and inventory holding expenses. The inventory balance is maintained through constraints (2.8). Constraints (2.9) establish a connection between the continuous production variables x_t and the binary setup variables y_t, ensuring that a setup cost is incurred only when production occurs in period t. Lastly, constraints (2.10) and (2.11) enforce integrality and non-negativity conditions.

The total supply chain cost under the RMI policy is given by:

$$SC_{RMI} = R_{RMI}^* + V_{RMI}^* \tag{2.12}$$

where

$$R^*_{RMI} = \sum_{t\in\mathcal{T}} h_t^{r(r)} I_t^{r(r)*} + \sum_{t\in\mathcal{T}} C_t^r v_t^{r*} \tag{2.13}$$

and

$$V^*_{RMI} = \sum_{t\in\mathcal{T}} P_t x_t^* + \sum_{t\in\mathcal{T}} S_t y_t^* + \sum_{t\in\mathcal{T}} h_t^v I_t^{v*} \tag{2.14}$$

For each period $t \in \mathcal{T}$, the variables $I_t^{r(r)*}$ and v_t^{r*} (respectively, x_t^*, y_t^*, and I_t^{v*}) represent the optimal solutions derived from the retailer's decision model $[\mathbf{RMI} - \mathbf{R}]$ and the vendor's decision model $[\mathbf{RMI} - \mathbf{V}]$. The corresponding optimal costs for these models are denoted as R^*_{RMI} for the retailer and V^*_{RMI} for the vendor.

ii) VMI MODEL WITH DIFFERENT CONTRACTS This section presents the integrated mathematical model for VMI, which autonomously selects the most cost-effective contract. In a VMI framework, responsibility for inventory management and transportation is assigned to a single partner. To formalize this allocation, we introduce the following decision variables: α: A binary variable that equals 1 if the vendor is responsible for inventory management and 0 if the retailer assumes this responsibility.
β: A binary variable that equals 1 if the vendor covers transportation costs and 0 if the retailer bears these costs.

The values of α and β define four distinct VMI contract structures:

1. VMI-IVTV: The vendor is responsible for both inventory holding and transportation costs ($\alpha = 1$, $\beta = 1$).

2. VMI-IVTR: The vendor manages inventory costs, while the retailer covers transportation expenses ($\alpha = 1$, $\beta = 0$).

3. VMI-IRTV: The retailer is responsible for inventory costs, whereas the vendor bears transportation costs ($\alpha = 0$, $\beta = 1$).

4. VMI-IRTR: The retailer is responsible for both inventory and transportation costs ($\alpha = 0$, $\beta = 0$).

$$\min\ SC_{VMI} = \sum_{t\in\mathcal{T}} P_t x_t + \sum_{t\in\mathcal{T}} S_t y_t + \sum_{t\in\mathcal{T}} h_t^v I_t^v + \sum_{t\in\mathcal{T}} h_t^{r(v)} I_t^{r(v)} + \sum_{t\in\mathcal{T}} C_t^v v_t^v + \sum_{t\in\mathcal{T}} h_t^{r(r)} I_t^{r(r)} + \sum_{t\in\mathcal{T}} C_t^r v_t^r \tag{2.15}$$

$$I_t^v = I_{t-1}^v + x_t - q_t, \quad \forall t \in \mathcal{T} \tag{2.16}$$

$$I_t^{r(v)} + I_t^{r(r)} = I_{t-1}^{r(v)} + I_{t-1}^{r(r)} + q_t - D_t, \quad \forall t \in \mathcal{T} \tag{2.17}$$

$$I_t^{r(v)} + I_t^{r(r)} \leq U, \quad \forall t \in \mathcal{T} \tag{2.18}$$

$$I_t^{r(v)} \leq M' \times \alpha, \quad \forall t \in \mathcal{T} \tag{2.19}$$

$$I_t^{r(r)} \leq M' \times (1-\alpha), \quad \forall t \in \mathcal{T} \tag{2.20}$$

$$x_t \leq y_t \sum_{t' \in \mathcal{T}} D_{t'}, \quad \forall t \in \mathcal{T} \tag{2.21}$$

$$q_t \leq M \times (v_t^v + v_t^r), \quad \forall t \in \mathcal{T} \tag{2.22}$$

$$v_t^v \leq \beta, \quad \forall t \in \mathcal{T} \tag{2.23}$$

$$v_t^r \leq 1 - \beta, \quad \forall t \in \mathcal{T} \tag{2.24}$$

$$y_t, v_t^r, v_t^v \in 0, 1, \quad \forall t \in \mathcal{T} \tag{2.25}$$

$$\alpha, \beta \in 0, 1 \tag{2.26}$$

$$x_t, I_t^v, I_t^{r(v)}, I_t^{r(r)}, q_t \geq 0, \quad \forall t \in \mathcal{T} \tag{2.27}$$

The objective function (2.15) aims to minimize the overall cost, which includes production, setup, and inventory holding costs at the vendor, along with inventory holding and transportation costs at the retailer. The allocation of these latter costs depends on the selected VMI contract, determined by the binary decision variables α and β.

Constraints (2.16) and (2.17) define the inventory balance equations for the vendor and retailer, ensuring that stock levels are correctly updated. Constraint (2.18) enforces the retailers storage capacity limits. Constraints (2.19) and (2.20) ensure that only one inventory ownership structure is selected, aligning with the chosen VMI contract. Constraint (2.21) links the production decision variable x_t to the setup variable y_t, ensuring that production can only occur if a setup has been executed in the respective period. Similarly, constraint (2.22) connects the order quantity variable q_t to the delivery decision variables $(v_t^r + v_t^v)$, ensuring that transportation costs are only incurred when a positive order quantity is shipped. Constraints (2.23) and (2.24) enforce the assignment of transportation cost responsibility, specifying whether the vendor or the retailer bears these costs under the selected contract. Finally, constraints (2.25)-(2.27) impose integrality and non-negativity requirements on the decision variables.

Beyond determining the optimal total supply chain cost, the model also allows for calculating individual cost components for both the vendor and the retailer, as outlined below:

$$V^{VMI} = \sum_{t \in \mathcal{T}} P_t x_t^* + \sum_{t \in \mathcal{T}} S_t y_t^* + \sum_{t \in \mathcal{T}} h_t^v I_t^{v*} + \sum_{t \in \mathcal{T}} h_t^{r(v)} I_t^{r(v)} + \sum_{t \in \mathcal{T}} C_t^v v_t^v \tag{2.28}$$

Similarly, the total cost incurred by the retailer under the VMI framework is given by:

$$R^{VMI} = \sum_{t \in \mathcal{T}} h_t^{r(r)} I_t^{r(r)} + \sum_{t \in \mathcal{T}} C_t^r v_t^{r*} \tag{2.29}$$

iii) SELECTION PROCESS The approach for selecting between VMI and RMI in this study is inspired by the framework proposed by C. C. Lee and Chu (2005). Their research suggests that VMI adoption is only feasible when both supply chain partners agree and when its implementation results in improved outcomes for both parties. However, VMI can still be considered if the overall cost savings for the supply chain surpass those under RMI, even if one partner benefits more significantly than the other. In this study, a VMI contract is selected only if it results in a lower total supply chain cost than RMI and ensures that neither partner incurs higher costs under VMI than they would under RMI.

The four previously defined VMI contracts (VMI-IVTV, VMI-IVTR, VMI-IRTV, VMI-IRTR) establish different cost-sharing arrangements between the vendor and the retailer. Under the VMI framework, the vendor assumes responsibility for production, setup, and its own inventory costs, while the allocation of transportation and inventory holding costs depends on the chosen contract. The selection process is formally outlined in Algorithm 2.1, which systematically determines the most suitable contract based on cost minimization criteria. Although the "All-or-

Algorithm 2.1 The "All-or-Nothing" Selection process

```
Input: R*_RMI, R*_VMI, V*_RMI, V*_VMI, SC_RMI, SC_VMI
Output: VMI or RMI
if SC_VMI < SC_RMI then
    if V*_VMI ≤ V*_RMI AND R*_VMI ≤ R*_RMI then
        Adopt VMI
    else
        Adopt RMI
    end if
else
    Adopt RMI
end if
```

Nothing" model provides a structured decision-making framework for selecting between VMI and RMI, it presents certain limitations. The model identifies the VMI contract that yields the lowest total cost and compares it with the RMI outcome, emphasizing overall cost reduction. However, this approach does not sufficiently consider the individual cost implications for each partner. To address this limitation, we introduce the Best-VMI model, which aims to facilitate VMI adoption by selecting a contract that not only reduces total supply chain costs but also ensures that both the vendor and retailer experience cost savings compared to the RMI scenario.

Numerical example:
To illustrate the selection process between RMI and VMI, consider a supply chain consisting of a single vendor and a single retailer. For simplicity, only two VMI contracts are available. The cost breakdown for each scenario is as follows:

- RMI: Total cost = 3500; Vendor cost = 2000; Retailer cost = 1500
- VMI Contract 1: Total cost = 3000; Vendor cost = 2500; Retailer cost = 500
- VMI Contract 2: Total cost = 3200; Vendor cost = 2000; Retailer cost = 1200

In the All-or-Nothing approach, the selection process identifies the VMI contract with the lowest overall cost, Contract 1, which has a total cost of 3000. However, when comparing this to RMI, the model favors RMI because Contract 1 increases the vendors cost from 2000 to 2500, making it less desirable for the vendor despite the reduction in total supply chain costs.

Conversely, the Best-VMI model selects Contract 2, where the total cost (3200) is still lower than RMI (3500), and the retailer benefits from a reduced cost (1200 compared to 1500 in RMI). Importantly, the vendor's cost remains unchanged at 2000. This model is more effective in facilitating VMI adoption, as it ensures cost reduction for at least one partner while preventing additional financial burdens on the other.

The key difference between the two models lies in their selection mechanisms. The All-or-Nothing approach makes independent decisions at each stage, prioritizing total cost minimization. In contrast, the Best-VMI model integrates the selection process into the optimization framework, ensuring that the chosen contract provides a balanced benefit to both supply chain partners.

2.3.2 *The Best-VMI model*

This model aligns closely with the All-or-Nothing approach, with the primary distinction being the absence of a selection process. The mathematical formulation is structured as follows:

In the RMI model, the objective is to minimize the retailer's cost, as defined by the objective function in Equation (2.1), subject to constraints (2.2) to (2.6). Simultaneously, the vendor's cost is minimized, as specified by the objective function in Equation (2.7), subject to constraints (2.8) to (2.11). The outcomes of the RMI model, denoted as R_{RMI} and V_{RMI}, serve as inputs for the VMI model. The VMI model aims to minimize the total supply chain cost, as represented by the objective function in Equation (2.15), subject to constraints (2.16) to (2.27). Furthermore, the following conditions must be satisfied:

$$SC_{VMI} < SC_{RMI} \tag{2.30}$$

$$V_{VMI} \leq V_{RMI} \tag{2.31}$$

$$R_{VMI} \leq R_{RMI} \tag{2.32}$$

These constraints (2.30), (2.31) and (2.32) ensure that the VMI model identifies a solution that outperforms the RMI model in terms of total supply chain cost, while also guaranteeing that neither the vendor nor the retailer incurs higher costs compared to the RMI scenario. If the model fails to identify such a solution, the RMI contract remains the default option.

2.4 NUMERICAL EXPERIMENTS

This section presents an evaluation and comparative analysis of the impact of various demand characteristicsincluding demand patterns, variability, volume, and intermittent demandon the performance of the All-or-Nothing and Best-VMI models. The mathematical formulations were implemented in Python and solved using CPLEX Solver version 12.8. Computational experiments were conducted on a system equipped with an AMD Ryzen 7 3700U processor (2.30 GHz), operating on a 64-bit Windows 10 platform with 8 GB of RAM. To ensure consistency in execution, a maximum computation time of 3600 seconds was imposed for each model.

2.4.1 *Data Generation*

To assess the performance of the proposed models, computational tests were conducted by varying both transportation and inventory costs incurred by the vendor. Each cost parameter was classified as either high or low. A total of 8,000 instances were generated for each model and cost category, with the average results analyzed and discussed.

The test instances were inspired by previous research, including Brahimi and Aouam (2016) and Archetti et al. (2011), among others. A planning horizon of 12 periods was considered, with parameters generated uniformly within predefined intervals. For consistency, costs were assumed to be time-independent. Notably, transportation and inventory holding costs were assigned a broad range of values to evaluate their impact on decision-making.

The parameters were defined as follows:

Production cost:

$$P_t = \text{uniform}[10, 50]$$

Setup cost:

$$S_t = \text{uniform}[1000, 5000]$$

Inventory holding cost at the vendor:

$$h_t^v = \text{uniform}[3, 10]$$

Maximum inventory capacity at the retailer:

$$U = \text{uniform}[2, 6] \times \bar{D}, \quad \text{where} \quad \bar{D} = \frac{\sum_{t \in \mathcal{T}} D_t}{T}$$

Variation in Transportation Costs
When analyzing the impact of transportation costs paid by the vendor, the following classifications were used: Low transportation cost:

$$C_t^v = \text{uniform}[1, 500], \quad C_t^r = \text{uniform}[100, 200]$$

High transportation cost:

$$C_t^v = \text{uniform}[1, 3000], \quad C_t^r = \text{uniform}[1000, 2000]$$

Inventory holding cost at the retailer (paid by vendor):

$$h_t^{r(v)} = \text{uniform}[1, 20]$$

Inventory holding cost at the retailer (paid by retailer):

$$h_t^{r(r)} = \text{uniform}[1, 20]$$

Variation in Inventory Holding Costs at the Retailer
When evaluating the impact of inventory holding costs at the retailer, two scenarios were considered: Low inventory holding cost:

$$h_t^{r(v)} = \text{uniform}[0.5, 13], \quad h_t^{r(r)} = \text{uniform}[5, 8]$$

High inventory holding cost:

$$h_t^{r(v)} = \text{uniform}[0.5, 25], \quad h_t^{r(r)} = \text{uniform}[10, 15]$$

Transportation costs paid by vendor and retailer:

$$C_t^v = \text{uniform}[1, 1000], \quad C_t^r = \text{uniform}[1, 1000]$$

Demand Variations

To analyze the impact of demand fluctuations, the following variations were considered:

1. Demand patterns: Four different patternsconstant, increasing, decreasing, and seasonalwere tested, with an average demand value of 25.

2. Demand volume: The average demand values were set to $\{3, 10, 25, 50, 250\}$ and remained constant over the planning horizon.

3. Demand variability: The demand fluctuated around an average value of 50, with variations of $\pm\{5, 10, 15, 25, 50\}$.

4. Intermittent demand: A random demand structure was introduced, characterized by a significant proportion of zero demand periods. The degree of intermittence ranged from 10% to 100%, represented by an intermittence parameter varying between 0.1 and 1.0.

2.4.2 *Results and Discussion*

This section outlines the outcomes derived from the All-or-Nothing and Best-VMI models under deterministic demand conditions. The analysis focuses on two key aspects: i) evaluating shifts in transportation and retailer inventory costs borne by the vendor, considering factors such as demand patterns, variability, volume fluctuations, and intermittent demand; and ii) comparing the performance of the two models. The rate of VMI adoption is calculated using Equation (2.33), where N^{VMI} represents the total instances in which a VMI contract was selected, and N^{VRMI} indicates the total instances where an RMI contract was preferred.

$$Rate_{VMI} = \frac{N_{VMI}}{N_{VMI} + N_{RMI}}, \tag{2.33}$$

2.4.2.1 *Evaluation of Models Under Varied Demand Patterns*

This section examines the rate of VMI adoption across all instances identified by the models, focusing on scenarios where the vendor's costs vary while the retailer's costs remain within predefined ranges. The analysis considers four distinct demand patterns: constant, increasing, decreasing, and seasonal. First, the scenario in which the vendor covers transportation costs is evaluated, followed by an assessment of the case where the vendor is responsible for the retailer's inventory costs.

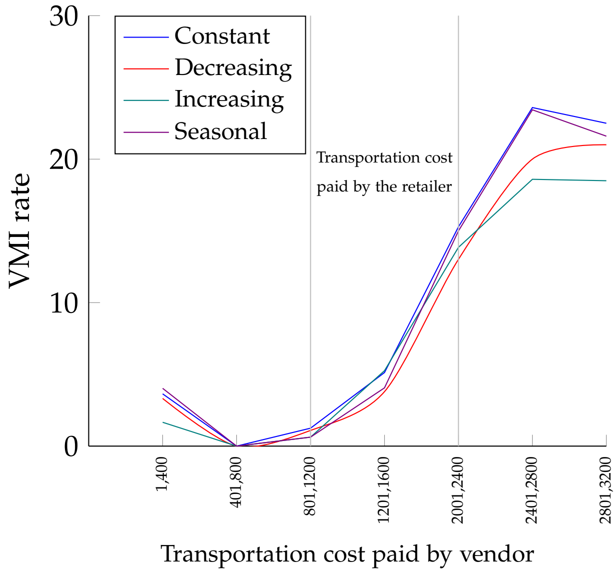
i)ANALYSIS OF VARIATIONS IN VENDOR INVENTORY AND TRANSPORTATION COSTS In this section, we assess the impact of varying transportation and inventory costs borne by the vendor on VMI adoption rates. The transportation costs incurred by the vendor range between [1,500] for low costs and [1,3000] for high costs, while the retailer's transportation expenses are set within [100,200] for low costs and [1000,2000] for high costs. Additionally, we examine scenarios where the retailers inventory cost, when covered by the vendor, fluctuates between [0.25,25], while the retailers inventory cost varies between [5,10] for low-cost cases and [10,15] for high-cost cases.

Figure 2.1 illustrates VMI adoption rates across different transportation cost scenarios. The results are divided into two cost categories: high and low. The outcomes of the All-or-Nothing model are displayed in Figures 2.1(a) (high costs) and 2.2(a) (low costs), while those of the Best-VMI model appear in Figures 2.1(b) (high costs) and 2.2(b) (low costs).

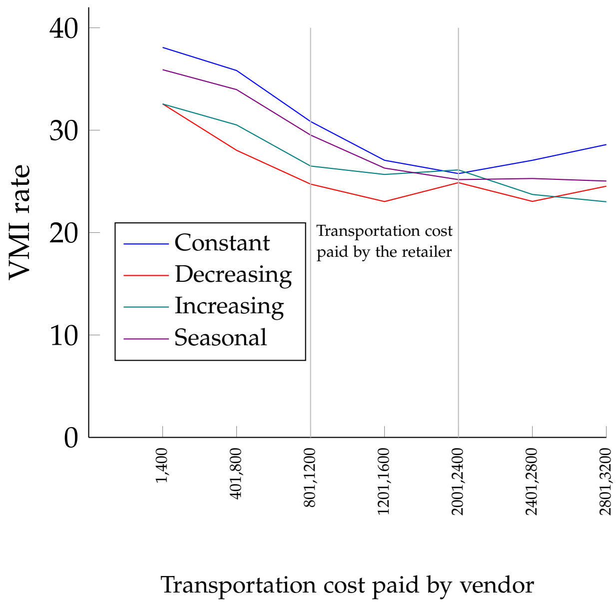
The findings reveal that VMI adoption rates range from 0% to 38% under the All-or-Nothing model, whereas the Best-VMI model achieves significantly higher adoption rates, varying between 23% and 90%. This confirms that VMI does not always surpass RMI in cost efficiency, as previously highlighted in Section 2.2. Despite some variations in values, the overall trend remains consistent across different demand patterns and cost categories.

The VMI adoption pattern under the All-or-Nothing model when transportation costs are high follows three distinct phases: "before", "during", and "after" the interval where the retailer bears the transportation cost. A detailed breakdown of VMI contract selections is presented in Tables A.1 and A.2 in the Appendix.

- Before the Interval [1000,2000]: The model selects the lowest-cost option, even when VMI adoption is not optimal. In this phase, the vendor's transportation costs are lower than the retailer's, prompting the model to allocate transportation expenses to the vendor. Additionally, the model assigns inventory responsibility to the party with the lower associated cost, whether the vendor or retailer.
  - During the Interval [1,400]: The model selects two contracts: VMI-IVTV and VMI-IRTV. VMI is only adopted if the vendor's transportation and inventory costs are considerably lower than those of the retailer, ensuring that the vendor does not incur excessive losses. For the VMI-IRTV contract, if the vendor's inventory cost is equal to or exceeds its internal inventory holding cost, and the retailers inventory cost is at least 20% lower, the retailer assumes inventory responsibilities. Additionally, the vendor only accepts transportation costs if they are reduced by at least 90% compared to what the retailer would have incurred.



(a) All-or-Nothing model under high transportation cost

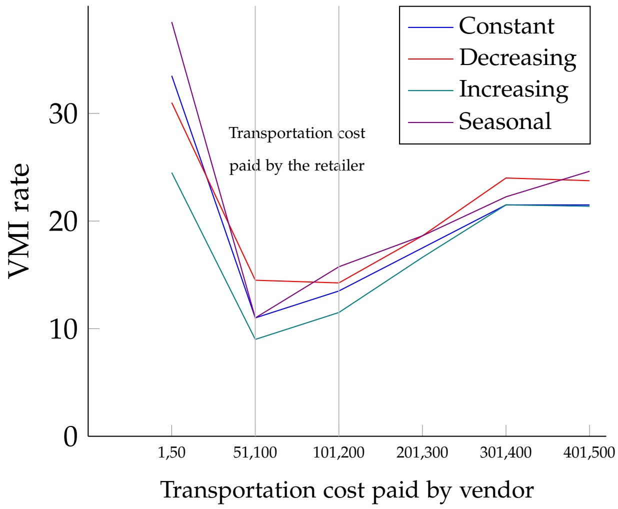


(b) Best-VMI model under high transportation cost

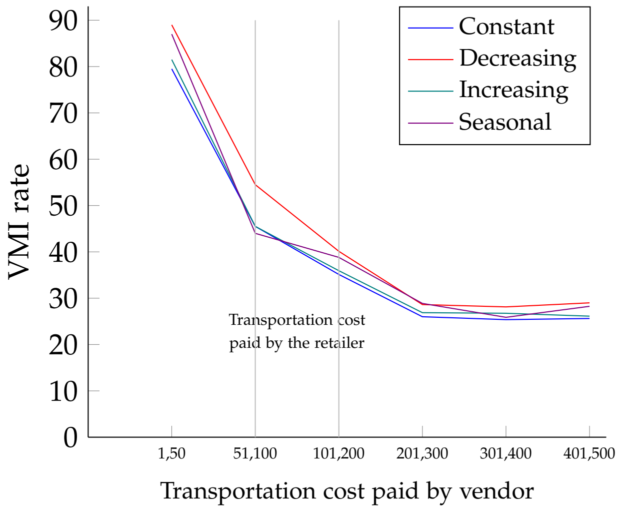
Figure 2.1: VMI adoption rate under high transportation cost and for different demand patterns.

  – During the Interval [400,800]: No VMI contracts are selected in this range. The model enforces a rule that the vendor should assume transportation and inventory costs if they are lower than the retailers. However, at this stage, transportation costs remain high, discouraging VMI adoption due to potential losses for the vendor.

- During the interval [1000-2000]: When transportation costs are high, the vendor is reluctant to bear them. VMI adoption occurs only when both



(a) All-or-Nothing model under low transportation cost



(b) Best-VMI model under low transportation cost

Figure 2.2: VMI adoption rate under low transportation cost and for different demand patterns.

partners can avoid substantial losses compared to RMI. In instances where the retailer assumes responsibility for transportation and inventory, the resulting scenario mirrors RMI, making VMI infeasible. The selection of the VMI-IVTR contract leads to an increase in VMI adoption during this phase, as the vendor prefers to store inventory at the retailers site when its holding costs are lower than its internal inventory costs. Meanwhile, the retailer covers transportation expenses. This results in significant cost savings for VMI compared to RMI.

- After the Interval [1000-2000]: Both partners continue using the strategies established in the previous phase. However, in some cases, the model selects the VMI-IRTR contract, facilitating further coordination between partners to optimize inventory costs.

For cases with low transportation costs, the same phases apply as in the high-cost scenario, with similar reasoning but varying adoption rates. The higher VMI adoption rate in the low-cost scenario can be attributed to the greater likelihood of both partners finding a cost-effective VMI contract, as reduced costs make optimization easier. Additional details on these results can be found in Tables A.5 and A.6 in Appendix.

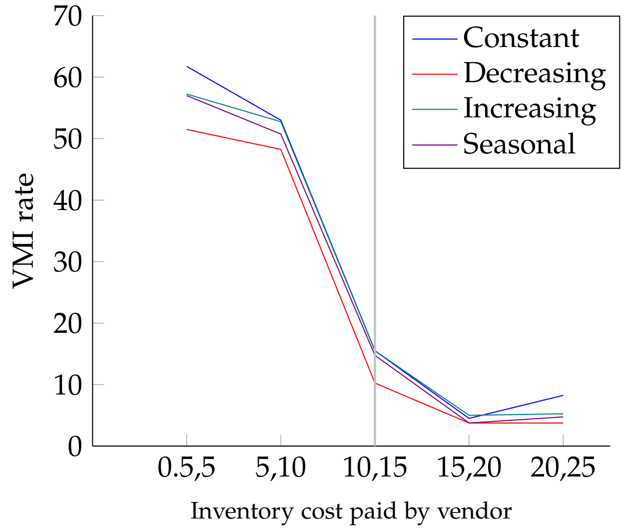
Figures 2.3 and 2.4 shows the adoption rates of VMI derived from various models as inventory costs borne by the vendor fluctuate, maintaining a structure analogous to that of Figure 2.1. Each graph includes a vertical line that represents the inventory cost borne by the retailer.

The results align with those of Figure 2.1, demonstrating that VMI does not consistently outperform RMI. Although the Best-VMI model achieves full VMI adoption at low inventory costs, this rate diminishes as inventory costs rise. In addition, demand patterns in both models follow comparable trends in both the low and high inventory cost ranges.

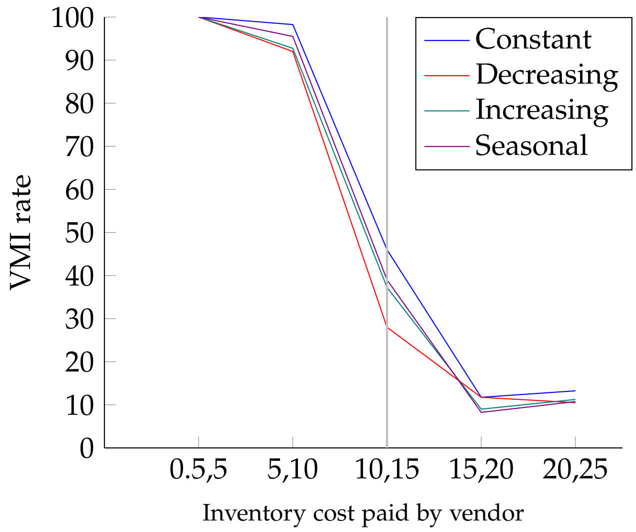
The behavior of the All-or-Nothing model under high inventory costs is analyzed using VMI contract rates presented in Table A.3 in Appendix. This analysis is divided into two phases: "before" and "after" the interval where the retailer bears inventory costs.

In the "before" phase, the model identifies the minimum cost even when the adoption of VMI is not possible. Here, the vendors inventory cost for the retailer is lower than the retailers own cost, prompting the model to favor solutions where the vendor covers inventory expenses. The model prioritizes options with the lowest transportation costs, regardless of whether they are borne by the vendor or retailer.

Within the interval [0.5, 10], two VMI contractsVMI-IVTV and VMI-IVTRare selected. These contracts are adopted only if the vendor satisfies one of two conditions. The first condition requires the vendors transportation costs to be substantially lower (by an average of 75%) than the retailers, coupled with the vendors inventory cost being 40% lower than the retailers and below the vendors internal inventory cost. The second condition specifies that the vendors inventory cost must be 80% lower than the retailers and below the vendors internal inventory cost, while transportation costs are 15% lower than the retailers. Under the second contract, the vendor assumes the retailers inventory costs if they are lower than both the vendors internal costs and the retailers costs. In this scenario, the vendor stores products at the retailers storage to minimize expenses, with the retailer covering transportation costs. Detailed outcomes are provided in Tables A.7 and A.8 in Appendix.



(a) All-or-Nothing model under high inventory holding cost

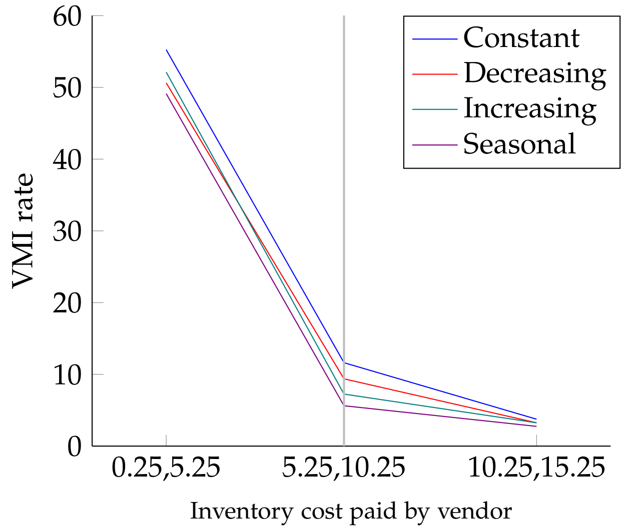


(b) Best-VMI model under high inventory holding cost

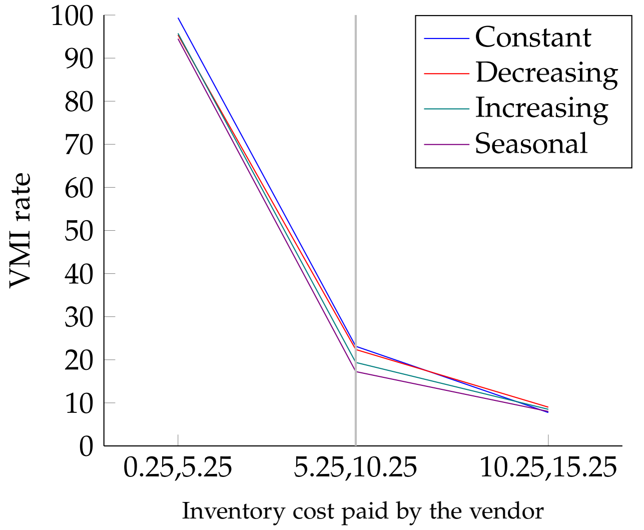
Figure 2.3: VMI adoption rate under high inventory cost and for different demand patterns.

In the interval [10, 15], inventory costs for both the vendor and retailer, as well as the vendors internal inventory costs, drop within the same range [10, 15]. Under these conditions, it becomes difficult for the vendor to adopt VMI while covering both inventory and transportation costs. Achieving the minimum total cost is feasible only if both costs are exceptionally low, with a probability of 0.3%. However, if the partners share the cost limitation, VMI adoption becomes possible at a combined rate of 13.5% through the selection of VMI-IVTR and VMI-IRTV contracts.

In the "after" phase, the vendor's internal inventory cost is lower than the retailer's inventory cost covered by the vendor, making it impractical for the vendor



(a) All-or-Nothing model under low inventory holding cost



(b) Best-VMI model under high inventory holding cost

Figure 2.4: VMI adoption rate under low inventory cost and for different demand patterns.

to cover inventory expenses. The vendor may cover transportation costs only if they are significantly lower than the retailers, compelling the retailer to bear inventory costs. Although other VMI adoption scenarios exist, the model prioritizes achieving the optimal total cost.

For low inventory costs, as shown in Figure 2.4(a) and (b), the phases reflect those for high inventory costs. In the "before" phase, VMI adoption occurs only if the vendor covers inventory costs and shares transportation costs with the retailer, leading to the selection of VMI-IVTV and VMI-IVTR contracts at rates of 3.5% and 48.2%, respectively. In the "after" phase, the retailer bears inventory costs, while the vendor covers transportation costs if they are significantly lower than its internal inventory cost, resulting in the selection of VMI-IRTV and VMI-IRTR contracts at rates of 3% and 0.2%, respectively. These percentages are detailed in Table A.4 in Appendix.

In summary, the findings highlight three key insights. First, they confirm the studies suggesting that VMI does not always surpass RMI, even when multiple VMI contracts are considered. Second, the cost distribution between the vendor and retailer significantly influences VMI adoption rates and contract selection across both models. Third, demand patterns do not affect VMI adoption rates.

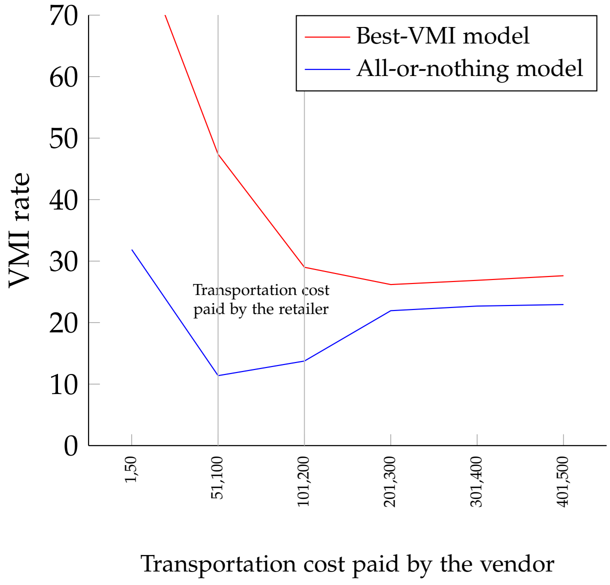
Given the third insight, demand patterns have been aggregated to facilitate model performance comparisons, which will be explored in the subsequent section.

ii) COMPARATIVE EVALUATION OF THE ALL-OR-NOTHING AND BEST-VMI MODELS To facilitate a comprehensive comparison between the two models, we aggregate the previous findings, particularly examining variations in inventory and transportation costs assumed by the vendor. Figure 2.6 illustrates this comparative analysis, presenting the impact of high and low transportation costs (Figures 2.5(a) and (b)) as well as high and low inventory costs (Figures 2.6(a) and (b)).

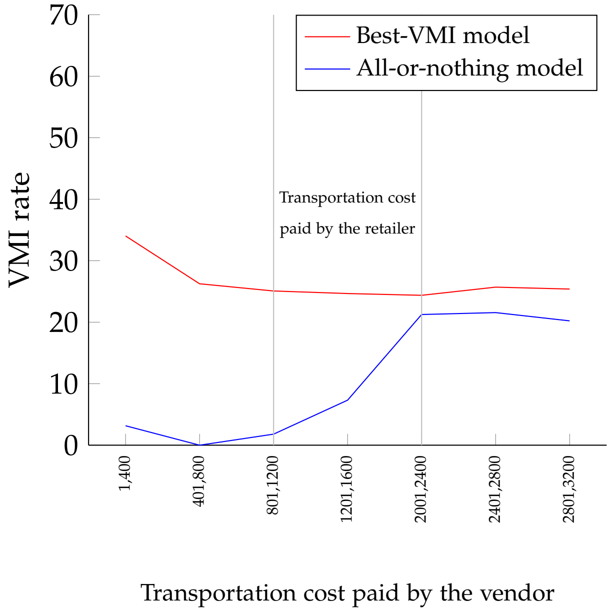
The findings depicted in Figure 2.5 and 2.6 highlight the superior performance of the Best-VMI model compared to the All-or-Nothing model in all scenarios involving variations in vendor-covered costs. This advantage is particularly evident in the "Before" phase, where, however, the Best-VMI model consistently achieves a higher VMI adoption rate. both models exhibit convergence during the "After" phase. The Best-VMI model stems from its integrated selection process within the VMI framework, which enables the identification of a viable VMI contract even when the total cost is not minimized.

These outcomes can be further comprehended by analyzing the VMI contracts chosen by each model, as illustrated in Figures 2.7 and 2.8. These figures display the distribution of selected VMI contracts, using distinct colors as indicated in the legend, across varying levels of transportation and inventory costs borne by the vendor. The contract selection patterns are analyzed separately for each model, where Model1 corresponds to the All-or-Nothing approach and Model2 represents the Best-VMI model.

Figures 2.7 and 2.8 illustrate that both models tend to select similar VMI contracts, although the Best-VMI model consistently achieves higher adoption rates. The key distinction arises in the "Before" phase, where the Best-VMI model modifies contracts not selected in the All-or-Nothing model by adjusting inventory levels or transportation schedules. This flexibility allows the model to adopt VMI contracts that may have initially appeared less cost-effective but become viable through these adjustments. Conversely, in the "After" phase, both models con-



(a) Low transportation cost

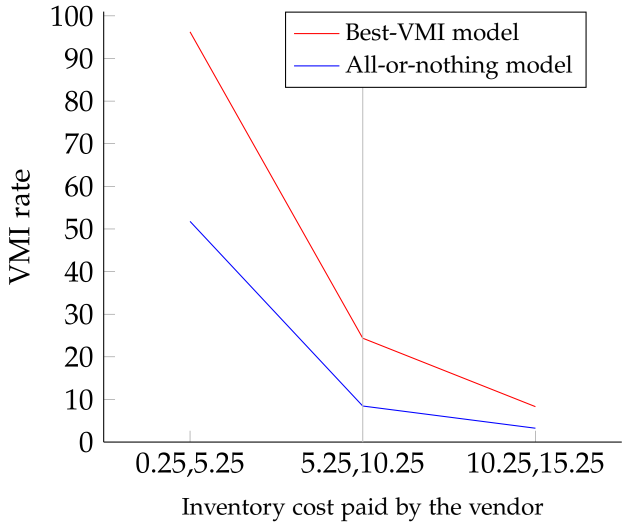


(b) High transportation cost

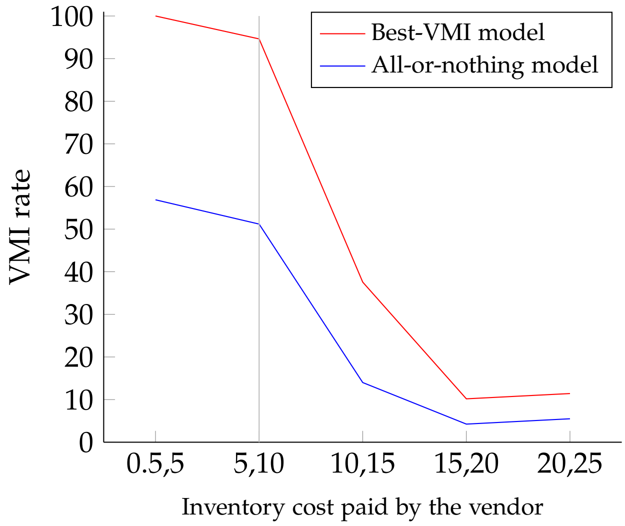
Figure 2.5: Comparison between models when changing transportation costs.

verge toward selecting the same contracts, as these represent the most cost-efficient choices. The difference between the models diminishes in this phase since only a limited number of solutions require adjustments in the Best-VMI model to increase VMI adoption.

In conclusion, the Best-VMI model demonstrates superior performance compared to the All-or-Nothing model in all scenarios, regardless of whether vendor-incurred inventory and transportation costs are high or low. This advantage is



(a) Low inventory holding cost



(b) High inventory holding cost

Figure 2.6: Comparison between models when changing inventory costs.

primarily due to the models ability to refine unselected contracts from the All-or-Nothing approach, optimizing inventory and transportation strategies to facilitate VMI adoption and enhance collaboration between supply chain partners.

iii) ANALYSIS OF CONTRACT SELECTION This section evaluates the adoption rates of individual VMI contracts and compares their respective percentages. Figure 2.9 displays the proportion of contracts selected by the All-or-Nothing and Best-VMI models. Both models predominantly favored the VMI-IVTR contract, as it enables the vendor to store products in the retailers location, achieving cost efficiencies while requiring the retailer to cover only transportation costs rather than

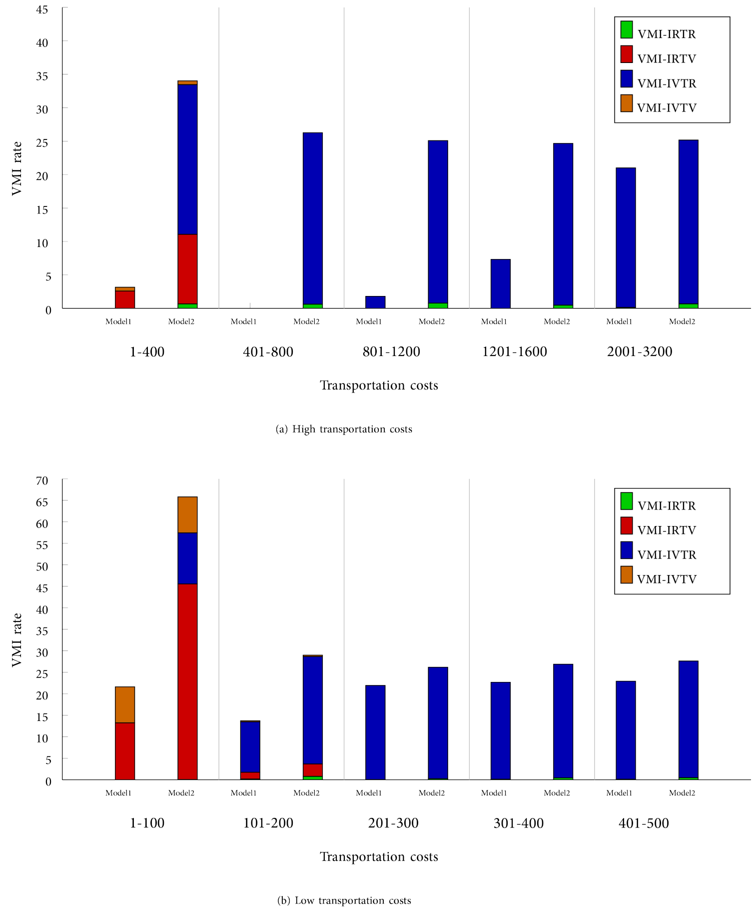
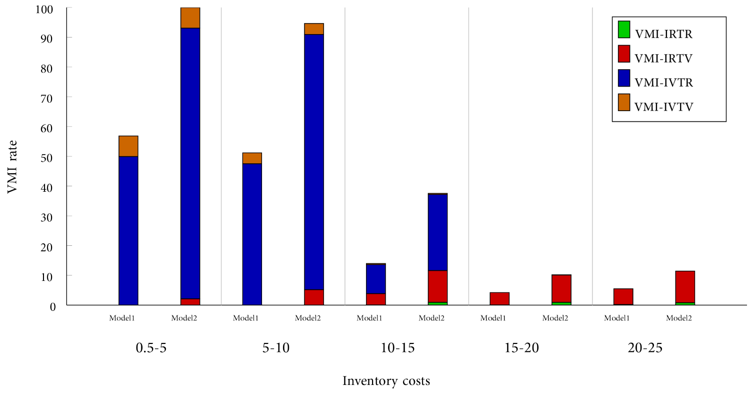
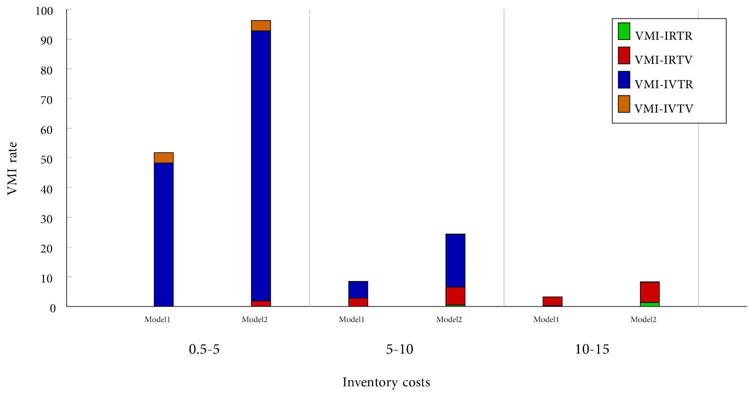


Figure 2.7: Selected VMI contracts from both models with transportation costs
Model1: All-or-Nothing, **Model2**:Best-VMI models

both inventory and transportation expenses. The VMI-IRTR contract was chosen in 0.37% and 1.35% of cases by the two models, respectively. This suggests that, when compared to the traditional VMI contract (VMI-IRTR), RMI often emerges as a more favorable option, as there is limited motivation to adopt VMI under such conditions. These findings underscore the significance of innovative VMI contracts that incorporate cost-sharing mechanisms between partners, which can incentivize and enhance VMI adoption rates. This section presents several key findings derived from the analysis. First, the results indicate that VMI does not consistently outperform RMI, aligning with certain prior studies (e.g., C. C. Lee and Chu 2005). Second, the distribution of inventory and transportation costs between the vendor and retailer plays a critical role in determining both the VMI adoption rate and the specific type of VMI contract selected. Third, demand patternswhether constant, decreasing, increasing, or seasonalwere found to have no significant effect on VMI adoption rates. Fourth, the Best-VMI model demonstrates superior performance over the All-or-Nothing model in terms of VMI implementation rates across all scenarios, irrespective of whether the vendor bears high or low transportation or inventory costs. This advantage stems from the models incorporation of a selection

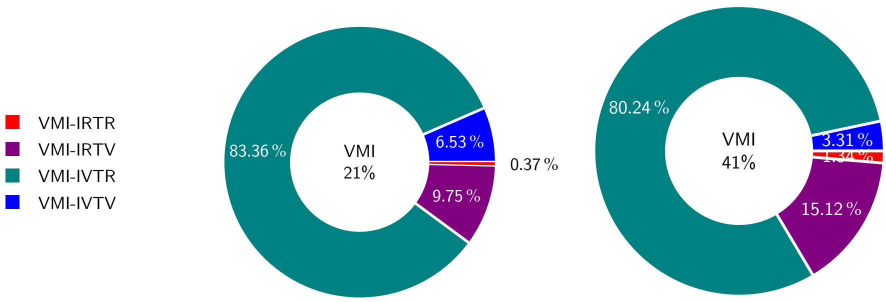


(a) High inventory costs



(b) Low inventory costs

Figure 2.8: Selected VMI contracts from both models with inventory costs
Model1: All-or-Nothing model, **Model2**: Best-VMI model



(a) All-or-Nothing model (b) Best-VMI model

Figure 2.9: Distribution of selected VMI contract types across both models. Labels indicate which party among vendor (V) and retailer (R) bears inventory (I) and transportation (T) costs.

process within its VMI framework. Finally, the traditional VMI contract, represented as VMI-IRTR in this study, exhibited a notably low selection rate compared to innovative VMI contracts, underscoring the importance of these newer agreements in enhancing VMI adoption.

Given that demand patterns did not influence VMI selection, the analysis was extended to explore the effects of demand volumes, demand variability, and intermittent demand on the outcomes.

2.4.2.2 *Evaluation of Models Under Varying Demand Volumes*

This section examines the effects of fluctuating demand volumes on VMI adoption rates for both the All-or-Nothing and Best-VMI models. The analysis utilizes average demand values of $\{3, 10, 25, 50, 250\}$, with demand held constant over the planning horizon. A total of 4,000 instances were generated for each demand volume, as depicted in Figure 2.10. The results reveal a modest rise in VMI adoption rates as demand volumes increase from 3 to 50, followed by a more pronounced surge from 50 to 250, particularly for the Best-VMI model. This trend is primarily observed in two VMI contracts: VMI-IRTR and VMI-IVTR. Higher demand volumes lead to larger lot sizes, which reduce the frequency of setups. This dynamic benefits the vendor by enabling product storage at the retailers facilities, thereby minimizing delivery requirements.

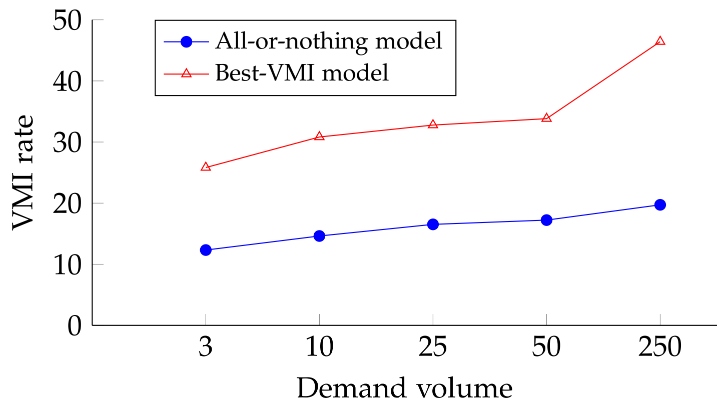


Figure 2.10: VMI rate with changes of demand volume

Given the marginal increase in VMI adoption rates for demand volumes ranging from 3 to 50 under constant demand, the subsequent section will explore the influence of demand variability on the selection of VMI contracts.

2.4.2.3 *Evaluation of Models Under Fluctuating Demand Variability*

This section investigates the influence of demand variability on VMI adoption rates for the All or Nothing and Best VMI models. Demand variability is varied around an average of 50, with levels set at $\pm\{5, 10, 15, 25, 50\}$. The analysis is conducted using 4,000 generated instances for each variability level, as illustrated in Figure 2.11. The results indicate that VMI adoption rates remain stable at low variability levels (5 and 10), rise to a peak at moderate variability (15), and then decline for higher variability levels (25 and 50), eventually stabilizing at a reduced rate.

At a variability level of 15, all VMI contracts are selected more frequently compared to other levels, suggesting that moderate demand fluctuations provide favorable conditions for VMI implementation. This is likely because both models can effectively balance inventory management and transportation decisions under moderate uncertainty. However, as variability increases beyond this point (25 and 50), the complexity of managing stock levels and transportation schedules reduces the efficiency of VMI for both approaches. These findings demonstrate

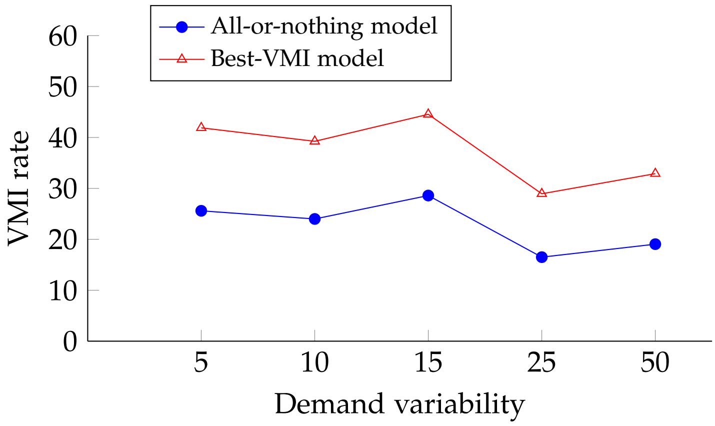


Figure 2.11: VMI rate with changes of demand varibility

that demand variability significantly impacts VMI adoption rates. Building on this analysis, the subsequent section will explore the effects of intermittent demand to further evaluate its influence on VMI contract selection.

2.4.2.4 *Evaluation of Models Under Intermittent Demand Conditions*

Intermittent demand refers to a demand pattern marked by a high proportion of zero values (Tian et al. 2021). In this study, the probability of intermittent demand is varied between 0.1 and 1, as depicted in Figure 2.12. At a probability of 0.1,

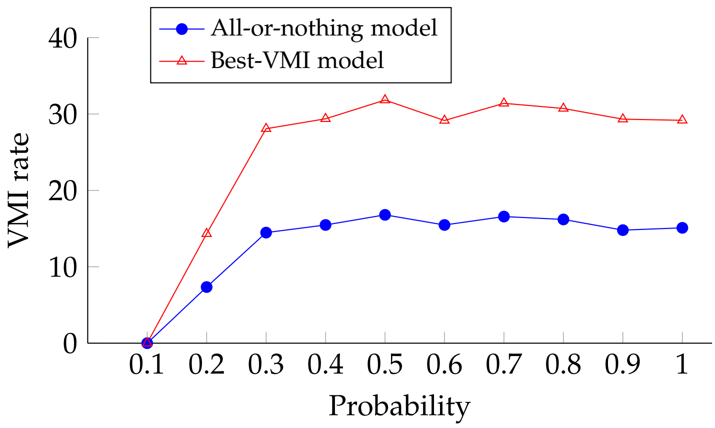


Figure 2.12: VMI rate with intermittent demands

neither model identifies viable solutions for VMI implementation. This is because, within a 12-period planning horizon, a 0.1 probability implies that only one period exhibits non-zero demand, providing insufficient motivation for either party to adopt VMI.

When the probability increases to 0.2, a marginal rise in VMI adoption rates is observed, corresponding to two periods with non-zero demand. Under these conditions, the retailer in the RMI model opts to minimize inventory costs by scheduling deliveries exclusively for periods with non-zero demand, resulting in an increased number of setups. Conversely, the VMI model generates solutions similar to RMI, particularly when non-zero demand periods are widely spaced (a scenario prevalent in most instances), leading to elevated inventory costs. However, when non-zero demand periods are closely clustered, the VMI model enables the vendor to store products either in their own inventory or the retailers, reducing setup frequency and generating cost savings for both parties.

As the probability rises from 0.3 to 1, the VMI adoption rate increases and stabilizes. This suggests that having at least three periods with non-zero demand results in an approximate 30% VMI adoption rate. Beyond a probability of 0.3, intermittent demand ceases to exert a significant influence on VMI adoption rates.

2.5 CONCLUSION AND FUTURE WORKS

This chapter presents a decision-making framework to assist vendors and retailers in evaluating the adoption of VMI or RMI contracts. By developing mathematical models for production and distribution planning, the study introduces two approaches: the All or Nothing and Best VMI models. These models are designed to determine the most suitable contract by evaluating various mechanisms, enabling partners to choose from RMI and four newly proposed VMI contracts. The proposed VMI contracts are grounded in principles of inventory ownership and shared transportation costs, drawing inspiration from INCOTERMS guidelines.

The findings highlight the superior performance of the proposed VMI contracts compared to traditional VMI agreements. Key factors influencing VMI adoption include the distribution of transportation and inventory costs between the vendor and retailer, demand volume, variability, and intermittency. Notably, demand patterns were found to have no significant effect on the selection process.

In practical applications, the All or Nothing and Best VMI models offer a single contract for adoption, which may not always align with the vendors preferences. A more flexible approach, such as a ranking tool, could provide vendors with a prioritized list of contracts, allowing them to select the most appropriate option. Furthermore, the model could be enhanced by incorporating additional conditions, such as sustainability considerations (e.g., remanufacturing) and constraints imposed by vendors and retailers, including payment schedules and penalty costs. Even though our MILP model is straightforward to implement and efficiently evaluates multiple instances within a manageable computational timeframe, the inclusion of additional conditions imposed by the partners necessitates the development of a heuristic or metaheuristic approach. Such methods would enable faster and more effective problem-solving in future studies. Additionally, deterministic models have yielded robust results in this study, stochastic or robust optimization methods may be more suitable for real-world scenarios, ensuring reliable solutions under uncertainty.

3 RANKING TOOL FOR VMI AND RMI CONTRACTS

This chapter introduced a ranking tool designed to evaluate and rank contracts, enabling vendors to consider alternative options beyond the top-ranked choice. Unlike the models presented in the previous chapter, which offer partners only a single recommended contract, this ranking tool provides greater flexibility. The proposed tool has been evaluated under both deterministic and stochastic conditions. Specifically, we developed a two-stage stochastic programming model to handle demand uncertainties and integrated a risk-averse model to address vendor risk preferences.

The remainder of this chapter is structured as follows: Section 3.1 provides an introduction to the chapter. Section 3.2 reviews relevant literature. Section 3.3 outlines the problem and presents the mathematical models utilized to address it. Section 3.4 discusses the outcomes of numerical experiments and computational analyses. Finally, Section 5.6 concludes the chapter and suggests directions for future research.

3.1 INTRODUCTION

Despite advancements in information technology and the growing adoption of VMI, the specific benefits of this approach still require further exploration. Organizations frequently encounter challenges when deciding whether VMI or RMI agreements offer greater cost-effectiveness under their particular conditions. As discussed in the preceding chapter, a decision-support tool can help partners select the most beneficial inventory management strategy. However, the All-or-nothing and Best-VMI models limit flexibility by recommending only a single option, often failing to account for vendors' and retailers' diverse preferences and operational constraints.

To address these limitations, this chapter proposes a comparative ranking tool that enables vendors to systematically evaluate different VMI agreements, as presented earlier, as well as RMI options. Our study introduces an innovative ranking tool that facilitates the exploration of alternative agreements beyond the top-ranked option, allowing consideration of second or third-ranked choices that may better align with their specific needs. Additionally, the framework incorporates models for both demand scenarios, with an added consideration of the vendors varying degrees of risk aversion to provide a practical and robust solution.

3.2 LITERATURE REVIEW

As discussed in Chapter 2 (Section 2.2), existing literature emphasizes the importance of coordination and integration in supply chain management to reduce inefficiencies, particularly those arising from the bullwhip effect (Krajewski and Malhotra 2022; Chopra and Meindl 2013). VMI is one widely adopted strategy for improving coordination, with studies frequently highlighting its potential for efficiency gains (Andel 1996). Various VMI contracts have been explored, notably inventory ownership arrangements like vendor-managed consignment inventory (Bichescu and Fry 2009; Gümü et al. 2008). However, most prior research on VMI has primarily examined overall supply chain cost reductions, with limited attention to individual partner impacts over time (Dong and Xu 2002). Additionally, the effectiveness of VMI compared to RMI is context-dependent, with some studies indicating scenarios where RMI may perform better, particularly in simplified settings such as newsvendor or deterministic EOQ models (C. C. Lee and Chu 2005; Pasandideh et al. 2010).

Given this uncertainty, Chapter 2 introduces a decision-making tool designed to help vendors and retailers select the most suitable option among four VMI contracts and RMI. However, these tools typically recommend only the single most cost-effective solution, which may not adequately address the diverse preferences and operational constraints of supply chain partners. This chapter extends the previous discussion by presenting a ranking tool. The proposed tool allows vendors to systematically evaluate multiple contract options, including second or third-ranked choices that might better meet their specific needs. The framework accommodates both deterministic and stochastic demand conditions and incorporates different levels of vendor risk aversion, providing a flexible and practical approach.

3.3 PROBLEM DESCRIPTION AND FORMULATIONS

This section introduces the "Ranking Tool" methodology under both deterministic and stochastic demand conditions. Additionally, a risk-averse two-stage stochastic programming framework is integrated to account for uncertainty in decision-making.

The supply chain under consideration consists of two partners: the vendor and the retailer. The sets, parameters, and decision variables remain consistent with those defined in Chapter 2, Section 2.3. The primary objective of the ranking tool is to evaluate and compare different contracts based on total and individual costs, offering a structured approach for vendors to assess their options beyond purely quantitative factors. This tool provides a more flexible decision-making process, as a vendor may find a second-ranked contract preferable due to reduced responsibilities, such as inventory management or transportation logistics.

The ranking tool comprises six components: the RMI model, four VMI models corresponding to different contracts (defined in Chapter 2) structures, and the ranking mechanism. The VMI models account for different cost allocation strategies, as follows:

1. VMI-IVTV: The vendor assumes responsibility for both inventory holding and transportation costs.
2. VMI-IVTR: The vendor covers inventory holding costs, while the retailer manages transportation expenses.
3. VMI-IRTV: The vendor is responsible for transportation costs, while the retailer bears inventory holding costs.
4. VMI-IRTR: The retailer takes full responsibility for both inventory and transportation costs.

The RMI model for the retailer is as follows:

$$[\mathbf{RMI}-\mathbf{R}] \quad \min R_{RMI} = \sum_{t\in\mathcal{T}} h_t^{r(r)} I_t^{r(r)} + \sum_{t\in\mathcal{T}} C_t^r v_t^r \tag{3.1}$$

$$I_t^{r(r)} = I_{t-1}^{r(r)} + q_t - D_t, \quad \forall t \in \mathcal{T} \tag{3.2}$$

$$I_t^{r(r)} \leq U, \quad \forall t \in \mathcal{T} \tag{3.3}$$

$$q_t \leq M \times v_t^r, \quad \forall t \in \mathcal{T} \tag{3.4}$$

$$v_t^r \in \{0,1\}, \quad \forall t \in \mathcal{T} \tag{3.5}$$

$$q_t, I_t^{r(r)} \geq 0, \quad \forall t \in \mathcal{T} \tag{3.6}$$

The aim of the objective function (3.1) is to reduce the retailer's expenses related to inventory management and transportation. The inventory balance conditions at the retailer are represented by constraints (3.2). The notation $I_t^{r(r)}$ is used to emphasize that the retailer manages its own inventory, with the initial inventory level assumed to be zero ($I_0^{r(r)} = 0$). Constraints (3.3) enforce the retailer's inventory capacity limits. Constraints (3.4) connect the continuous order variables q_t to the binary delivery variables v_t^r, ensuring that transportation costs are only incurred when there is a non-zero order in period t. Here, the big M is defined as the total

demand across the planning horizon, T. Lastly, constraints (3.5) and (3.6) impose integrality and non-negativity requirements on the relevant variables.
The retailer model determines the shipment quantity q_t, which serves as input for the vendor model ($\hat{q}_t$). The RMI model for the vendor is as follows:

$$[\mathbf{RMI-V}]\ \min V_{RMI} = \sum_{t\in\mathcal{T}} P_t x_t + \sum_{t\in\mathcal{T}} S_t y_t + \sum_{t\in\mathcal{T}} h_t^v I_t^v \tag{3.7}$$

s.t.

$$I_t^v = I_{t-1}^v + x_t - \hat{q}_t, \quad \forall t \in \mathcal{T} \tag{3.8}$$

$$x_t \leq y_t \sum_{t'\in\mathcal{T}} \hat{q}_{t'}, \quad \forall t \in \mathcal{T} \tag{3.9}$$

$$y_t \in 0,1, \quad \forall t \in \mathcal{T} \tag{3.10}$$

$$x_t, I_t^v \geq 0, \quad \forall t \in \mathcal{T} \tag{3.11}$$

The objective function (3.7) is designed to minimize the vendor's overall costs, encompassing production, setup, and inventory-related expenses. The inventory balance conditions at the retailer are governed by constraints (3.8). Constraints (3.9) establish a relationship between the continuous production variables x_t and the binary setup variables y_t, ensuring that a setup cost is incurred only when there is a positive production quantity in period t. Additionally, constraints (3.10) and (3.11) enforce integrality and non-negativity conditions on the respective decision variables.

The VMI-IVTV model is presented below:

$$[\mathbf{VMI-IVTV}] \quad \min \sum_{t\in\mathcal{T}} \left(P_t x_t + S_t y_t + h_t^v I_t^v + h_t^{r(v)} I_t^{r(v)} + C_t^v v_t^v \right) \tag{3.12}$$

Subject to Constraints, (2.16), (2.21), (2.25), (2.27), and

$$I_t^{r(v)} = I_{t-1}^{r(v)} + q_t - D_t, \quad \forall t \in \mathcal{T} \tag{3.13}$$

$$I_t^{r(v)} \leq U, \quad \forall t \in \mathcal{T} \tag{3.14}$$

$$q_t \leq M \times v_t^v, \quad \forall t \in \mathcal{T} \tag{3.15}$$

In this VMI-IVTV model, the objective function (3.12) minimizes the inventory costs at the retailer and transportation costs paid by the vendor. Constraints (3.13) are inventory balance equations for the retailer. Constraints (3.14) impose a limit on the level of inventory at the retailer. Constraints (3.15) link the continuous variables q_t to the binary variables v_t^v.

Model VMI-IVTR is obtained by replacing C_t^v with C_t^r and v_t^v with v_t^r in the VMI-IVTV model. The VMI-IRTV model is obtained by replacing $h_t^{r(v)}$ and $I_t^{r(v)}$ with $h_t^{r(r)}$ and $I_t^{r(r)}$, respectively in the VMI-IVTV model. Finally, model VMI-IRTR replaces $h_t^{r(v)}$, $I_t^{r(v)}$, v_t^v, and C_t^v with $h_t^{r(r)}$, $I_t^{r(r)}$, v_t^r, and C_t^r, respectively, in the VMI-IVTV model.

3.3.1 *Ranking Process*

The ranking procedure evaluates each VMI contract i ($i \in \{VMI-IVTV, VMI-IRTV, VMI-IVTR, VMI-IRTR\}$) by comparing the total supply chain cost under VMI (SC_i^{VMI}) with the total cost under RMI (SC^{RMI}). Note that SC_i^{VMI} and SC^{RMI} are determined using equations (2.15) and (2.12), respectively. If the VMI contract yields a lower total cost than RMI, the next step involves analyzing the individual cost savings for both the vendor and the retailer. This analysis compares the vendor's costs under RMI and VMI (V^{RMI} and V_i^{VMI}) as well as the retailer's costs under RMI and VMI (R^{RMI} and R_i^{VMI}).

A VMI contract is categorized as "Better" if both the vendor and the retailer achieve cost reductions relative to RMI. Conversely, if this condition is not met, the contract is classified as "Worse".

After evaluating all VMI contracts, the ranking process proceeds as follows: First, contracts in the "Better" category are sorted in ascending order of total cost. Next, the RMI contract is included in the ranking. Finally, contracts in the "Worse" category are also ranked in ascending order of total cost. The detailed steps of this process are presented in Algorithm 3.1.

3.3.2 *Two-Stage Stochastic Programming Models*

In real-world supply chain operations, demand uncertainty arises from factors such as market competition and weather conditions, making it a critical consideration in planning. To address this uncertainty in the VMI contract selection problem, we employ a two-stage stochastic programming framework. In this model, demand is represented as a random variable with a finite set of possible realization $\omega = 1, \ldots, \Omega$, called scenarios. Each scenario assigned a probability ρ_ω, where $\rho_\omega > 0$ and $\sum_{\omega=1}^{\Omega} \rho_\omega = 1$.

The two-stage stochastic programming approach introduces recourse decisions, dividing decision variables into two categories: first-stage and second-stage variables. First-stage decisions are made prior to the realization of uncertain demand,

Algorithm 3.1 Ranking process

Input: $R^{RMI}, R_i^{VMI}, V^{RMI}, V_i^{VMI}, SC^{RMI}, SC_i^{VMI}, Better = \emptyset, Worse = \emptyset$
Output: *Ranking*
for $i \in$ Number of VMI contracts **do**
  if $SC_i^{VMI} < SC^{RMI}$ **then**
    if $V_i^{VMI} \leq V^{RMI}$ AND $R_i^{VMI} \leq R^{RMI}$ **then**
      $VMI_i \rightarrow Better$
    else
      $VMI_i \rightarrow Worse$
    end if
  else
    $VMI_i \rightarrow Worse$
  end if
end for
Rank the *Better* set contract in ascending order based on SC_i^{VMI}
Add RMI contract to the end of *Better* set
Rank the *Worse* set in ascending order based on SC_i^{VMI}
Concatenate the *Better* set with the *Worse* set

based on limited information about future conditions. Once a specific demand scenario ω is observed, second-stage decisions are implemented as adaptive measures to optimize the system's performance (Birge and Louveaux 2011).

In the proposed decision-making framework, first-stage variables represent setup and delivery decisions made prior to demand realization. Second-stage variables include inventory levels at both the vendor and retailer, as well as backorder quantities. Due to demand uncertainty, unmet demand may arise, necessitating the inclusion of backorder variables to capture shortages (Adulyasak et al. 2015a).

The objective of the two-stage stochastic programming model is to identify an optimal first-stage decision that performs robustly across all possible demand scenarios. To this end, penalty costs for backorders are introduced: σ_t^v denotes the penalty per unit of backordered quantity $e_{t\omega}^v$ at the vendor in period t, and σ_t^r represents the penalty per unit of backordered quantity $e_{t\omega}^r$ at the retailer. By incorporating these stochastic elements, the model aims to optimize decision-making under demand uncertainty.

$$[\mathbf{Sto-RMI-R}] \quad \min \sum_{t \in \mathcal{T}} (C_t^r v_t^r + \sum_{\omega \in \Omega} \rho_\omega (h_t^r I_{t\omega}^{r(r)} + \sigma_t^r e_{t\omega}^r)) \tag{3.16}$$

$$s.t.$$

$$I_{t\omega}^{r(r)} + e_{t-1,\omega}^r = I_{t-1,\omega}^{r(r)} + e_{t,\omega}^r + q_{t\omega} - D_{t\omega} \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.17}$$

$$I_{t\omega}^{r(r)} \leq U \qquad \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.18}$$

$$q_{t\omega} \leq M \times v_t^r \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.19}$$

$$v_t^r \in \{0,1\} \quad \forall\, t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.20}$$

$$q_{t\omega}, I_{t\omega}^{r(r)}, e_{t,\omega}^r \geq 0 \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.21}$$

The objective function (3.16) is designed to minimize the expected total cost, incorporating first-stage transportation costs and second-stage expenses associated with inventory holding and backorders. Constraints (3.17) to (3.21) retain the same structure as their deterministic model but are applied across all demand scenarios ω.

$$[\mathbf{Sto-RMI-V}] \quad \min \sum_{t\in\mathcal{T}} (S_t y_t + \sum_{\omega\in\Omega} \rho_\omega (P_t x_{t\omega} + h_t^v I_{t\omega}^v + \sigma_t^v e_{t\omega}^v)) \tag{3.22}$$

$s.t.$

$$I_{t\omega}^v + e_{t-1,\omega}^v = I_{t-1,\omega}^v + e_{t\omega}^v + x_{t\omega} - \hat{q}_{t\omega}^r \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.23}$$

$$x_{t\omega} \leq y_t \sum_{t'=t}^{T} \hat{q}_{t',\omega}^r \quad \forall\, t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.24}$$

$$y_t \in \{0,1\} \quad \forall\, t \in \mathcal{T} \tag{3.25}$$

$$x_{t\omega}, I_{t,\omega}^v, e_{t,\omega}^v \geq 0 \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.26}$$

The objective function (3.22) is designed to minimize the expected total cost, incorporating first-stage setup costs and second-stage expenses related to production, inventory, and backorder. Constraints (3.23) to (3.26) retain the same structure as their deterministic equivalents, but are applied in all demand scenarios ω.

$$[\mathbf{Sto-VMI-IVTV}] \quad \min \sum_{t\in\mathcal{T}} (S_t y_t + C_t^v v_t^v) + \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega} \rho_\omega \left(P_t x_{t\omega} + h_t^v I_{t\omega}^v + h_t^{r(v)} I_{t\omega}^{r(v)} + \sigma_t^v e_{t\omega} \right) \tag{3.27}$$

s. t.

$$I_{t\omega}^v = I_{t-1\omega}^v + \ x_{t\omega} - q_{t\omega} \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.28}$$

$$I_{t\omega}^{r(v)} + e_{t-1,\omega} = I_{t-1\omega}^{r(v)} + e_{t\omega} + q_{t\omega} - D_{t\omega} \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.29}$$

$$I_{t\omega}^{r(v)} \leq U \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.30}$$

$$x_{t\omega} \leq y_t \sum_{t'=t}^{T} D_{t'\omega} \forall\, t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.31}$$

$$q_{t\omega} \leq M \times v_t^v \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.32}$$

$$y_t, v_t^v \in \{0,1\} \forall\, t \in \mathcal{T} \tag{3.33}$$

$$x_{t\omega}, I_{t\omega}^v, I_{t\omega}^{r(v)}, q_{t\omega} \geq 0 \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{3.34}$$

The objective function (3.27) aims to minimize the total expected cost, which includes both first-stage and second-stage costs. First-stage costs are related to setup and transportation costs borne by the vendor. Second-stage costs include production costs, vendor inventory costs, backorder costs, and retailer inventory costs covered by the vendor. Constraints (3.28)-(3.34) correspond to their deterministic versions but must be satisfied for every scenario ω. We obtain the Sto-VMI-IVTR, Sto-VMI-IRTV, and Sto-VMI-IRTR models by applying to the Sto-VMI-IVTV the same transformations operated on their deterministic counterparts. Hence, to obtain model Sto-VMI-IVTR, C_t^v and v_t^v are replaced with C_t^r and v_t^r, respectively. The Sto-VMI-IRTV model is obtained by replacing $h_t^{r(v)}$ and $I_t^{r(v)}$ with $h_t^{r(r)}$ and $I_t^{r(r)}$, respectively. Finally, model Sto-VMI-IRTR replaces $h_t^{r(v)}$, $I_t^{r(v)}$, v_t^v, and C_t^v with $h_t^{r(r)}$, $I_t^{r(r)}$, v_t^r, and C_t^r, respectively.

3.3.3 *Risk-Averse Two-Stage Stochastic Models*

Risk-averse stochastic models focus on minimizing outcome variability across scenarios by incorporating a variable that enhances model robustness (Macedo et al. 2016, Boutarfa et al. 2024). This approach integrates risk deviation measures into the objective function to evaluate fluctuations in second-stage variables. One commonly used metric is the upper partial mean, which quantifies the expected value of positive deviations (Mulvey et al. 1995).

To model this variability, a new decision variable, δ_ω, is introduced to measure the deviation between the total cost in scenario ω and the expected total cost. This variable is incorporated into the constraints and penalized in the objective function using a risk-aversion parameter $\lambda > 0$, which balances expected cost and risk. A similar approach has been successfully applied in Macedo et al. (2016) for lot-sizing problems with remanufacturing, showcasing its effectiveness in reducing sensitivity to scenario variations and mitigating risks associated with second-stage cost fluctuations.

The risk-averse formulations for both RMI and VMI models are outlined below.

In the case of the RMI model, risk aversion is considered only for the vendor's perspective.

$$[\mathbf{Sto}\breve{} - \mathbf{RMI} - \mathbf{V}] \quad \min \sum_{t \in \mathcal{T}} \left(S_t y_t + \sum_{\omega \in \Omega} \rho_\omega (P_t x_{t\omega} + h_t^v I_{t\omega}^v + \sigma_t^v e_{t\omega}^v) \right) + \lambda \sum_{\omega \in \Omega} \rho_\omega \delta_\omega \tag{3.35}$$

Subject to constraints (3.23) - (3.26), and:

$$\delta_\omega \geq \sum_{t \in \mathcal{T}} (P_t x_{t\omega} + h_t^v I_{t\omega}^v + \sigma_t^v e_{t\omega}^v) - \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega} \rho_\omega \left(P_t x_{t\omega} + h_t^v I_{t\omega}^v + \sigma_t^v e_{t\omega}^v \right), \quad \forall \omega \in \Omega \tag{3.36}$$

$$\delta_\omega \geq 0, \quad \forall \omega \in \Omega \tag{3.37}$$

$$[\mathbf{Sto}\breve{} - \mathbf{VMI} - \mathbf{IVTV}] \quad \min \sum_{t \in \mathcal{T}} (S_t y_t + C_t^v v_t^v + \sum_{\omega \in \Omega} \rho_\omega (P_t x_{t\omega} + h_t^v I_{t\omega}^v + h_t^{r(v)} I_{t\omega}^{r(v)} + \sigma_t^v e_{t\omega})) + \lambda \sum_{\omega \in \Omega} \rho_\omega \delta_\omega \tag{3.38}$$

Subject to constraints (3.28) - (3.34), and:

$$\delta_\omega \geq \sum_{t \in \mathcal{T}} \left(P_t x_{t\omega} + h_t^v I_{t\omega}^v + h_t^{r(v)} I_{t\omega}^{r(v)} + \sigma_t^v e_{t\omega} \right) - \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega} \rho_\omega \left(P_t x_{t\omega} + h^v t I t \omega^v + h_t^{r(v)} I_{t\omega}^{r(v)} + \sigma_t^v e_{t\omega} \right), \quad \forall \omega \in \Omega \tag{3.39}$$

$$\delta_\omega \geq 0, \quad \forall \omega \in \Omega \tag{3.40}$$

The objective functions (3.35) and (3.38) aim to minimize the first-stage cost, the second-stage cost, and the associated risk. They address two conflicting goals: minimizing the expected total cost and minimizing risk. These objectives are balanced using the values of λ. When $\lambda = 0$, it indicates that the decision-maker is risk-free. Conversely, as λ increases, the decision-maker becomes more risk-averse, showing a greater willingness to accept a higher expected total cost in exchange for solutions with reduced variability. Constraints (3.36) and (3.39) specify the upper partial mean deviation relative to the expected total second-stage cost, a measure similar to the standard deviation across scenarios. By definition, this deviation measure is non-negative, as indicated in constraints (3.37) and (3.40).

Models Sto$_\lambda$-VMI-IVTR, Sto$_\lambda$-VMI-IRTV, and Sto$_\lambda$-VMI-IRTR are obtained from $Sto_\lambda - VMI - IVTV$ in the same manner presented in Section 3.3 for the deterministic models and Section 3.3.2 for the stochastic models.

3.4 NUMERICAL EXPERIMENTS

This section examines the influence of demand variations under deterministic and stochastic scenarios. In the deterministic case, two numerical examples are analyzed to evaluate the effectiveness of the ranking tool. For the stochastic case, the models within the ranking tool are assessed by considering demand uncertainty, with a focus on measuring the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS). These metrics quantify the benefits of perfect information and the advantages of integrating uncertainty into the model, respectively. Additionally, the study explores how the models adjust to varying levels of risk aversion in uncertain environments, offering insights into robust decision-making strategies.

The mathematical formulations were implemented in Python and solved using CPLEX Solver version 12.8. Computational experiments were performed on a system equipped with an AMD Ryzen 7 3700U processor (2.30 GHz), running a 64-bit Windows 10 operating system with 8 GB of RAM. To ensure computational feasibility, the runtime for each model was restricted at 3600 seconds.

3.4.1 *Data Generation*

The instances are categorized into two groups based on demand characteristics: deterministic and stochastic. For deterministic demand scenarios, the data corresponding to each numerical example are presented in the subsequent section. The stochastic demand data are derived from the methodology outlined in Macedo et al. (2016). The model parameters are defined as follows:

- Production cost: P_t is uniformly distributed in the range $[3, 5]$.
- Setup cost: S_t follows a uniform distribution between $[200, 2000]$.
- Vendor's inventory holding cost: h_t^v is uniformly sampled from $[1, 5]$.
- Retailer's maximum inventory level: U is calculated as uniform$[2, 6] \times \bar{D}$, where $\bar{D}$ represents the average demand across all periods and scenarios, computed as $\bar{D} = \left(\sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega} D_{t\omega}\right) / |T|$.
- Retailer's inventory holding cost (vendor-paid): $h_t^{r(v)}$ is uniformly distributed in $[1, 5]$.

- Retailer's inventory holding cost (retailer-paid): $h_t^{r(r)}$ is also uniformly sampled from $[1,5]$.
- Vendor's transportation cost: C_t^v is uniformly distributed in $[1,1000]$.
- Retailer's transportation cost: C_t^r is uniformly sampled from $[1,1000]$.
- Vendor's penalty cost: σ_t^v is determined as uniform$[2,10] \times P$.
- Retailer's penalty cost: σ_t^r is calculated as uniform$[2,10] \times P_t$.
- Number of scenarios: $\Omega = 20$, with each scenario assigned an equal probability of $\rho_\omega = 0.05$.
- Demand generation: Demand values are randomly generated from a uniform distribution in the range $[1,1000]$.
- Risk-aversion parameter: For the risk-averse models, the parameter λ is varied from 1 to 10 in increments of 1

3.4.1.1 *Evaluation of Decision-Making Tool Rankings Under Deterministic Demand*

This section evaluates the ranking results generated by the decision-making tool through illustrative example scenarios.

i) SCENARIO 1: Consider a three-period example with demand levels of [150, 200, 170]. Production cost is established at 20, setup cost at 2000, and the vendor's inventory holding cost at 7. The retailer incurs an inventory holding cost of 4 if borne directly, and 3 if covered by the vendor. Inventory capacity is restricted to 300 units. Transportation expenses amount to 1000 when paid by the retailer and 600 when covered by the vendor.

Findings: The decision-making tool ranks the contractual options in the following order: 1) **VMI-IRTV**, 2) **VMI-IVTR**, 3) **RMI**, 4) **VMI-IVTV**, 5) **VMI-IRTR**.

In this particular scenario, the VMI-IRTV contract is identified as the optimal solution primarily due to its cost-sharing structure. As presented in Table 3.1, although VMI-IRTV does not yield the absolute lowest total cost (achieved by VMI-IVTV), it significantly reduces individual expenses for both the vendor and the retailer. When the vendor agrees to cover transportation or inventory-related costs, both parties benefit from cost savings compared to the RMI arrangement. Conversely, if the vendor opts not to cover additional costs, the RMI model may ultimately represent the most beneficial option for both entities.

ii) SCENARIO 2: Consider a three-period scenario with demand values of [100, 150, 200]. The production cost is fixed at 30, the setup cost at 3000, and the vendor's

Table 3.1: Contracts' costs of Example 1

	IRTR	IRTV	IVTR	IVTV	RMI
Vendor's cost	12890	14090	14300	15500	14400
Retailer's cost	3880	1880	2000	0	2680
Total cost	16770	15970	16300	15500	17080

inventory holding cost at 7. The retailer faces an inventory holding cost of 4 if incurred directly and 9 if borne by the vendor. The inventory capacity constraint is set to 200 units. Transportation costs are consistently set at 1000, regardless of whether the retailer or the vendor covers them.

Findings: The decision-making tool ranks the contractual alternatives as follows: 1) **RMI**, 2) **VMI-IRTR**, 3) **VMI-IRTV**, 4) **VMI-IVTR**, 5) **VMI-IVTV**.

In this scenario, the RMI contract appears as the optimal choice, implying limited cost advantages from adopting VMI contracts for either partner. According to Table 3.2, while the RMI contract does not yield the absolute lowest total costs (a feature shared by VMI-IRTR and VMI-IRTV), it remains the most advantageous option overall. Nevertheless, if the partners opt for a VMI arrangement, the following outcomes are observed:

- If the vendor accepts no additional costs, total costs remain equivalent to RMI, resulting in no cost-saving advantage.
- If the vendor bears transportation expenses, the vendor incurs higher costs, but the overall supply chain costs remain equal to RMI.
- If the vendor covers both inventory and transportation expenses, the vendor's costs rise, although the total supply chain costs are reduced compared to RMI.
- If the vendor takes responsibility for inventory costs, vendor expenses increase, and the total supply chain costs remain lower than under RMI.

Table 3.2: Contracts' costs of Example 2

	IRTR	IRTV	IVTR	IVTV	RMI
Vendor's cost	19300	21300	20650	22650	19300
Retailer's cost	2600	600	2000	0	2600
Total cost	21900	21900	22650	22650	21900

3.4.2 *Two-stage Stochastic Models Results*

This section outlines the findings from two-stage stochastic models, structured into three key components. First, we evaluate the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS). Following this, we explore the model's performance under different degrees of risk aversion. Lastly, an illustrative example of the decision-making tool is presented.

3.4.2.1 *Expected Value of Perfect Information and the Value of Stochastic Solution*

Stochastic programming techniques are valuable for addressing problems influenced by uncertain parameters, such as fluctuating demand driven by unpredictable market conditions (Birge and Louveaux 2011). Among these techniques, the two-stage stochastic model is one of the most widely adopted frameworks (Cunha et al. 2017). In the proposed two-stage models, first-stage decisionssuch as setups and transportation plansare determined as optimal choices, independent of demand variations. This approach raises two critical questions:

1. How essential is it to incorporate uncertainty through scenario-based modeling?

2. Can simpler deterministic methods effectively approximate the stochastic solution?

To address these questions, two key metrics are employed: the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS). These metrics assess the influence of uncertainty on decision-making outcomes.

- EVPI quantifies the potential cost savings achievable if all uncertainties are resolved. It is computed as the difference between the "Wait-and-See (WS)" solutionobtained by solving deterministic problems for each scenario $\omega \in \Omega$ and averaging the expected costsand the "here-and-now" solution from the Recourse Problem (RP). A low EVPI suggests that randomness has minimal impact, making deterministic approaches adequate. Conversely, a high EVPI indicates that uncertainty significantly affects decisions and must be explicitly modeled.

- VSS measures the cost of disregarding uncertainty. It is calculated by first solving the "expected value (EV)" problem, where random variables are replaced by their mean values. The "expected value of the wait-and-see solutions (EEV)" is then determined by solving each scenario $\omega \in \Omega$ while fixing the first-stage variables from the EV problem. The VSS is the difference between EEV and RP. A small VSS implies that deterministic methods provide a reasonable approximation, whereas a large VSS underscores the importance of accounting for uncertainty.

For a comprehensive discussion of EVPI and VSS, readers are directed to Birge and Louveaux (2011). Figure 5.5 depicts the EVPI and VSS values for different contracts (VMI-IVTV, VMI-IRTV, VMI-IVTR, VMI-IRTR, and RMI). The results demonstrate that access to perfect information could lead to cost savings exceeding 3,000 monetary unitsapproximately 5% of the total stochastic costacross all models. This underscores the critical role of uncertainty in shaping decision-making processes. Additionally, EVPI quantifies the potential expense a decision-maker would bear to acquire perfect information, with higher values emphasizing the benefits of employing a stochastic framework.

The VSS results further confirm this insight. Employing deterministic approximations increases costs by 10% for VMI contracts and 62% for the RMI contract. From a managerial standpoint, adopting a stochastic approach, especially for RMI, introduces a "cost of uncertainty." While this may raise expenses, it ensures feasible and robust decisions. In contrast, deterministic models risk producing infeasible production plans when faced with future uncertainties. In conclusion, the expected value (EV) problem does not serve as a suitable substitute for the stochastic problem; it is crucial to account for all scenarios concurrently.

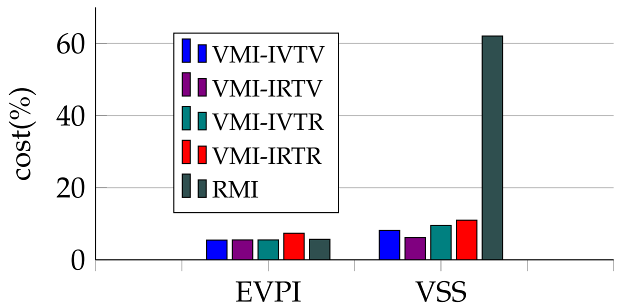


Figure 3.1: The Expected Value of Perfect Information and the Value of Stochastic Solution for the contracts

3.4.2.2 *Risk-Averse Model Results*

This section investigates the effects of the risk-aversion parameter λ, which modulates the compromise between expected costs and risk considerations, on several stochastic models characterized by risk aversion. Specifically, we examine the following models: $[Sto_\lambda - RMI - V]$, $[Sto_\lambda - VMI - IVTV]$, $[Sto_\lambda - VMI - IRTV]$, $[Sto_\lambda - VMI - IVTR]$, and $[Sto_\lambda - VMI - IRTR]$, as initially described in Section 3.3. Preliminary analyses demonstrate that by adjusting the risk-aversion parameter λ within a defined interval (0 to 10), it is possible to effectively balance the trade-off between reducing expected cost variations and limiting increases in total costs across both VMI and RMI frameworks.

Figures 3.2 (a) and (b) provide a visual representation of how expected costs change relative to the baseline scenario, defined at $\lambda = 0$:

$$CostRatio(\lambda) = \frac{Cost(\lambda)}{Cost(\lambda = 0)}$$

Furthermore, these figures illustrate the corresponding decrease in expected deviation, computed as the deviation at a specified λ relative to the baseline deviation at $\lambda = 0$:

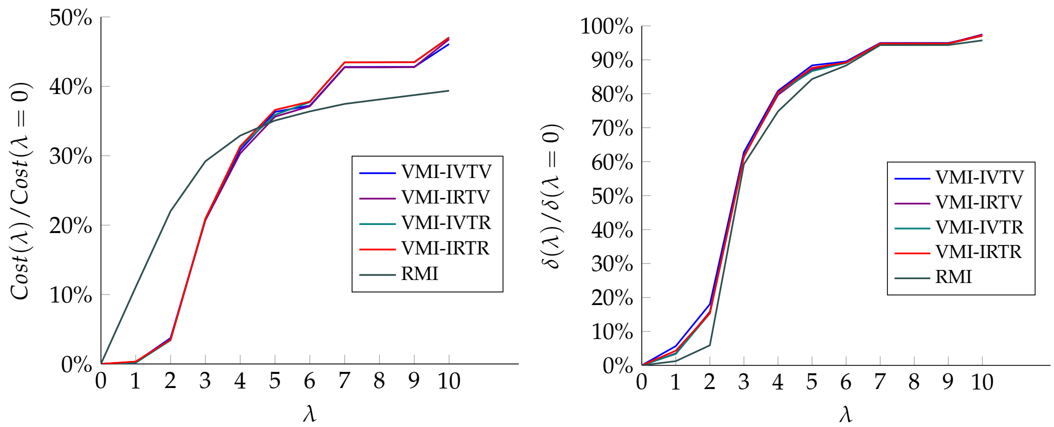
$$DeviationRatio(\lambda) = \frac{\delta(\lambda)}{\delta(\lambda = 0)}$$

As illustrated in Figure 3.2 (a) and (b), considerable reductions in risk can be accomplished with relatively minor increases in expected costs. For instance, in VMI models, assigning $\lambda = 1$ achieves risk reductions of approximately 35% compared to $\lambda = 0$, while the total cost increase remains minimal, between 0.170.34%. Starting from $\lambda = 7$, this trend becomes increasingly evident across all models, where substantial risk reduction corresponds to modest increments in expected total costs. Between $\lambda = 7$ and $\lambda = 10$, risk is reduced by over 95%, accompanied by an increase in total costs of about 37% for the RMI model and 42% for the VMI models.

Figure 3.2 reveals consistent patterns across all models, indicating that increased risk aversion correlates with rising expected costs and notable decreases in deviation. Decision-makers who become more risk-averse will consequently face higher expected total costs while significantly reducing uncertainty.

The total expected costs across varying degrees of risk aversion are depicted in Figure 3.3. When the risk aversion parameter λ is set between 0 and 6, the VMI models consistently yield the lowest total costs, positioning them as preferable solutions for supply chain participants. However, at higher levels of risk aversion ($\lambda = 7$ to $\lambda = 10$), the RMI model demonstrates slightly superior cost efficiency compared to VMI contracts. Practically, this indicates that vendors with greater risk aversion may favor the RMI model to achieve lower overall costs.

The comparative cost disadvantage observed in the RMI model relative to VMI contracts is primarily due to Equation (3.36). This equation calculates deviation costs considering only the vendor's production, inventory, and backorder expenses, resulting in lower costs than the corresponding calculations for VMI contracts represented by Equations (3.39), which additionally incorporate the retailers inventory costs.



(a) Increasing of the Expected Cost (b) Reduction of the Expected Deviation

Figure 3.2: Expected Cost and Expected Deviation evolution vs varying λ

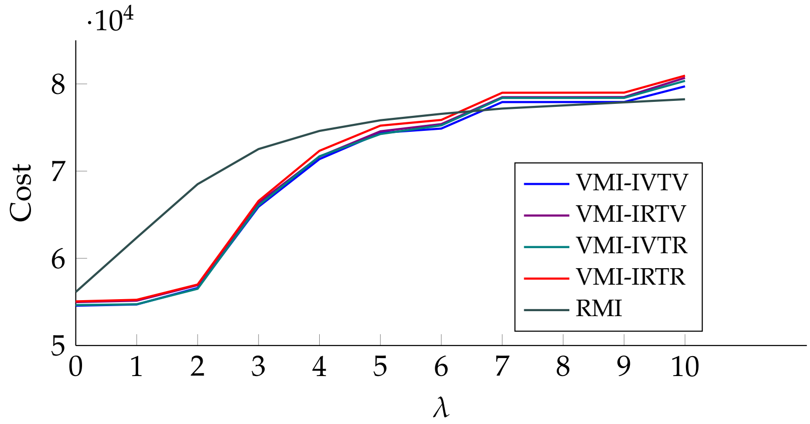


Figure 3.3: Average Expected Cost

3.4.2.3 *Numerical Example of the Ranking Tool under Stochastic Demand*

This section provides illustrative numerical examples to assess the performance of the ranking tool across multiple risk scenarios under stochastic demand conditions.

We consider a planning horizon consisting of 12 periods. The unit production cost is set at 4, and the setup cost is 1300. The vendor incurs an inventory holding cost of 1.5 per unit, while the retailer's inventory holding cost is 1 per unit when borne directly and 3.5 per unit if assumed by the vendor. Inventory capacity is limited to 3000 units. Transportation costs incurred by the retailer are 1100, which decrease to 825 if covered by the vendor. Demand values are generated randomly based on a uniform distribution ranging between 400 and 800 units. Additionally, a penalty cost of 39 per unit is charged for any unmet demand.

The numerical analysis is conducted under multiple risk-aversion parameters: 0 (risk-neutral), 1, 3, and 9.

RESULTS In this numerical example, the optimal contract choice varies with the vendor's risk aversion level, as summarized in Table 3.3. The table illustrates the relationship between different risk levels and the corresponding rankings, total costs, vendor costs, and retailer costs for each contractual arrangement. At a risk-neutral position (risk level 0), the optimal contract identified is RMI, even though it does not yield the lowest total cost. This preference arises due to significantly higher costs imposed on one partner when choosing certain VMI contracts. Nevertheless, adopting a VMI arrangement with lower total costs is possible if either the vendor assumes additional costs under the VMI-IRTV contract or if the retailer is prepared to bear greater expenses with the VMI-IRTR contract.

When the vendor adopts moderate risk aversion (risk levels 1 and 3), transitioning to a VMI contract becomes beneficial for both parties, although the specific optimal contract varies. At risk level 1, the preferred option is the VMI-IRTR contract, whereas, at risk level 3, the optimal choice shifts to the VMI-IRTV contract. Both these VMI arrangements result in reduced total costs compared to the RMI approach.

At a high-risk aversion level (risk level 9), the RMI contract reemerges as the most suitable option. However, two alternative VMI contractsVMI-IRTV and VMI-IRTRstill provide lower total costs. Adoption of these contracts may be preferable if either partner agrees to incur individually higher costs to achieve overall cost efficiency.

Table 3.3: Ranking Results for the Numerical Example

Risk	Ranking	Total Cost	Vendor's Cost	Retailer's Cost
0	RMI	47473	36027	11446
	VMI-IRTV	45823	40977	4846
	VMI-IRTR	47462	33381	14081
	VMI-IVTV	52409	52409	0
	VMI-IVTR	55682	49082	6600
1	VMI-IRTR	47473	36027	11446
	RMI	49806	38360	11446
	VMI-IRTV	45823	40977	4846
	VMI-IVTV	52409	52409	0
	VMI-IVTR	55709	42509	13200
3	VMI-IRTV	49710	42004	7706
	RMI	53699	42253	11446
	VMI-IRTR	51379	37179	14200
	VMI-IVTV	56601	56601	0
	VMI-IVTR	59901	46701	13200
9	RMI	55725	44279	11446
	VMI-IRTV	54006	45022	8984
	VMI-IRTR	55659	37552	18107
	VMI-IVTV	60841	60841	0
	VMI-IVTR	64140	50940	13200

3.5 CONCLUSION AND FUTURE RESEARCH

This chapter introduces a decision-support framework to assist vendors and retailers in determining whether to implement VMI or RMI contracts. By formulating mathematical models for production and distribution planning under both deterministic and stochastic demand conditions, the chapter provides a structured approach to contract selection. A ranking tool was developed to enable vendors to identify the most suitable contract based on preferences such as minimizing inventory management responsibilities or transportation costs.

Numerical experiments under deterministic and stochastic demand scenarios assessed the tool's effectiveness. A risk-averse optimization approach using a mean-risk model was included to address uncertainties in second-stage costs. The results demonstrate that VMI contracts are not universally optimal; their suitability depends on the allocation of inventory and transportation costs between the vendor and retailer. The decision-making tool proved valuable, revealing that VMI is viable when the vendor's risk aversion is below 7. Beyond this threshold, RMI becomes more favorable due to its lower overall cost.

In practice, additional constraints may affect the feasibility of the MILP model. While this approach is computationally efficient for solving various instances within a reasonable CPU time, increasing complexity from additional business constraints may necessitate heuristic or metaheuristic methods for faster and more scalable solutions. Furthermore, robust optimization techniques could complement stochastic models to provide more reliable solutions in real-world scenarios.

Future research could extend this framework to supply chains with multiple retailers, allowing for the simultaneous assessment of VMI adoption across different retailers. Additionally, the model could incorporate alternative inventory management strategies, such as Integrated Inventory Management (IIM) (Song and Dinwoodie 2008) and Customer Inventory Management (CIM) (Chen et al. 2024).

4

INTEGRATED PRODUCTION AND DISTRIBUTION PROBLEM

This chapter explores the integration of production and distribution planning in a multi-item and multi-trip direct shipment in the context of Vendor-Managed Inventory policy. The objective is to minimize total costs, including production, inventory, transportation, and vehicle utilization costs. To tackle this complex optimization problem, we propose a MILP formulation, followed by an efficient HSA algorithm. To assess the effectiveness of the proposed heuristic, we conduct extensive computational experiments and provide a comprehensive discussion of the results, offering key insights derived from this analysis.

The remainder of this chapter is organized as follows: Section 5.1 presents an introduction to the chapter. Section 4.2 presents a review of relevant literature. Section 4.3 describes the problem and introduces the mathematical model used to address it. Section 4.4 outlines the HSA algorithm employed for optimization. Section 4.5 presents the results of numerical experiments and computational analysis. Finally, Section 4.6 provides the conclusion of this chapter and highlights future research directions.

4.1 INTRODUCTION

Effective supply chain management is crucial in today's competitive business landscape, where companies face growing customer expectations and pressure to minimize operational costs. A significant challenge is determining optimal batch sizes and scheduling decisions to control production, inventory, and shipping costs efficiently. Traditionally, these decisions are addressed separately, resulting in inefficiencies and increased expenditures.

This chapter focuses on an integrated approach to solving industrial lot-sizing and direct shipment problems within a VMI system. Under VMI, inventory replenishment decisions at retailers are centralized, allowing better coordination between production planning and distribution activities (Neves-Moreira et al., 2019). Specifically, we address the operational complexities faced by a company manufacturing personal protective equipmentcurrently the sole medical glove producer in the country.

Recently, the company encountered substantial operational difficulties due to a disconnected production and distribution strategy, coupled with retailer-managed inventories. These issues intensified during the COVID-19 pandemic as demand surged, highlighting inefficiencies in transportation management and reliance on manual planning processes. To overcome these challenges, this research proposes an integrated production-distribution planning framework that simultaneously determines multi-item production lot sizes and direct shipment schedules involving multiple trips. The framework explicitly accounts for production capacities, inventory levels, delivery timing requirements, and vehicle constraints.

The primary contribution of this chapter lies in formulating this complex real-world scenario as a mixed-integer linear programming (MILP) model. The problem is solved by utilizing both a commercial solver and a hybrid simulated annealing metaheuristic. Computational experiments illustrate that the proposed integrated approach significantly outperforms the company's existing manual practices, demonstrating enhanced efficiency and cost-effectiveness.

4.2 LITERATURE REVIEW

This study investigates the integration of lot-sizing and direct shipment to optimize costs related to production, setup, inventory, and transportation within a predefined planning horizon. This topic is particularly relevant in VMI systems, where suppliers oversee inventory management on behalf of retailers (Neves-Moreira et al. 2019). It is illustrated in Figure 4.1. To contextualize our research, this section reviews the key literature on lot-sizing models, their integration with distribution decisions, and relevant solution approaches.

4.2.1 *Lot Sizing Problem*

The DLSP has been extensively studied in operations research and supply chain management (Brahimi et al. 2017). Early foundational works include those of Wagner and Whitin (1958) and Manne (1958), which introduced models for single-item and multi-item lot-sizing, respectively. Over time, research in this domain has grown significantly, with surveys such as Brahimi et al. (2017) compiling over 300 contributions. Other key reviews include those by B. Karimi et al. (2003), Jans and Degraeve (2007), Robinson et al. (2009), and Díaz-Madroñero et al. (2014).

In recent decades, researchers have increasingly focused on extending lot-sizing models to incorporate additional supply chain constraints. These include quality inspection (Bettayeb et al. 2018), production scheduling (Gomez Urrutia et al.

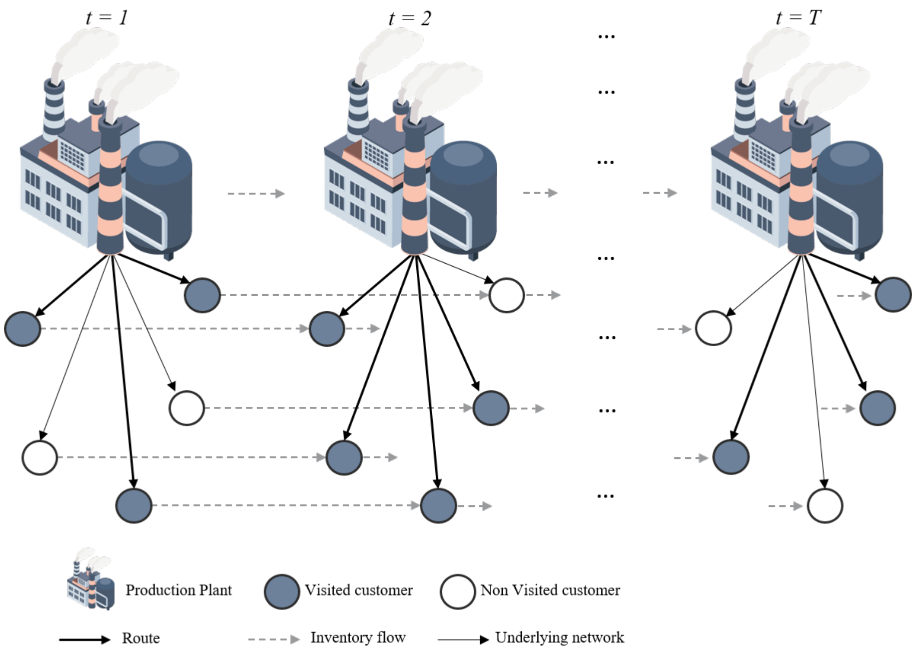


Figure 4.1: Representation of lot-sizing with Direct shipment

2014), cutting stock problems (Melega et al. 2018), and distribution planning. Our research aligns with this latter category, particularly in integrating lot-sizing with direct shipment decisions.

4.2.2 *Integrated Lot Sizing with Direct Shipment*

The integration of lot-sizing and distribution decisions was first studied by Chandra and Fisher (1994), who demonstrated that a coordinated approach can reduce total supply chain costs by 3% to 20% compared to treating these problems separately. The literature in this field can be broadly divided into production-routing models and direct shipment models, as reviewed by Adulyasak et al. (2015b). More recently, Berghman et al. (2023) provided an updated review of research from 2010 to 2022 on scheduling and outbound vehicle routing.

Several studies have examined different lot-sizing and direct shipment settings. Blumenfeld et al. (1985) analyzed the trade-offs between inventory holding, production setup, and transportation costs to determine cost-efficient shipping strategies. Research by Herer and Tzur (2001) explored full truckload shipments with transshipment between customers. C.-L. Li et al. (2004) investigated suppliers' choices between truckload (TL) and less-than-truckload (LTL) transportation, while Jaruphongsa et al. (2007) developed a dynamic programming model for optimizing truckload cost structures. Additionally, Chand et al. (2007) extended the problem to multiple customers, considering the possibility of backlogging.

4.2.3 *Multi-Trip Vehicle Routing problem*

An important extension of lot-sizing and distribution models is multi-trip vehicle routing, where a vehicle can make multiple deliveries within a single planning period. This aspect is especially critical in urban logistics, where road restrictions often necessitate smaller-capacity vehicles that return to depots multiple times per day. Cattaruzza et al. (2016) emphasize the importance of multi-trip routing for improving fleet utilization and addressing congestion-related challenges.

Despite its practical importance, multi-trip vehicle routing remains relatively underexplored. Fleischmann (1990) was among the first to study this problem, introducing the concept of multiple vehicle usage within a planning period. Brandão and Mercer (1998) later developed a heuristic approach to solve the multi-trip vehicle routing problem, while Hernandez et al. (2014) proposed a set-covering formulation with trips as decision variables. Further advancements include Cattaruzza et al. (2014), who introduced an Iterated Local Search (ILS) algorithm, and Crainic et al. (2015), who explored zone-based routing strategies for customer segmentation.

4.2.4 *Solution Approaches*

The complexity of integrated lot-sizing and direct shipment problems often necessitates metaheuristic methods such as genetic algorithms, tabu search, and simulated annealing (Talbi et al. 2016). In this study, we adopt SA due to its efficiency in handling combinatorial optimization problems and its ability to escape local optima while seeking globally optimal solutions (Kirkpatrick 1984; Bertsimas and Tsitsiklis 1993).

SA has been applied to lot-sizing problems, with Kuik and Salomon (1990) employing SA-based heuristics for multi-level lot sizing. Özdamar and Barbarosoglu (2000) integrated Lagrangian relaxation with SA for solving large-scale problems, while Vasant (2010) introduced a hybrid Hybrid Simulated Annealing and Genetic Algorithm (HSAGA) for production planning.

For vehicle routing problems, SA has also been widely utilized. Koç and Karaoglan (2016) proposed a branch-and-bound SA hybrid for green vehicle routing, while Lai et al. (2010) integrated SA with tabu search to optimize pickup and delivery routes. Vincent et al. (2022) further applied SA to problems involving heterogeneous fleets and parcel locker-based delivery systems.

Although SA has been used in integrated production distribution problems, research in this area remains low. One notable contribution is by Yamur and Kesen (2021), who developed a hybrid memetic algorithm incorporating SA for solving scheduling and vehicle routing problems. Our study extends this line of research by developing an efficient SA-based algorithm to address the integrated lot-sizing with direct shipment problem in a real-world industrial setting.

4.2.5 *Contribution*

This chapter explores the integration of lot-sizing, direct shipment, and multi-trip vehicle routing, a topic of significant practical relevance, as demonstrated by our case study. Despite its importance, our review indicates a lack of existing research that simultaneously incorporates these three components into a unified framework. To address this gap, we develop a MILP formulation tailored to a real-world industrial context. Additionally, we propose an efficient SA algorithm to solve the problem and evaluate different operational scenarios. By comparing our proposed strategies with the company's existing approach, we provide valuable insights and strategic recommendations to enhance decision-making and improve overall supply chain efficiency.

4.3 PROBLEM DESCRIPTION

This study is motivated by a real-world challenge faced by a manufacturing company specializing in Personal Protective Equipment (PPE). The company operates a processing center, where products are produced and subsequently transported to a warehouse and two customers. The warehouse acts as an intermediary, receiving customer orders and transmitting them to the production facility.

The production system consists of two distinct units, each responsible for manufacturing different product categories. Setup costs are incurred whenever production is initiated in either unit. A homogeneous fleet of vehicles is used for deliveries, with each truck assigned to transport goods directly to a single customer per period. The manufactured products can either be immediately loaded onto delivery vehicles or stored at the production site for future dispatch. However, storage capacity is limited at the plant, warehouse, and customer locations, requiring careful inventory management. Figure 4.2 provides an illustration of this distribution network. In the past, the company managed deliveries efficiently with a limited customer base of three clients and the available fleet size was sufficient to meet demand. However, with the recent addition of 20 high-demand customers, the current logistics setup has become inadequate. The single-trip vehicle usage strategy is no longer viable, as the fleet size is insufficient to meet all delivery requirements. To overcome this challenge, we introduce the multi-trip concept, where each vehicle is allowed to complete up to three trips within a single period. This strategy ensures that existing resources are utilized more effectively, reducing

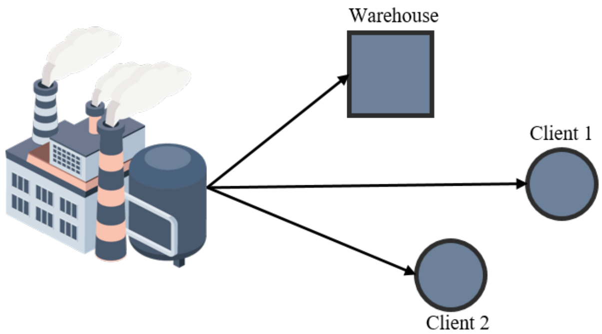


Figure 4.2: A general description of the investigated network

the need for additional expansion of the fleet. Figure 4.3 illustrates this multi-trip scheduling approach.

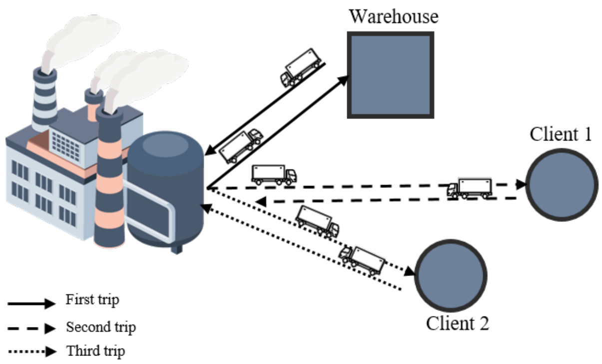


Figure 4.3: Representation of the multi-trip concept (Benfedel et al. 2025a)

To model the companys operational dynamics, we establish the following assumptions:

- The planning horizon is divided into T discrete periods.
- Product demand is deterministic and must be fully satisfiedno customer orders can be left unfulfilled.
- The production facility is capable of manufacturing multiple product types simultaneously.
- Constraints are imposed on production capacity, storage limits, and vehicle availability to reflect real-world operational restrictions.
- Direct shipment is used for deliveries, meaning that vehicles transport goods directly to customers without intermediate stops.
- The inventory policy follows a maximum-level approach, ensuring that storage constraints are never exceeded.

- The multi-trip strategy allows vehicles to make multiple deliveries per period, which is crucial for handling the increased demand from the expanded customer base while operating within existing fleet limitations.

The sets and variables that make up our model are presented below:
Sets and parameters:

- Sets:
  $\mathcal{T} = \{1, .., T\}$: set of time periods in the planning horizon.
  $\mathcal{N} = \{1, .., N\}$: set of nodes (processing unit, warehouses, and customers).
  $\mathcal{P} = \{1, .., P\}$: set of products.
  $\mathcal{R} = \{1, .., R\}$: set of trips.

- Production data:
  p_j : Production cost of product j.
  f_j : Product setup cost j.
  C_j : Production capacity for product j.

- Inventory data:
  h_{ijt} : Inventory Holding cost of product j at location i at period t.
  U_{ij} : Storage capacity of product j at location i.
  d_{ijt} : Customer demand of item j at location i in period t.

- Transport data:
  c_i : Travel cost between manufacturing plant and customer i.
  Q : Vehicle capacity.
  C_U : Vehicle utilization cost.
  V : Number of homogeneous vehicles.

Decision variables:

- Production:
  x_{jt} : Amount of product j manufactured at period t.
  y_{jt} : Binary setup variable equal to 1 if a production of product j occurs at the plant in period t and 0 otherwise.

- Inventory:
  I_{ijt} : Inventory level of product j at location i in period t.

- Transport:
  v_{itr} : Number of vehicles sent to customer i in trip r in period t.
  q_{ijtr} : Amount of product j shipped to customer i in trip r in period t
  Z_t : Total number of vehicles used at period t

The MILP model suggested to depict the problem is presented as follows:

$$[\mathbf{IPD-PPE}] \quad \min\Big(\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{P}} b_j x_{jt} + \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{P}} f_j y_{jt} + \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{P}}\sum_{t\in\mathcal{T}} h_{ijt} I_{ijt} + \sum_{r\in\mathcal{R}}\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}\setminus\{1\}} c_i v_{itr} + \sum_{t\in\mathcal{T}} C_U Z_t\Big) \tag{4.1}$$

Subject to

$$I_{1jt} = I_{1j,t-1} + x_{jt} - \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}\setminus\{1\}} q_{ijtr} \quad \forall j\in\mathcal{P},\ \forall t\in\mathcal{T} \tag{4.2}$$

$$I_{ijt} = I_{ij,t-1} + \sum_{r\in\mathcal{R}} q_{ijtr} - d_{ijt} \quad \forall i\in\mathcal{N}\setminus\{1\},\ \forall j\in\mathcal{P},\ \forall t\in\mathcal{T} \tag{4.3}$$

$$I_{ijt} \le U_{ij} \quad \forall i\in\mathcal{N},\ \forall t\in\mathcal{T} \tag{4.4}$$

$$x_{jt} \le C_j \quad \forall j\in\mathcal{P},\ \forall t\in\mathcal{T} \tag{4.5}$$

$$x_{jt} \le y_{jt}\sum_{t'\in\mathcal{T}}\sum_{i\in\mathcal{N}\setminus\{1\}} d_{ij,t'} \quad \forall j\in\mathcal{P},\ \forall t\in\mathcal{T} \tag{4.6}$$

$$\sum_{j\in\mathcal{P}} q_{ijtr} \le Q\times v_{itr} \quad \forall i\in\mathcal{N}\setminus\{1\},\ \forall t\in\mathcal{T} \tag{4.7}$$

$$\sum_{i\in\mathcal{N}\setminus\{1\}} v_{itr} \le V \quad \forall r\in\mathcal{R},\ \forall t\in\mathcal{T} \tag{4.8}$$

$$\sum_{i\in\mathcal{N}\setminus\{1\}} v_{itr} \le H_t \quad \forall r\in\mathcal{R},\ \forall t\in\mathcal{T} \tag{4.9}$$

$$y_{jt} \in \{0,1\} \quad \forall j\in\mathcal{P},\ \forall t\in\mathcal{T} \tag{4.10}$$

$$x_{jt},\ I_{ijt} \ge 0 \quad \forall i\in\mathcal{N},\ \forall j\in\mathcal{P},\ \forall t\in\mathcal{T} \tag{4.11}$$

$$q_{ijtr},\ v_{itr},\ Z_t \ge 0 \quad \forall i\in\mathcal{N},\ \forall j\in\mathcal{P},\ \forall t\in\mathcal{T},\ \forall r\in\mathcal{R} \tag{4.12}$$

The objective function (4.1) aims to minimize the total cost, which comprises various components: production costs, production setup costs, inventory holding costs at the manufacturing facility, warehouse, and customer locations, as well as transportation costs, including both delivery expenses and vehicle utilization costs. The model ensures inventory balance at the manufacturing unit and customer locations through Constraints (4.2) and (4.3), respectively. The warehouse demand, d_{ijt}, is determined as the aggregate demand of its associated customers. The limitations of the inventory capacity are enforced by constraint (4.4), while the limitations of the production capacity are imposed by constraint (4.5). Additionally, Constraint (4.6) establishes the relationship between production and transportation decisions to guarantee demand fulfillment. Constraint (4.7) ensures that vehicle capacity is not exceeded, while Constraint (4.8) restricts the number of vehicles used per trip in each period to remain within the available fleet size. The deployment of vehicles across different periods is controlled by Constraint (4.9). Finally, Constraints (4.10)-(4.12) specify the nature and types of decision variables in the model.

4.4 SOLUTION APPROACH

The IPD-PPE problem is classified as NP-hard. This complexity is evident as it extends the well-known one-warehouse multi-retailer problem, which has been proven to be NP-hard (e.g., Arkin et al. 1989). Given the need for timely decision-making in real-world applications, companies seek efficient solutions for both short-term operational adjustments and long-term strategic planning, where evaluating multiple scenarios is essential.

This section introduces a HSA algorithm designed to effectively address the IPD-PPE problem. Additionally, its performance is assessed in comparison to a state-of-the-art commercial solver to evaluate its efficiency and practical applicability.

4.4.1 *The hybrid simulated annealing algorithm*

SA is a widely used metaheuristic to solve combinatorial optimization problems. It operates by iteratively refining a solution through minor local modifications, continuing this process until no further enhancement is possible or a predefined stopping criterion is met. The term “simulated annealing" is derived from the analogy between the physical annealing of materials and the optimization process of the algorithm. One of SAs key strengths is its ability to escape local optima, making it particularly effective for complex optimization problems. Unlike many other metaheuristics, SA generates only a single neighboring solution at each iteration.

If the newly generated solution outperforms the current one, it is immediately accepted. However, if the new solution is inferior, it may still be adopted with a certain probability, allowing the algorithm to explore a broader solution space and avoid premature convergence.

The neighborhood structure used in the proposed SA approach is influenced by the tabu search (TS) algorithm introduced by Armentano et al. (2011) to solve the PRP. The main idea is to shift the quantity of delivery assigned to the customer i from one period t to another period t' while ensuring that the inventory restrictions are not violated. If customer i is not scheduled for a visit in period t', the delivery is reassigned to period t using the cheapest insertion technique. Notably, this approach allows for temporary infeasible solutions, which enhances exploration and diversification during the search process.

To ensure the efficiency of the SA algorithm, various parameters must be carefully tuned. These include solution representation, the definition of the objective function, neighborhood structure design, and constraint-handling mechanisms. The following sections provide a detailed discussion of these components.

4.4.1.1 *Solution Structure*

There are two types of decision variables in each solution:
(*i*) binary decision variables:

- setup variable y_{jt}.

and (*ii*) continuous decision variables:

- production amount x_{jt}.
- Inventory quantity I_{ijt} .
- Number of vehicles v_{itr}.
- Delivery quantity q_{ijtr}.
- Total number of vehicles used Z_t.

4.4.1.2 *Initial Solution*

The initial solution is obtained in three steps are as follows:

- **Step 1:** At the beginning of the planning horizon, customer demand is fulfilled using the available initial inventory. Asnning progresses, deliveries are scheduled to match the exact demand for each period. For instance, consider a scenario with a single customer over three periods ($T = 3$) where the demand is 10 units per period. If the initial inventory available to the customer is 15 units, we first utilize this stock to cover early demands. The remaining requirements for each period are then computed by deducting the initial inventory from the cumulative demand, resulting in $[0, 5, 10]$. Consequently, the delivery quantities $q_{i,j,t}$ are set to $[0, 5, 10]$, ensuring demand fulfillment while effectively utilizing the initial stock.
- **Step 2:** Determine the required number of vehicles needed to serve each customer. This is determined using the following equation:

$$V_{it} = \lceil \frac{\sum_{j \in \mathcal{P}} q_{i,j,t}}{QR} \rceil \qquad \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{4.13}$$

- **Step 3:** Utilize the Wagner-Whitin algorithm Wagner and Whitin (1958) to establish the production schedule for each item. The algorithm has been adapted to ensure that the generated plan remains within the available capacity constraints.

4.4.1.3 *Neighborhood Structure*

The neighborhood of a given solution is generated by applying three different moves, which are detailed below.

move 1 This move involves transferring a maximum of the delivered quantity, denoted as $z_{i,j,t,t_1} \leq q_{i,j,t}$, from one period t to another period t_1. Two periods, t and t_1, are randomly selected for this adjustment. Based on Constraints (4.2) and (4.3), shifting a quantity r_{jktt_1} from period t to an earlier period $t_1 < t$ leads to a reduction in inventory levels $I_{j0\tau}$ and an increase in $I_{jk\tau}$ for $\tau = t_1, ..., t-1$, both by an amount equal to r_{jktt_1}. Here, k denotes the vehicle index, while j represents the product index, as defined in Section 4.3. Similarly, transferring r_{jktt_1} from period t to a later period $t_1 > t$ results in an increase in inventory levels $I_{j0\tau}$ while simultaneously decreasing $I_{jk\tau}$ for $\tau = t, ..., t_1$ by the same quantity. if $t_1 < t$:

$$z_{ijt,t_1} = \min\Big\{q_{ijt}\ ,\ \Big(t_1 \times C_j - \sum_{i \in \mathcal{N}} \sum_{t_2=1}^{t_1} q_{ij,t_2}\Big)\ ,\ \Big(Q \times R \times V - \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{P}} q_{ij,t_1}\Big)\ ,\ \min_{t_2}\{U_{ij} - I_{ij,t_2}, I_{1,j,t_2}\}\Big\} \tag{4.14}$$

and if $t_1 > t$:

$$z_{ijt,t_1} = \min\Big\{q_{ijt}\ ,\ \Big(t_1 \times C_j - \sum_{i \in \mathcal{N}} \sum_{t_2=1}^{t_1} q_{ij,t_2}\Big)\ ,\ \Big(Q \times R \times V - \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{P}} q_{i,j,t_1}\Big)\ ,\ \min_{t_2}\{I_{ij,t_2}\}\Big\} \tag{4.15}$$

This move is applied to each product j and each customer i.

move 2 Modify the delivery schedule by adding the quantity z_{ijt,t_1} to q_{ijt_1}, thereby creating an updated delivery plan. Following this adjustment, it is necessary to recalculate the number of trucks and trips required to transport goods to customers.

move 3 Apply the Wagner-Whitin algorithm Wagner and Whitin (1958) to determine the revised production plan. Shifting z_{ijt,t_1} from period t to t_1 modifies the delivery quantity as follows: $q_{ij,t_1} = q_{ij,t_1} + z_{ijt,t_1}$. To ensure feasibility, a production capacity constraint is enforced during this adjustment:

$$q'_{jt} = \sum_{i \in N} \max\Big\{0, q_{ijt} - \max\{0, I_{1,j,0} - \sum_{t_1=1}^{t-1} q_{ij,t_1}\}\Big\} \tag{4.16}$$

production quantity shifted must respect production and inventory capacities:

$$x_{j,t_2} = \min\Big\{\Big(C_j - (x_{j,t_2} + q'_{jt})\Big), \Big(U_{1,j} - (I_{1,j,t_2} + q'_{jt})\Big)\Big\} \qquad \forall t_2 \in \{1, t-1\} \tag{4.17}$$

For each product j and customer i, the first move is generated randomly. Following the execution of Moves 2 and 3, the solution S is evaluated to ensure capacity constraints are met using the following function:

$$g(S) = \sum_{t \in \mathcal{T}} \max\Big\{\sum_{j \in \mathcal{P}} x_{jt} - C_j, 0\Big\} \tag{4.18}$$

$$h(S) = \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{P}} \sum_{i \in \mathcal{N}} \max\Big\{I_{ijt} - U_{ij}, 0\Big\} \tag{4.19}$$

$$l(S) = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}} \max\Big\{\sum_{r \in \mathcal{R}} v_{itr} - R \times V, 0\Big\} \tag{4.20}$$

$$C(S) = g(S) + h(S) + l(S) \tag{4.21}$$

if $C(S) = 0$ we accept the solution as a feasible solution.

4.4.2 *Pseudo-code of the heuristic*

The SA algorithm begins by generating an initial solution S_0 (as described in the previous section), which serves as the starting point. Using the three neighborhood moves outlined in Section 4.4.1.3, the algorithm iteratively explores new solutions by modifying the current one. The overall procedure is summarized in Algorithm 4.1.

The algorithm takes three input parameters: the initial temperature τ_0, the final temperature τ_f, and the cooling rate μ (line 1). The process starts with the generation of the initial solution S_0 (line 2). The current temperature τ is then initialized to τ_0 (line 3), and both the current solution S and the best-known solution are set to S_0 (line 4). The main loop runs until the temperature τ falls below τ_f, ensuring the search continues while a feasible solution is being explored (lines 523). In each iteration, the current solution S is modified to create a neighboring solution S' using the three predefined moves (Move 1, Move 2, and Move 3) (line 7). The cost difference Δ between the new solution S' and the existing solution S is then computed (line 8). If Δ is negative, meaning the new solution is better, it is immediately accepted as the current solution (lines 910). Otherwise, the new solution may still be accepted based on a probability α, allowing the algorithm to escape local optima (lines 1117). If the updated solution is both feasible and has a lower cost than the best-known solution, it is stored as the new best solution (lines 1819). Finally, the temperature τ is reduced according to the cooling rate μ (line 22).

Different probability acceptance functions can be used within Algorithm 4.1. As part of the numerical experiments, an alternative acceptance probability $\alpha = e^{((-100/\tau)\times(\Delta/C(S)))}$, proposed by Adulyasak et al. (2014b), has also been evaluated to compare its impact on solution performance.

Algorithm 4.1 Simulated annealing algorithm

1: *the parameters* τ_0, τ_f, μ
2: *Generate initial solution* S_0
3: *Determine current temperature as the initial temperature* $\tau = \tau_0$
4: *Set* S = *Best Solution* $= S_0$
5: **while** $\tau > \tau_f$ **do**
6:     **do**
7:         **for** $i = 1$ to *number of iterations* **do**
8:             *Generate Neighbor solution* S' *for* S : *use move* 1 *then move* 2 *then move* 3
9:             $\Delta = cost(S') - Cost(S)$
10:             **if** $\Delta \leq 0$ **then**
11:                 $S = S'$
12:             **else**
13:                 $a = Uniform[0, 1]$
14:                 $\alpha = e^{(-\Delta/\tau)}$
15:                 **if** $a \leq \alpha$ **then**
16:                     $S = S'$
17:                 **end if**
18:             **end if**
19:             **if** $cost(S) \leq cost(Best\ solution)$ *and* $C(S) == 0$ **then**
20:                 *Best solution* $= S$
21:             **end if**
22:         **end for**
23:     **while** $C(Best\ Solution) > 0$
24:     $\tau = \tau \times \mu$
25: **end while**

4.5 NUMERICAL EXPERIMENTS

This section serves two primary objectives. First, we evaluate the computational efficiency of the proposed HSA heuristic by benchmarking it against established instances from the PRP literature. To validate its performance, the heuristic was initially tested on a set of PRP benchmark instances introduced by Archetti et al. (2011). Additionally, we applied it to randomly generated datasets with varying characteristics, including different planning horizons, numbers of items, and logistics network sizes.

The second objective of this section is to analyze the obtained results from a managerial perspective. We discuss the practical implications of our approach and compare its outcomes with existing decision-making strategies employed by the case company.

For the exact resolution of the IPD-PPE problem, we formulated it as a MILP model and solved it using CPLEX 12.8. The HSA heuristic was implemented in Python 3.7 using JetBrains PyCharm 2018. Although Python is generally slower than compiled languages like C or C++, it was chosen for its ease of use, extensive libraries, and flexibility in implementation.

All computational experiments were conducted on a personal computer equipped with an AMD Ryzen 7 3700U processor (2.3 GHz) and 8 GB of RAM. The key parameters of the simulated annealing algorithminitial temperature (τ_0), final temperature (τ_f), and cooling rate (μ)were set to 140, 1, and 0.9, respectively, following recommendations from Shaabani and Kamalabadi (2016). CPLEX was executed with its default configuration settings.

4.5.1 *Archetti et al. benchmark instances*

Given the limited availability of datasets for lot-sizing problems with direct shipments, we assessed the performance of our algorithm on benchmark instances designed for the PRP with a single product and vehicle. These instances were introduced by Archetti et al. (2011) and are publicly available on their website (http://orbrescia.unibs.it/instances).

The dataset consists of four categories of PRP instances, where variations were introduced by modifying production costs, transportation costs, and retailer holding costs (set to zero in some cases). Each category includes instances with 19, 50, and 100 retailers. In this study, we focused on solving instances from the first, second, and fourth categories, specifically those with 19 retailers.

The original problem formulation assumes a constant product demand and an unconstrained production facility. To align our approach with these assumptions, we introduced modifications to our algorithm:

- In Move 2: After incorporating the quantity z_{ijt,t_1} into the delivery plan q_{ijt_1}, we applied the LKH implementation Helsgaun (2017) to solve the Traveling Salesman Problem (TSP). This step optimizes the delivery route by minimizing the total travel distance required to serve all customers.

- In Move 3: The Wagner and Whitin (1958) algorithm was employed to address the lot-sizing problem. This ensures an optimal production schedule that minimizes total costs while meeting demand across the planning horizon.

Table 4.1: Performance of the proposed metaheuristic against heuristic of Archetti et al. (2011)

	CLASS I			CLASS II			CLASS IV		
Instances	HSA	Archetti	GAP %	HSA	Archetti	GAP %	HSA	Archetti	GAP %
1	50275	**50196**	0.16	312912	**312697**	0.07	35518	**35262**	0.73
2	**50798**	51541	-1.46	**313268**	313482	-0.07	37421	**37002**	1.13
3	55445	**55127**	0.58	317885	**317564**	0.10	40497	**39690**	2.03
4	**105354**	105756	-0.38	**803413**	804832	-0.18	**87738**	87957	-0.25
5	**106171**	106598	-0.40	**804454**	805432	-0.12	91388	**91076**	0.34
6	113362	**113307**	0.05	**812527**	813020	-0.06	98244	**96970**	1.31
7	**41247**	41590	-0.83	**302428**	302816	-0.13	39538	**38926**	1.57
8	**42303**	42377	-0.17	**303369**	303396	-0.01	41609	**40714**	2.20
9	45275	**44404**	1.96	307234	**306425**	0.26	45904	**44957**	2.11
10	**93720**	96379	-2.84	**789960**	792716	-0.35	91939	**91621**	0.35
11	**96419**	98202	-1.85	792922	**792495**	0.05	96089	**94878**	1.28
12	102804	**101926**	0.86	800324	**799695**	0.08	103822	**102257**	1.53
13	**55672**	55808	-0.24	**316884**	317176	-0.09	296540	**296352**	0.06
14	**56257**	56967	-1.26	**318070**	318307	-0.07	298349	**298092**	0.09
15	61731	**60800**	1.53	324356	**323217**	0.35	302283	**301360**	0.31
16	**110450**	110738	-0.26	**807356**	807621	-0.03	**784090**	784197	-0.01
17	**112097**	112878	-0.70	809149	**808535**	0.08	787536	**787316**	0.03
18	119052	**118542**	0.43	818379	**817462**	0.11	795316	**794564**	0.09
19	45425	**45304**	0.27	**305546**	306556	-0.33	300640	**300016**	0.21
20	47089	**46594**	1.06	**307192**	307684	-0.16	303189	**301804**	0.46
21	50859	**49682**	2.37	313004	**311435**	0.50	309244	**306536**	0.88
22	**97935**	100143	-2.25	**794192**	796475	-0.29	**787832**	787861	-0.003
23	101342	**101244**	0.10	797505	**797484**	0.002	792293	**791118**	0.15
24	108657	**106932**	1.61	806414	**804940**	0.18	801968	**800032**	0.24
Average			-0.07			-0.004			0.7

Table 4.1 presents a performance comparison between our algorithm and the best solutions reported by Archetti et al. (2011) for the first, second, and fourth instance categories with 19 retailers. Our approach achieved superior results for the first and second instance sets, improving upon the best-known solutions by an average of 0.07% and 0.004%, respectively. However, for the fourth category, our method resulted in an average objective value that was 0.7% higher than the best-known solution.

It is important to acknowledge that the problem formulation in Archetti et al. (2011) differs slightly from our optimization model. Nevertheless, the results demonstrate that our algorithm remains competitive, performing well across multiple instance sets despite these differences.

4.5.2 *Generated data-sets*

To construct the datasets used in this study, we combined data from both the literature (Senoussi et al. 2016, Armentano et al. 2011) and real-world instances from the case company. The currency used in this study is denoted as CU, where 1 USD is approximately equivalent to 100 CUs. Various parameters were adjusted to create different numerical test cases.

The dataset generation process considered variations in the number of nodes (N), products (P), and time periods (T), with values set as follows: $N = \{4, 10, 20\}$, $P = \{2, 4, 8\}$, and $T = \{3, 6, 9\}$. The company processes products at two different rates, leading to two potential production capacities defined as $C_j = \{1.5A, 2A\}$, where $A = \sum_{j\in\mathcal{P}} \sum_{t\in\mathcal{T}} d_{j,t}/T$. The corresponding setup costs are $f_j = 1000$ for $C_j = 1.5A$ and $f_j = 3000$ for $C_j = 2A$, for all $j \in \mathcal{P}$.

Vehicle capacity was set as $Q = \{1.5B, 2B\}$, where $B = \max_{t\in\mathcal{T}}(\sum_{j\in\mathcal{P}} d_{j,t})/V$. The associated fixed costs per vehicle were either $C_U = 10,000$ for $Q = 1.5B$ or $C_U = 20,000$ for $Q = 2B$.

By combining these five key parametersthe number of nodes, products, periods, production capacities, and vehicle capacitieswe generated $3 \times 3 \times 3 \times 2 \times 2 = 108$ different configurations. For each configuration, five instances were randomly created based on parameter variations outlined in Table 4.2, leading to a total of 540 instances.

It is important to note that the company initially rented three trucks from a Third-party Logistics (3PL) provider with a total fleet of eight trucks. As the customer base expanded, these three trucks became insufficient to meet demand, prompting the company to adjust the number of rented trucks for each delivery. To capture the flexibility of multi-trip deliveries with a varying fleet size, we represented this variability as an interval in our experiments. This approach aligns with certain studies on the Multi-Trip Vehicle Routing Problem, where the number of available vehicles is sometimes limited to one to enforce multi-trip operations (Cattaruzza et al. 2018).

4.5.3 *Analysis of the performance of HSA heuristic*

To evaluate the efficiency of the HSA heuristic, we compare its results with those obtained using the CPLEX solver. The solver is run with a maximum computational time of 3600 seconds. The performance gap between the heuristic solution

Table 4.2: Instance generation.

Parameters	
Customers demand	$d_{i,j,t} \in [0, 25]$
Inventory capacity at the plant	$U_{1,j} = \text{uniform}[1, T/2] \times \bar{d}_j$,,
	where $(\bar{d}_j = (\sum_{i \in \mathcal{N}-1} \sum_{t \in \mathcal{T}} d_{i,j,t}/T \times (N-1))$,
Inventory capacity at the customers	$U_{i,j} = \text{uniform}[1, T/2] \times \bar{d_{i,j}}$,
	where $(\bar{d_{i,j}} = (\sum_{t \in \mathcal{T}} d_{i,j,t}/T)$
Initial inventory at the plant	$I_{1,j,0} = 0$
Initial inventory at the customers	$I_{i,j,0} \in [0, U_{i,j}]$
Unit inventory cost at the plant	$h_{1,j} = 0.5$
Unit inventory cost at the customers	$h_{i,j} \in [0.3, 1]$
Unit production cost	$b_j \in [1500, 2500]$
Processing time	$P_j = 1$
Transportation cost	$C_i = \sqrt{(x_i - x_1)^2 + (y_i - y_1)^2}$
Coordinates	$x_i \in [0, 500], y_i \in [0, 1000]$
Number of vehicles	$V \in [2, 8]$
Number of trips allowed	$R \in [(N/V) + 1, 2 \times (N/V)]$

and the best-known solution from CPLEX is computed using the following formula:

$$\text{Gap} = 100 \times \frac{Sol^{HSA} - UB^{CPLEX}}{UB^{CPLEX}} \tag{4.22}$$

where Sol^{HSA} represents the best solution found by the HSA heuristic, and UB^{CPLEX} denotes the best upper bound obtained by CPLEX for the IPD-PPE formulation. It is important to note that for instances where CPLEX fails to reach optimality within the given time limit, UB^{CPLEX} may not necessarily correspond to the optimal solution, as the solver may stop with a positive integrality gap.

The rest of this section is organized as follows. First, we provide a discussion on the selection of the acceptance probability in the HSA algorithm. Then, the computational performance of the heuristic is assessed in two stages:

1. For smaller instances, we compare the best solution found by the HSA heuristic to the optimal solution obtained using CPLEX. 2. For larger instances where CPLEX does not reach optimality within the 3600-second limit, we compare the best solutions obtained by both methods, ensuring that the solver's runtime is restricted to the convergence time of the HSA heuristic.

This comparative analysis is presented in Section 4.5.3.3.

4.5.3.1 *Choice of acceptance probability*

To evaluate the impact of different acceptance probabilities on the performance of the HSA heuristic, we conducted simulations using two probability functions, denoted as α_1 and α_2, defined as follows:

$$\alpha_1 = e^{-\Delta/\tau} \tag{4.23}$$

$$\alpha_2 = e^{(-100/\tau)\times(\Delta/f(s))} \tag{4.24}$$

where Δ represents the cost difference between the current solution and its neighboring solution, i.e., $\Delta = C(S') - C(S)$. The probability function α_2 was previously introduced by Adulyasak et al. (2014b).

To compare the performance of these two probabilities, we define the gap between the solutions obtained using HSA and the optimal solutions from CPLEX as follows:

$$\text{Gap}(\alpha_i) = 100 \times \frac{Sol^{HSA(\alpha_i)} - UB^{CPLEX}}{UB^{CPLEX}} \tag{4.25}$$

where $i \in \{1, 2\}$, $Sol^{HSA(\alpha_i)}$ is the best solution obtained by HSA using α_1 or α_2, and UB^{CPLEX} represents the optimal solution found by CPLEX. Unlike the general gap calculation in Equation (4.22), this comparison is limited to instances where CPLEX successfully reached optimality.

The evaluation was performed on 20 instances generated for the scenario where $T = 6$, $N = 4$, and $P = 4$. The results, illustrated in Figure 4.4, indicate that the average gap values for α_1 and α_2 were 2.68% and 3.68%, respectively, suggesting that the use of α_1 generally leads to better outcomes. However, for instances 1, 3, 12, 14, and 19, the HSA heuristic with α_2 performed slightly better, with gap differences of 0.6, 0.2, 0.01, 0.05, and 0.39 percentage points, respectively. Despite these cases, the higher standard deviation observed with α_2 (1.73 compared to 1.25 for α_1) suggests that it may be less stable.

Regarding computational time, Figure 4.5 shows that the HSA heuristic takes an average of 17 seconds to find the best solution. However, for instances 3, 5, and 19, HSA with α_1 required slightly more time. This is likely because the algorithm allows infeasible solutions in the search space, making it more challenging to quickly identify a feasible solution. In contrast, the use of α_2 facilitates the identification of feasible solutions earlier in the search process.

Although α_2 yielded better solutions in certain instances, the overall performance of the HSA heuristic depends on multiple factors, including the problem structure, neighborhood selection, and cooling schedule. Additional testing may be required to refine the acceptance probability and optimize other algorithm parameters.

Based on these findings, α_1 was selected as the acceptance probability for the subsequent experiments.

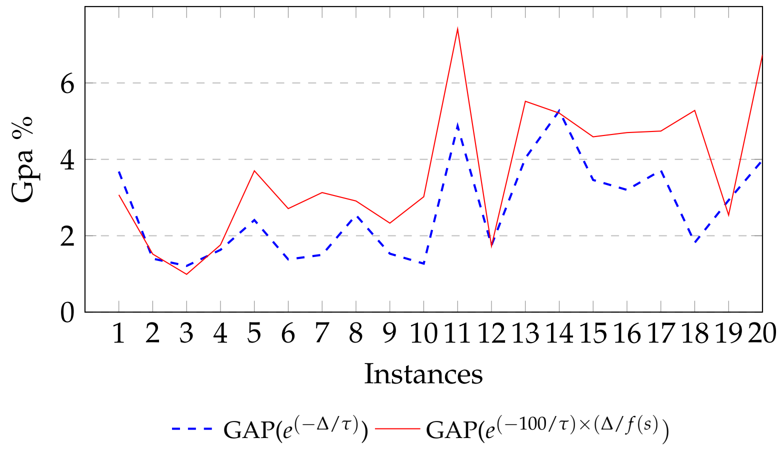


Figure 4.4: Comparing Gaps of different acceptance probability in Simulated Annealing for $(T, N, P) = (6, 4, 4)$

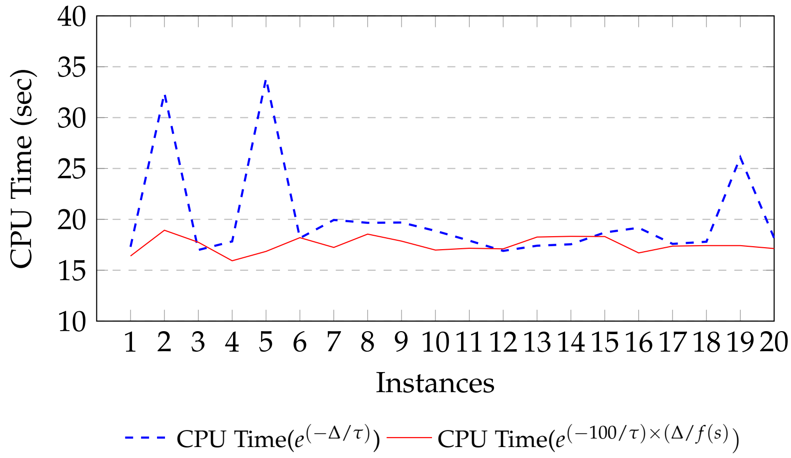


Figure 4.5: Comparing CPU time of different acceptance probability in Simulated Annealing

4.5.3.2 *Computational Performance of Hybrid Simulated Annealing on Small Instances*

Table 4.3 presents a comparison between the solutions obtained using the HSA heuristic and the CPLEX solver for small-sized instances. These instances were generated based on a full factorial combination of parameters: $T \in \{3, 6, 9\}$, $N \in \{4, 10, 20\}$, and $P \in \{2, 4, 8\}$, as described in Section 4.5.2. Since the focus here is on small-scale problems, the objective function values, computational time, and solution gaps were averaged over 20 instances for each parameter combination (T, N, P).

An analysis of Table 4.3 reveals that the HSA heuristic demonstrates strong performance across all instances, achieving an average optimality gap of just 1.44%. The gap remains consistently low regardless of instance size, ranging from 0.24% for the smallest case ($T = 3, N = 4, P = 2$) to 2.52% for the largest considered instance ($T = 9, N = 20, P = 2$). The highest observed deviation from the optimal solution was 4.2%, which was obtained in just 13.54 secondsapproximately 1/30th of the time required by CPLEX (381.47 seconds) to reach optimality.

For instances where $T = 3$, CPLEX was able to determine optimal solutions faster than the HSA heuristic, requiring only 6.32 seconds compared to 40.50 seconds for the heuristic. However, as the problem size increased, particularly for instances with $T = 9$, the solvers computational time increased significantly, reaching a peak of 1738.56 seconds. In contrast, HSA maintained a more stable performance, becoming particularly competitive when the number of nodes was at least four. The heuristic exhibited a more consistent runtime, with an average CPU time of 37.07 seconds, significantly lower than the solvers average of 153.63 seconds.

These findings indicate that the hybrid simulated annealing approach is a viable and efficient method for solving the production and distribution problem, particularly in terms of computational speed.

4.5.3.3 *Computational Performance of Hybrid Simulated Annealing on large Instances*

In this section, we further assess the effectiveness of the HSA heuristic by focusing on large instancesspecifically, those for which CPLEX is unable to reach an optimal solution within the 3600-second time limit. To ensure a fair comparison, the solvers execution time is restricted to match the CPU time required by the HSA heuristic.

Table 4.4 presents the solutions obtained by both HSA and CPLEX, along with the gap between them and the corresponding computational times. The results indicate that CPLEX begins to struggle with finding high-quality feasible solutions for instances with at least 6 periods, 10 nodes, and 4 products. This appears to mark a complexity threshold where CPLEXs performance becomes significantly constrained.

The HSA heuristic demonstrates superior performance across almost all instances, except two cases ($T = 6, N = 10, P = 4$ and $T = 9, N = 4, P = 8$), where CPLEX produced slightly better solutions. The most notable advantage of HSA is observed in the largest instances ($T = 9, N = 20$), where it achieved solutions that were 3.74% and 3.19% better than those obtained by CPLEX.

The HSA heuristic proves highly competitive, consistently outperforming CPLEX across mediumand large-scale instances. On average, the heuristic im-

Table 4.3: Performance of the proposed metaheuristic for small instances against CPLEX

			CPU Time		GAP
T	N	P	HSA	CPLEX	(HSA - Optim)
3	4	2	2.84	0.24	0.24
3	4	4	7.93	0.45	0.18
3	4	8	25.26	0.53	0.36
3	10	2	16.34	0.60	0.45
3	10	4	51.59	1.20	0.74
3	10	8	54.25	3.15	0.42
3	20	2	58.20	2.92	0.95
3	20	4	64.13	6.89	0.44
3	20	8	84.05	40.89	0.18
6	4	2	7.23	0.69	1.89
6	4	4	20.10	2.81	2.68
6	4	8	57.95	4.02	1.80
6	10	2	32.31	9.35	2.26
6	20	2	34.56	28.18	2.49
9	4	2	13.54	381.47	4.20
9	10	2	49.01	389.72	2.80
9	20	2	50.79	1738.56	2.52
Average			37.07	153.63	1.44

proved solutions by 0.71% across the entire dataset. These findings highlight the potential of HSA as a strong alternative for solving complex production and distribution problems, offering both efficiency and high-quality solutions.

4.5.4 *The case study: Comparing Company's Strategy with our proposed approach*

This section presents a case study of the PPE company, which operates two production units, each specializing in a specific type of medical gloves. The company follows a distribution system where products are either dispatched directly to customers upon production or temporarily stored in a warehouse before being distributed. The transportation network includes deliveries to a central hospital pharmacy and a major wholesaler, in addition to the warehouse, which serves both as a storage facility and a redistribution center. Given the considerable distances between customers and the high demand levels, the company currently employs a direct shipment strategy, delivering orders individually to each customer.

Table 4.4: Performance of the proposed metaheuristic for large instances against CPLEX

T	N	P	GAP	CPU Time
6	10	4	1.98	222
6	10	8	-2.17	67
6	20	4	-0.55	41
6	20	8	-0.06	98
9	4	4	-0.08	35
9	4	8	0.84	38
9	10	4	-0.03	44
9	10	8	-0.14	100
9	20	4	-3.75	61
9	20	8	-3.19	151
Average			-0.71	85.7

At present, production planning is managed independently from distribution decisions. The company follows an RMI policy, in which customers place their own orders, rather than the VMI approach, which has been shown to improve overall efficiency (Neves-Moreira et al., 2019, Mishra & Raghunathan, 2004). The production of each type of medical glove requires a setup process, and manufacturing operations are carried out in a single 8-hour shift, five days per week.

Product deliveries are scheduled by loading company-owned vehicles while ensuring that capacity limits are not exceeded. If customer demand surpasses the available fleets capacity, additional trucks are rented to meet delivery requirements. Delivery quantities match customer demand exactly, meaning that storage constraints at customer locations are not considered. Moreover, all orders must be fulfilled daily.

Currently, experienced planners perform production and distribution planning manually. While this approach often results in high-quality plans, it is time-consuming and prone to occasional errors. The company prioritizes production scheduling first, determining the timing and quantity of manufacturing, followed by distribution planning, which dictates when and how much to deliver to each customer.

Following the framework proposed by Absi et al. (2018), the company's current decision-making process can be classified as a "sequential RMI-based approach", in contrast to the integrated optimization model proposed in this study.

4.5.4.1 *Integrated vs. company's sequential approaches*

This section compares the company's sequential planning approach with our integrated method. While the companys current strategy separates production and distribution decisions, our proposed approach optimizes them simultaneously.

To evaluate both methods, we estimate the total production cost, which includes production expenses, setup costs, and inventory holding costs at the manufacturing facility, calculated as follows:

$$\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{P}} x_{j,t} \times b_j + \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{P}} y_{j,t} \times f_j + \sum_{j\in\mathcal{P}}\sum_{t\in\mathcal{T}} I_{1,j,t} \times h_{1,j,t} \tag{4.26}$$

Similarly, the total distribution cost is determined by summing transportation costs, vehicle utilization expenses, and inventory holding costs at retailers. However, since the company follows an RMI policy, retailer inventory costs are excluded from the evaluation:

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}-\{1\}} v_{i,t} \times c_i + \sum_{t\in\mathcal{T}} C_U \times Z_t \tag{4.27}$$

Sequential Approach
In the company's existing process, production planning is handled first to minimize costs associated with manufacturing, setup, and inventory storage. The demand values that must be met within each period are determined based on retailer consumption rates, assuming that deliveries align with consumption patterns. This approach allows for demand fulfillment while controlling production costs and follows standard practice in supply chain management.

To find exact solutions using this method, the capacitated lot-sizing problem is first solved using a standard MILP solver. The resulting production plan is then used as an input for distribution planning, which is also solved using an MILP solver.

Integrated Approach
Unlike the sequential method, the integrated approach simultaneously optimizes production and distribution decisions. This problem is formulated as an MILP and solved using the same solver. The comparison was conducted using real-world company data, with a planning horizon of $T = 5$ (weekly planning), two product types ($P = 2$), and three customers ($N = 4$). Our approach also allows for multiple trips per period, with two deliveries per time period considered in this study.

As the number of customers increases, the computational effort required to solve the problem also grows. To address this, both the integrated and sequential approaches can be solved using the HSA heuristic. In this case, the companys sequential approach is modeled as follows:

1. Production Planning: The capacitated lot-sizing problem is solved using Move 3 of the HSA heuristic.

2. Distribution Planning: The output of the lot-sizing solution is used as an input for Move 1 and Move 2 within HSA to optimize the distribution plan.

Figure 4.6 illustrates the comparison between the two methods. By integrating production and distribution planning and incorporating multiple trips, our approach achieved a cost reduction of 15.8% compared to the company's sequential strategy. These results highlight the advantages of integrating supply chain decisions, demonstrating the potential for significant cost savings and operational efficiency improvements.

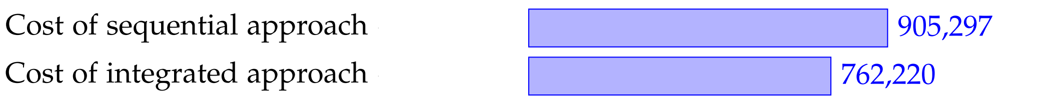


Figure 4.6: comparison of integrated and sequential approaches

4.5.4.2 *Integrated vs. sequential approaches in a VMI context*

As part of the VMI strategy, we explore two possible implementation options for the company:

- Zero Initial Inventory: This scenario assumes that customer inventory levels start at zero, with all demand fulfilled through deliveries under the VMI policy.

- Including Initial Inventory: Here, the initial inventory levels of customers are considered, allowing existing stock to be used before additional deliveries are made.

For both options, we compare the sequential approach with two trips per period against the integrated approach with one and two trips per period. Unlike the RMI context, the total distribution cost in this case also accounts for customer inventory holding costs, computed as follows:

$$\text{Cost} = \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}-\{1\}} \sum_{r\in\mathcal{R}} v_{itr} \times c_i + \sum_{t\in\mathcal{T}} C_U \times Z_t + \sum_{i\in\mathcal{N}-1} \sum_{j\in\mathcal{P}} \sum_{t\in\mathcal{T}} I_{ijt} \times h_{ijt} \tag{4.28}$$

In the sequential approach with initial inventory, determining the production plan requires estimating demand over time. This is done by assuming that deliveries coincide with consumption periods. The effective demand for production planning is adjusted by subtracting initial inventory levels from each retailers demand, as proposed by Absi et al. (2018):

$$\hat{d}_{ijt} = \max\left\{0, d_{ijt} - \max\{0, I_{ij0} - \sum_{t'=1}^{t-1} d_{ij,t'}\}\right\} \quad \forall i \in \mathcal{N}, j \in \mathcal{P}, t \in \mathcal{T} \quad (4.29)$$

To assess the impact of these different VMI configurations, we conducted a series of experiments comparing the two approaches under both implementation options. The total costs obtained for each method are illustrated in Figure 4.7, providing valuable insights for the company on the potential benefits of transitioning to a VMI-based system.

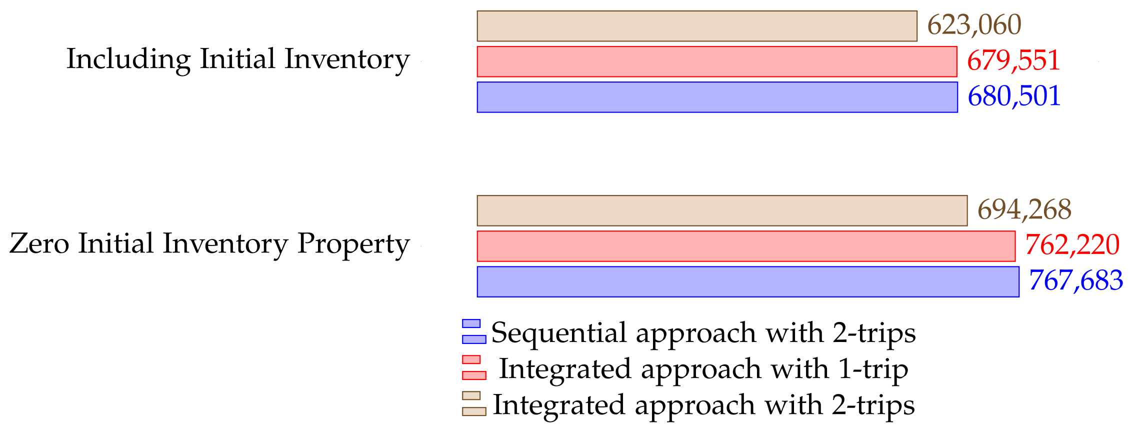


Figure 4.7: Total cost comparison of two VMI policy options in Integrated and Sequential Approaches

The findings indicate that the most cost-effective strategy involves integrating production and distribution decisions, adopting a VMI policy, allowing multiple trips, and accounting for initial inventory levels. This approach resulted in a 22% cost reduction compared to the integrated model with a single trip and 23% savings compared to the sequential model with two trips, both under a VMI framework with zero initial inventory. Additionally, it led to a 45.29% cost reduction when compared to the sequential approach under the RMI policy, which reflects the company's current strategy.

From these results, the company has two viable alternatives:

- By maintaining its sequential approach while using multi-trips, it can reduce costs by 17.92%.
- A more significant 33% cost reduction can be achieved by the integrated approach and incorporating initial inventory levels into the planning process, relative to the sequential RMI-based approach currently in use.

These insights highlight the potential advantages of shifting to an integrated planning strategy. By coordinating production and distribution decisions, the company can improve efficiency and reduce operational costs. However, transitioning to a VMI policy and incorporating multiple-trip deliveries may introduce additional complexities in the planning process, requiring adjustments to current management practices.

4.6 CONCLUSION AND FUTURE WORKS

This chapter explored the integration of production and distribution planning for multiple products over a finite planning horizon. The objective was to minimize total costs, including production, inventory (at both the manufacturing facility and customer locations), distribution, and vehicle utilization costs. To solve this complex problem, a HSA algorithm was developed, inspired by the work of Armentano et al. (2011). The heuristic iteratively adjusts delivery quantities across periods, determines the number of vehicles required to meet customer demand, and updates the production schedule accordingly. The approach was tested on randomly generated instances, demonstrating strong performance for small instances and outperforming CPLEX for medium and large-scale cases in terms of computational efficiency and solution quality.

A real-world case study was also examined, focusing on a company specializing in PPE manufacturing. The study considered various operational constraints, including multiple production units and multi-trip deliveries. By incorporating a VMI policy, accounting for initial customer inventory, and allowing multiple trips per period, the proposed approach achieved cost reductions of 45.29% compared to the companys current sequential strategy.

Despite these promising results, several limitations remain. First, routing decisions were not included, meaning the model assumes direct deliveries rather than optimizing routes for multiple customers. As the customer base expands, route optimization could become a critical factor. Additionally, inter-site transfers were not considered, even though they could be valuable given the nature of PPE products. Another limitation is the assumption that all data is deterministic, particularly transportation costs, which are influenced by fluctuating fuel prices and uncertain travel times.

Future research will focus on addressing these limitations. A key priority is to develop models that account for uncertainty in transportation costs, as well as potential variations in processing times and production yields. To handle such uncertainties, advanced robust optimization techniques (Bertsimas et al. 2011) could be explored, ensuring that solutions remain effective under real-world variability.

5 TWO-ECHELON PRODUCTION ROUTING PROBLEM

This chapter builds upon the previous one by extending the Production Routing Problem (PRP) to a Two-Echelon Production Routing Problem with Mixed Delivery Modes (2E-PRP). The objective is to minimize total production, inventory, and transportation costs within a two-echelon supply chain, where the first echelon consists of production plants, and the second echelon includes distribution centers. This chapter aims to: (1) formulate an original and complex 2E-PRP as a MILP, (2) adapt an existing heuristic from the literature as an initial solution approach, (3) develop a more efficient heuristic that outperforms both the adapted heuristic and a commercial MILP solver, and (4) provide managerial insights based on the findings.

The remainder of this chapter is structured as follows: Section 5.1 introduces the chapter, while Section 5.2 reviews the relevant literature. Section 5.3 defines the problem and presents its mathematical formulation. Section 5.4 describes the expanded Two-Phase Iterative heuristic and the Simulated Annealing with Path Relinking (SA-PR) algorithm. Section 5.5 presents numerical experiments, analyzing both computational performance and managerial implications. Finally, Section 5.6 concludes the chapter with key findings and directions for future research.

5.1 INTRODUCTION

Despite the benefits of integration, achieving full decision-making integration across all supply chain levels remains a significant challenge. As a result, research in Operations Research (OR) and Management Science (MS) often focuses on integrating specific processes or departments rather than the entire supply chain. One of the most complex and practically relevant problems in this domain is the PRP (Adulyasak et al., 2015b).

This chapter is motivated by real-world challenges faced by manufacturers in different industries, particularly a PPE manufacturer (analyzed in Chapter 4) and a soft drink company. These companies struggle with various inefficiencies, including a lack of integration between production and distribution planning, reliance on retailer-managed inventory systems leading to high costs, and manual planning processes that limit operational agility.

The research in this chapter develops and analyzes an extended version of the PRP: the 2E-PRP to address these issues. Despite its relevance to complex supply chains, the 2E-PRP has received limited attention in the literature. This problem seeks to minimize total costs associated with production, inventory, and transportation within a two-echelon supply chain, where the first echelon consists of production plants and the second echelon includes distribution centers. Key decisions involve production quantities, inventory levels, and vehicle routing. A distinguishing feature of this model is the incorporation of two delivery modes, reflecting common industry practices: high-priority customers receive direct shipments, while smaller customers are served through consolidated deliveries. This added complexity makes the 2E-PRP more challenging than the standard PRP while enhancing its applicability to real-world supply chain optimization.

5.2 LITERATURE REVIEW

This chapter reviews relevant literature by categorizing related studies into four key areas: the fundamentals of the PRP, research on single-echelon PRP, advancements in two-echelon PRP (2E-PRP), and commonly used solution approaches. This structured review helps position our study within the existing body of knowledge, highlight its contributions, and justify the chosen methodology.

The 2E-PRP originates from the broader class of PRP, which integrate elements from both LSP and VRP. Early foundational studies, such as those by Wagner and Whitin (1958) on lot-sizing and Dantzig and Ramser (1959) on vehicle routing, provided the basis for optimizing production and distribution decisions. Since then, extensive research has been conducted on both problems.

5.2.1 *The single echelon and the two-echelon PRP*

Before exploring the complexities of the 2E-PRP, it is essential to examine research focused on the Single-Echelon PRP. Early work in this area, such as Chandra (1993), aimed to minimize costs related to setup, production, inventory, and transportation. Later, Chandra and Fisher (1994) demonstrated that integrating production and routing decisions leads to significant cost reductions compared to treating LSP and VRP separately. More recently, Hrabec et al. (2022) quantified these benefits, showing that production-routing integration results in an average cost reduction of 11.1%.

Over the past two decades, the PRP has gained significant research attention, leading to numerous extensions, industry applications, and advancements in solution methodologies. Notable problem variations include single-item PRP (Archetti et al., 2011), multi-item PRP (Fumero & Vercellis, 1999), backordering PRP (Brahimi & Aouam, 2016), and remanufacturing PRP (Qiu et al., 2018). Practical applications have emerged across various industries, including fish feed production (Brekkå et al., 2022), meat processing (Neves-Moreira et al., 2019), petrochemicals (Schenekemberg et al., 2021), and home delivery services (Qiu et al., 2021). Additionally, Ramos et al. (2022) introduced an arc flow model for a multi-trip PRP with time-window constraints, further broadening the scope of PRP research. For an extensive review of PRP-related studies, refer to Adulyasak et al. (2015b) and Hrabec et al. (2022).

Research on the Two-Echelon Production Routing Problem (2E-PRP) remains scarce, with only a few contributions in the literature. Schenekemberg et al. (2021) introduced a 2E-PRP model for a vendor-managed inventory system in the petrochemical sector, incorporating production, inventory, pickups, and deliveries. Their approach optimized vehicle routes for transporting raw materials to production plants and finished goods to customers, with each vehicle restricted to one pickup or delivery per period. Their study also explored maximum level (ML) and order-up-to (OU) inventory policies.

Another relevant study by Qiu et al. (2021) investigated a 2E-PRP with cross-docking satellites, particularly applicable to e-commerce and urban logistics. In this model, products are transported from manufacturing facilities to cross-docking hubs, which then distribute them to customers without intermediate storage.

Our research builds upon these existing models by introducing a Two-Echelon Production Routing Problem with Mixed Delivery Modes (2E-PRP-MDM). Unlike prior studies, our model classifies customers into primary and secondary groups. In the first echelon, production plants send direct shipments to primary customers and warehouses, while in the second echelon, warehouses manage deliveries to secondary customers. This novel approach enhances practical relevance by accommodating different delivery priorities within a multi-echelon supply chain.

5.2.2 *Solution approaches*

Solution methods for the PRP can generally be categorized into exact algorithms and heuristic-based approaches, including decomposition heuristics, mixed-integer programming (MIP)-based heuristics, and metaheuristics. Due to the high computational complexity of PRPs, exact methods are primarily used to evaluate the performance of heuristics and metaheuristics rather than for practical large-scale applications.

5.2.2.1 *Exact Methods*

Most exact approaches for solving PRP rely on the Branch-and-Cut (B&C) algorithm, as seen in the works of Ruokokoski et al. (2010), Archetti et al. (2011), and Adulyasak et al. (2014a). These methods, however, require substantial computational time. For instance, the algorithm developed by Adulyasak et al. (2014a) required two hours to find optimal solutions for instances with 35 customers, three time periods, and three vehicles.

For the 2E-PRP, both Schenekemberg et al. (2021) and Qiu et al. (2021) employed B&C algorithms. Additionally, Schenekemberg et al. (2021) introduced a parallel computing-based exact algorithm with local search, while Qiu et al. (2021) proposed a matheuristic to generate an initial feasible solution. Although exact methods provide optimal solutions and allow for reliable managerial insights, their long computational times make them impractical for real-world applications.

5.2.2.2 *Decomposition Heuristics*

Several researchers have employed decomposition heuristics to solve PRP by breaking it into smaller, more manageable subproblems. Studies such as Chandra (1993), Chandra and Fisher (1994), Boudia et al. (2008), Neves-Moreira et al. (2019), and Chekoubi et al. (2022) follow this approach. Typically, these heuristics solve production and routing subproblems sequentially, refining the solution through an improvement heuristic. The uniqueness of each method lies in the sequence of solving subproblems and the refinement technique used.

5.2.2.3 *Mathematical Programming-Based Heuristics*

Mathematical programming heuristics have also been widely used in PRP research. Brahimi and Aouam (2016) applied a relax-and-fix heuristic combined with a local search procedure, solving lot-sizing decisions first, followed by routing. Another approach by Absi et al. (2015) introduced an iterative heuristic, where routing costs were approximated as fixed values in the PRP formulation. At each iteration, a MIP model was solved optimally, followed by a heuristic-based routing decision update. This iterative process continued until convergence.

5.2.2.4 *Metaheuristic Approaches*

Metaheuristics have proven to be effective in solving large-scale PRP instances by providing a balance between solution quality and computational efficiency. Notable metaheuristics used for PRP include Greedy Randomized Adaptive Search Procedure (GRASP) proposed by (Boudia et al., 2007), Memetic Algorithms (Boudia and Prins 2009), Tabu Search (Bard and Nananukul 2009; Shiguemoto and Armentano 2010), Adaptive Large Neighborhood Search (ALNS) (Adulyasak et al. 2014b).

According to Adulyasak et al. (2015b), ALNS provides an excellent trade-off between computational time and solution quality. However, Absi et al. (2015) demonstrated that mathematical programming-based heuristics can outperform ALNS regarding solution quality.

Simulated annealing (SA) has been widely applied to combinatorial optimization problems such as the lot-sizing problem (Ceschia et al., 2017), yet its use in PRP remains limited. Moreover, the combination of simulated annealing with path relinking (SA-PR) has not been explored for PRP. Given its potential, this study applies SA-PR to the 2E-PRP, demonstrating its efficiency through numerical experiments.

To solve the 2E-PRP-MDM, we developed a MILP model, as formulated in Section 5.3, and solved it using a commercial solver. Additionally, we propose a three-phase heuristic, extending the two-phase heuristic of Absi et al. (2015). Finally, we introduce a SA-PR algorithm.

5.3 PROBLEM DESCRIPTION AND FORMULATION

This study addresses a single-item two-echelon production routing problem, where a manufacturing plant produces and distributes goods through a multi-level supply chain network. The distribution system is represented as an undirected graph $G = (\mathcal{N}, A)$, where $\mathcal{N}$ consists of four distinct sets of nodes:

- $\mathcal{N}_0$: Manufacturing plant(s).
- $\mathcal{N}_c$: Customers directly served by the plant.
- $\mathcal{N}_w$: Warehouses that receive shipments from the plant.
- $\mathcal{N}_{c'}$: Customers supplied by the warehouses.

The edges A represent transportation links between these nodes. A schematic representation of the network is provided in Figure 5.1.

The problem operates over two echelons:

- First Echelon: The manufacturing plant (nodes in N_0) directly supplies products to both customers (N_c) and warehouses (N_w). This transportation is handled by an unlimited fleet of vehicles, each with a capacity of Q_0. The transportation cost from the plant to a receiving node $i \in (N_c \cup \mathcal{N}_w)$ is denoted as c_i.
- Second Echelon: Warehouses (N_w) are responsible for distributing products to secondary customers ($N_{c'}$). Unlike the first echelon, this stage uses a limited fleet of homogeneous vehicles (K), each with a capacity of Q. Following

prior research (e.g., Adulyasak et al. 2014a), each customer in $N_{c'}$ receives a single delivery per time period from a designated vehicle. Additionally, in line with Qiu et al. (2021), each customer can only be served by one warehouse per period. The transportation cost between warehouses and customers is represented as c_{ij}, where $i, j \in (N_w \cup \mathcal{N}_{c'})$.

Warehouses play a crucial role in aggregating demand across multiple locations and time periods. They contribute to economies of scale, reducing overall logistics costs. Additionally, due to storage constraints at the manufacturing plant, warehouses provide buffer storage for excess production. In certain cases, warehouses also offer lower inventory holding costs per unit due to specialized storage infrastructure.

The problem is defined over a finite planning horizon of T discrete time periods. The manufacturing plant has a limited production capacity of C units per period and incurs: production costs (p) per unit produced and setup costs (f) for initiating production in any given period. Each node $i \in \mathcal{N}$ has: inventory holding costs (h_i) per unit stored, an initial inventory level (I_{i0}) and a maximum inventory capacity (U_i). Customer demand for each period t is given as d_{it} for $i \in (N_c \cup \mathcal{N}_{c'})$.

For each time period, the following key decisions must be made:

- Production levels at the manufacturing plant
- Shipment quantities from:
  - The plant to warehouses and direct customers (N_c)
  - Warehouses to secondary customers ($N_{c'}$)
- Routing sequence for deliveries in the second echelon ($N_{c'}$)
- Inventory levels at the plant, warehouses, and customer locations at the end of each period

The objective is to minimize the total supply chain costs, which include production, inventory holding, setup, and transportation costs.

In this chapter, we focus on a single manufacturing plant. However, the model can be extended to accommodate multiple plants with minor modifications. We introduce the following notation.

Sets and parameters:

- Sets:
  $\mathcal{T} = \{1, .., T\}$: set of time periods in the planning horizon.
  $\mathcal{N} = \{\mathcal{N}_0 \cup \mathcal{N}_c \cup \mathcal{N}_w \cup \mathcal{N}_{c'}\}$: set of nodes (processing unit, customers in the first echelon, warehouses, and customers in the second echelon) where $\mathcal{N}_0 = \{0\}, \mathcal{N}_c = \{1, .., N_c\}, \mathcal{N}_w = \{N_c + 1, .., N_c + N_w\},$ *and* $\mathcal{N}_{c'} = \{N_c + N_w +$

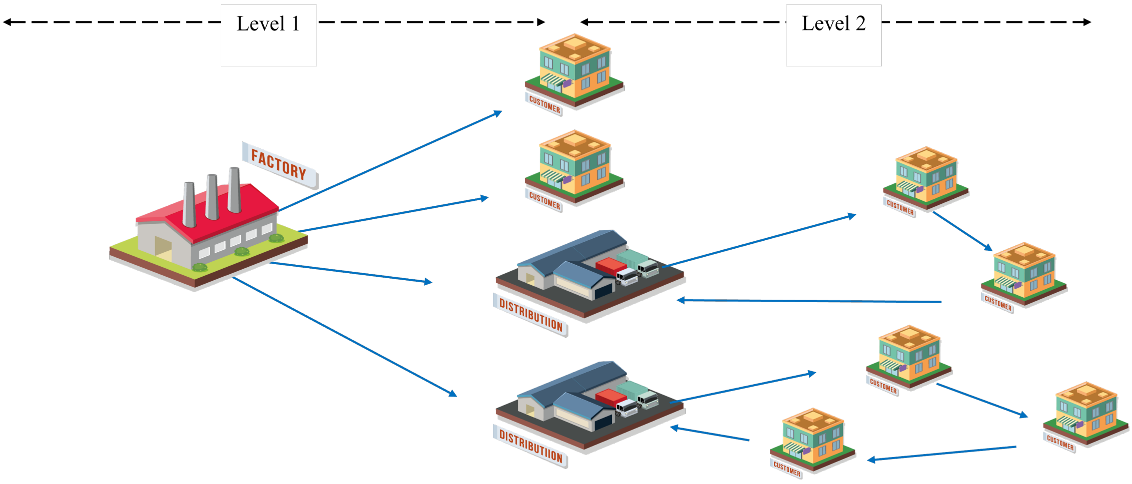


Figure 5.1: Graphical Illustration of Supply Network (Benfedel et al. 2025b)

$1, .., N_c + N_w + N_{c'}\}$.
$A = \{A_1 \cup A_2\}$: set of arcs, where $A_1 = \{(0, i) : i \in \mathcal{N}_c \cup \mathcal{N}_w\}$ and $A_2 = \{(i, j) : i, j \in \mathcal{N}_{c'}, i \neq j \vee i \in \mathcal{N}_{\rfloor'}, j \in \mathcal{N}_w \vee i \in \mathcal{N}_w, j \in \mathcal{N}_{c'}\}$.
$\mathcal{K} = \{1, .., K\}$: set of homogeneous vehicles at the warehouses.

- Production data:
  p : Production cost.
  f : Setup cost.
  C : Production capacity.

- Inventory data:
  h_i : Inventory Holding cost at node $i \in \mathcal{N}$.
  U_i : Storage capacity at location $i \in \mathcal{N}$.
  I_{0i} : Initial inventory at location $i \in \mathcal{N}$.
  d_{it} : Demand of customer $i \in \{\mathcal{N}_c \cup \mathcal{N}_{c'}\}$ in period t.

- Transport data:
  c_i : Travel cost from manufacturing plant to warehouses and customer $i \in \mathcal{N}_w \cup \mathcal{N}_c$.
  c_{ij} : Travel cost between $(i, j)\{i, j \in \mathcal{N}_w \cup \mathcal{N}_{c'}\}$.
  Q_0 : manufacturing plant's vehicle capacity.
  Q_w : warehouses vehicle capacity.

Decision variables:

- Production:
  x_t: Production level at period t.
  y_t: Binary setup variable equal to 1 if there is a production in period t and 0 otherwise.

- Inventory:
  I_{it}: Inventory level of the product at location i in period t.

- Transport:
  v_{it}: Number of vehicles sent to node $i \in \mathcal{N}_c \cup \mathcal{N}_w$ in period t.
  q_{it}: Amount of product shipped to node $i \in \mathcal{N}_c \cup \mathcal{N}_w$ in period t
  r_{ijkt}: Amount delivered from warehouse i to customer $j \in \mathcal{N}_{c'}$ by vehicle k at period t
  L_{wijkt}: amount of load carried by vehicle k of warehouse w when arc $(i,j) \in A_2$ is traversed in period t.
  H_{ijkt}: binary variable, equal to 1 if vehicle k serves customer $j \in \mathcal{N}_{c'}$ from warehouse i in period t, 0 otherwise.
  Y_{wijkt}: binary variable, equal to 1 if arc $(i,j) \in A2$ is traversed in period t by vehicle k of warehouse w, 0 otherwise.

The proposed MILP formulation is presented below.

$$[\mathbf{2E-PRP-MDM}] \min \sum_{t\in\mathcal{T}} (px_t + fy_t) + \sum_{i\in\mathcal{N}} \sum_{t\in\mathcal{T}} h_i I_{it} + \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_c\cup\mathcal{N}_w} c_i v_{it} + \sum_{(i,j)\in A_2} \sum_{w\in\mathcal{N}_w} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} c_{ij} Y_{wijkt} \quad (5.1)$$

Subject to:

$$I_{0t} = I_{0,t-1} + x_t - \sum_{i\in\mathcal{N}_c\cup\mathcal{N}_w} q_{it}, \quad \forall t \in \mathcal{T} \quad (5.2)$$

$$I_{it} = I_{i,t-1} + q_{it} - d_{it}, \quad \forall i \in \mathcal{N}_c, \forall t \in \mathcal{T} \quad (5.3)$$

$$I_{it} = I_{i,t-1} + q_{it} - \sum_{j\in\mathcal{N}_{c'}} \sum_{k\in\mathcal{K}} r_{ijkt}, \quad \forall i \in \mathcal{N}_w, \forall t \in \mathcal{T} \quad (5.4)$$

$$I_{jt} = I_{j,t-1} + \sum_{i\in\mathcal{N}_w} \sum_{k\in\mathcal{K}} r_{ijkt} - d_{jt}, \quad \forall j \in \mathcal{N}_{c'}, \forall t \in \mathcal{T} \quad (5.5)$$

$$I_{it} \leq U_i, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \quad (5.6)$$

$$x_t \leq y_t \times \min\{C, \sum_{t'\in\mathcal{T}} \sum_{i\in\mathcal{N}_c,\mathcal{N}_{c'}} d_{i,t'}\}, \quad \forall t \in \mathcal{T} \quad (5.7)$$

$$q_{it} \leq Q_0 \times v_{it}, \quad \forall i \in \mathcal{N}_c \cup \mathcal{N}_w, \forall t \in \mathcal{T} \quad (5.8)$$

$$\sum_{j\in\mathcal{N}_{c'}} r_{ijkt} \leq Q_i H_{ijkt}, \quad \forall i \in \mathcal{N}_w, \forall t \in \mathcal{T}, \forall k \in \mathcal{K} \quad (5.9)$$

$$r_{ijkt} \leq H_{ijkt} \times \min\left\{\sum_{t' \in \mathcal{T}} d_{j,t'}, U_j + d_{j,t}, Q_i\right\}, \quad \forall i \in \mathcal{N}_w, \forall j \in \mathcal{N}_{c'}, \forall t \in \mathcal{T}, \forall k \in \mathcal{K} \tag{5.10}$$

$$\sum_{i \in \mathcal{N}_w} \sum_{k \in \mathcal{K}} H_{ijkt} \leq 1, \quad \forall j \in \mathcal{N}_{c'}, \forall t \in \mathcal{T} \tag{5.11}$$

$$L_{wijkt} \leq Q_i Y_{wijkt}, \quad \forall w \in \mathcal{N}_w, \forall i, j \in \mathcal{N}_w \cup \mathcal{N}_{c'}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5.12}$$

$$\sum_{j \in \mathcal{N}_{c'}} L_{iijkt} = \sum_{j \in \mathcal{N}_{c'}} r_{ijkt}, \quad \forall i \in \mathcal{N}_w, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5.13}$$

$$\sum_{i \in \mathcal{N}_w \cup \mathcal{N}_{c'}} L_{wijkt} - \sum_{i \in \mathcal{N}_w \cup \mathcal{N}_{c'}} L_{wjikt} = r_{wjkt}, \quad \forall w \in \mathcal{N}_w, \forall j \in \mathcal{N}_{c'}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5.14}$$

$$\sum_{j \in \mathcal{N}_w \cup \mathcal{N}_{c'}} Y_{wijkt} + \sum_{j \in \mathcal{N}_w \cup \mathcal{N}_{c'}} Y_{wjikt} = 2H_{wikt}, \quad \forall w \in \mathcal{N}_w, i \in \mathcal{N}_w \cup \mathcal{N}_{c'} \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5.15}$$

$$\sum_{j \in \mathcal{N}_w \cup \mathcal{N}_{c'}, i \neq j} Y_{wijkt} = H_{wikt}, \quad \forall w \in \mathcal{N}_w, \forall i \in \mathcal{N}_w \cup \mathcal{N}_{c'}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5.16}$$

$$\sum_{i \in \mathcal{N}_w \cup \mathcal{N}_{c'}} \sum_{j \in \mathcal{N}_{c'}} Y_{wijkt} \leq \sum_{j \in \mathcal{N}_{c'}} r_{wjkt} / Q_w, \quad \forall w \in \mathcal{N}_w, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5.17}$$

$$x_t \geq 0, \quad \forall t \in \mathcal{T} \tag{5.18}$$

$$I_{it} \geq 0, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \tag{5.19}$$

$$q_{it}, v_{it} \geq 0, \quad \forall i \in \mathcal{N}_0 \cup \mathcal{N}_c \cup \mathcal{N}_w, \forall t \in \mathcal{T} \tag{5.20}$$

$$r_{wjkt} \geq 0, \quad \forall w \in \mathcal{N}_w, \forall j \in \mathcal{N}_{c'}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5.21}$$

$$L_{wijkt} \geq 0, \quad \forall w \in \mathcal{N}_w, \forall i, j \in \mathcal{N}_w \cup \mathcal{N}_{c'}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5.22}$$

$$y_t \in \{0, 1\}, \quad \forall t \in \mathcal{T} \tag{5.23}$$

$$H_{wjkt} \in \{0, 1\}, \quad \forall w \in \mathcal{N}_w, \forall j \in \mathcal{N}_{c'}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5.24}$$

$$Y_{wijkt} \in \{0, 1\}, \quad \forall w \in \mathcal{N}_w, \forall i, j \in \mathcal{N}_w \cup \mathcal{N}_{c'}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{5.25}$$

The objective function (5.3) aims to minimize total costs, which include production and setup costs, inventory holding costs at the manufacturing plant, warehouses, and customers, as well as transportation costs for both direct shipments in the first echelon and deliveries in the second echelon. The inventory balance constraints (5.2)(5.5) regulate stock levels at the manufacturing plant, first-echelon customers, warehouses, and second-echelon customers, ensuring consistency between incoming shipments, demand fulfillment, and inventory levels. Constraints (5.6) enforce inventory capacity limits at all storage locations. The production constraints (5.7) ensure that the total production volume does not exceed plant capacity. Additionally, these constraints establish a link between the continuous production variable x_t and the binary setup variable y_t, ensuring that a setup cost is incurred only when production occurs. Constraints (5.8) and (5.9) ensure that the total quantity delivered to first- and second-echelon customers does not exceed the respective vehicle capacities. Constraints (5.10) establish a connection between delivery quantities and customer visits, while constraints (5.11) enforce the rule that each customer can only be served by one warehouse and one vehicle per period. For vehicle routing, constraints (5.12) enforce vehicle load limits within the second-echelon network, while constraints (5.13) and (5.14) define the vehicle's load based on the delivered quantities. Constraints (5.15) and (5.16) ensure that a vehicle must arrive at a customer location before departing, maintaining route feasibility. Constraints (5.17) guarantee that the selected transportation arcs form a connected route, as proposed by Chandra and Fisher (1994). Constraints (5.13) - (5.17) are multi-stop adaptations of single-stop routing constraints. Finally, constraints (5.18)-(5.25) enforce integrality and non-negativity conditions.

5.3.1 *Numerical Example*

To illustrate the problem, we present a simplified example involving a single manufacturer, one warehouse, and three customers over a three-period planning horizon. The manufacturer directly supplies a primary customer and a warehouse, which in turn serves two additional customers.

The manufacturer incurs a production cost of 30 per unit and a setup cost of 3000 per production period. Inventory holding costs vary across locations: 3 per unit at the manufacturer and warehouse, 6 per unit for the primary customer, and 7 and 8 per unit for the two additional customers, respectively. The manufacturer has a production capacity of 100 units per period and a storage limit of 50 units, while the warehouse can store up to 40 units and operates a vehicle with a 50-unit capacity. Each customer has an inventory limit of 20 units. Demand is spread over three periods, with the primary customer requiring 10 units per period, while Customer 1 and Customer 2 require 7 and 13 units per period, respectively. Transportation costs between different nodes are as follows: 450 from the manufacturer to the warehouse, 162 from the manufacturer to the primary customer, 385 from the warehouse to Customer 1, 154 from the warehouse to Customer 2, and 267 between Customer 1 and Customer 2.

The optimal total cost for this scenario is 8385, with detailed results presented in Tables 5.1, 5.2, 5.3, and 5.4. The manufacturer produces 90 units in the first period to minimize setup costs, avoiding unnecessary production in later periods. The manufacturer preemptively supplies the primary customer with 30 units to further reduce transportation costs, allowing for future demand fulfillment. The customer holds 20 units in inventory in Period 1 and 10 units in Period 2, covering demand for Period 3.

Table 5.1: Production quantity and Setup times

	t1	t2	t3
Production	90	0	0
Setup	1	0	0

Table 5.2: Inventory quantity

	t1	t2	t3
Manufacturer	0	0	0
Warehouse	13	13	0
Primary customer	20	10	0
Customer 1	14	7	0
Customer 2	13	0	0

Table 5.3: Delivery quantity of first-level

	t1	t2	t3
Warehouse	60	0	0
Primary customer	30	0	0

Table 5.4: Delivery quantity of second-level

	t1	t2	t3
Customer 1	21	0	0
Customer 2	26	0	13

Simultaneously, the manufacturer delivers 60 units to the warehouse, ensuring inventory availability for Customer 1 in Period 1 and Customer 2 in both Periods 1 and 3. This coordinated approach balances production, inventory, and transportation costs while meeting customer demands efficiently.

5.4 SOLUTION APPROACH

This section introduces an enhanced version of the Two-Phase Iterative Heuristic, originally developed for the classic PRP by Absi et al. (2015), adapted to address the 2E-PRP with Mixed Delivery Modes (2E-PRP-MDM). Additionally, we propose a SA heuristic combined with a Path Relinking (PR) strategy to further improve solution quality. The performance of both heuristic approaches is assessed by comparing their results against those obtained using a commercial MILP solver applied to the 2E-PRP-MDM formulation.

5.4.1 *Three-Phase Iterative (3PI) Heuristic Approach*

As highlighted in the review by Adulyasak et al. (2015b), the two-phase iterative heuristic developed by Absi et al. (2015) is recognized as one of the most effective approaches for solving the PRP. Based on its proven efficiency, we extend this method to address the 2E-PRP-MDM, resulting in the development of the 3PI Heuristic. The heuristic consists of the following key phases:

- Phase I: *Echelons Coordination Phase* This phase incorporates estimated routing costs (SC_{wit}) to determine visit schedules for second-echelon customers and establish delivery quantities (r_{wit}). The decision-making process accounts for production capacity, vehicle capacity, and inventory limitations.

- Phase IIa: *Lot Sizing with Direct Shipment Phase* At this stage, production quantities, delivery schedules, and vehicle assignments for warehouses and first-echelon customers are determined. Constraints related to production limits, vehicle capacities, and inventory restrictions are considered to ensure feasible planning.
- Phase IIb: *Routing Phase* This phase refines the delivery routes based on the results obtained in Phase I, addressing the VRP to enhance transportation efficiency.

After executing these three phases for a defined number of iterations (i_{max}), a diversification mechanism is introduced to explore alternative solutions. This iterative process continues until the predefined iteration threshold (j_{max}) is reached. Figure 5.2 presents a flowchart illustrating the overall structure of the heuristic. Detailed explanations of each phase and the diversification mechanism are provided in the following subsections.

5.4.1.1 *Phase I: Echelons Coordination Phase*

This phase focuses on optimizing the flow of goods from the manufacturer to warehouses and scheduling the distribution of products to second-echelon customers. The cost associated with serving a second-level customer i from warehouse w in period t is denoted as SC_{wit}. The primary objective is to minimize total system costs, which include inventory holding costs at both warehouses and second-level customers, transportation costs for shipments from the manufacturer to warehouses, and the additional costs incurred when integrating second-echelon customers into vehicle routes. The first phase model (PhI-MILP) is:

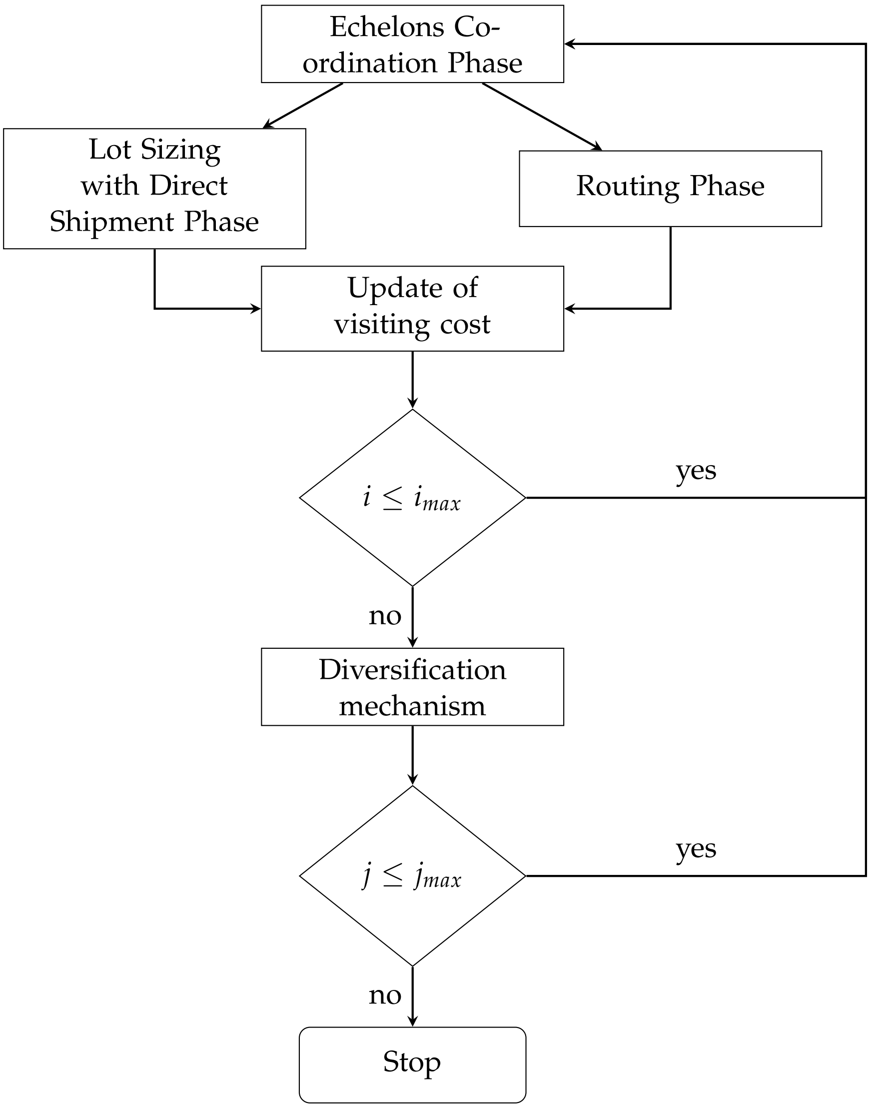


Figure 5.2: Flowchart of the proposed Three-Phase Iterative (3PI) Heuristic Approach.

$$[\mathbf{PhI-MILP}] \quad \min \sum_{i \in \mathcal{N}_w \cup \mathcal{N}_{c'}} \sum_{t \in \mathcal{T}} h_i I_{it} + \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}_w} c_i v_{it} + \sum_{w \in \mathcal{N}_w} \sum_{i \in \mathcal{N}_{c'}} \sum_{t \in \mathcal{T}} SC_{wit} H_{wit} \tag{5.26}$$

Subject to Constraints (5.4) - (5.6), and

$$q_{it} \leq Q_0 \times v_{it}, \quad \forall i \in \mathcal{N}_w, \forall t \in \mathcal{T} \tag{5.27}$$

$$\sum_{j \in \mathcal{N}_{c'}} r_{ijt} \leq \lambda_{it} \times K \times Q_i, \quad \forall i \in \mathcal{N}_w, \forall t \in \mathcal{T}, \forall k \in \mathcal{K} \tag{5.28}$$

$$r_{ijt} \leq H_{ijt} \times \min\{\sum_{t' \in \mathcal{T}} d_{j,t'}, U_j + d_{j,t}, Q_i\}, \quad \forall i \in \mathcal{N}_w, \forall j \in \mathcal{N}_{c'}, \forall t \in \mathcal{T} \tag{5.29}$$

$$\sum_{i\in\mathcal{N}_w} H_{ijt} \leq 1, \quad \forall j \in \mathcal{N}_{c'}, \forall t \in \mathcal{T} \tag{5.30}$$

$$\sum_{i\in\mathcal{N}_w} q_{it} + \sum_{i\in\mathcal{N}_c} \delta_{it} \leq C, \quad \forall t \in \mathcal{T} \tag{5.31}$$

$$I_{it} \geq 0, \quad \forall i \in \mathcal{N}_w \cup \mathcal{N}_{c'},\ t \in \mathcal{T} \tag{5.32}$$

$$q_{it}, v_{it} \geq 0, \quad \forall i \in \mathcal{N}_w,\ t \in \mathcal{T} \tag{5.33}$$

$$r_{ijt} \geq 0, \quad \forall i \in \mathcal{N}_w, \forall j \in \mathcal{N}_{c'}, t \in \mathcal{T} \tag{5.34}$$

$$H_{ijt} \in \{0,1\}, \quad \forall i \in \mathcal{N}_w, \forall j \in \mathcal{N}_{c'}, \forall t \in \mathcal{T} \tag{5.35}$$

Constraints (5.28) ensure that the total quantity delivered does not exceed the available vehicle capacity at any given time. A scaling factor λ_{it} is introduced, initially set to 1, and later adjusted following specific rules that will be detailed at the end of this section. Constraints (5.29) and (5.30) serve the same function as constraints (5.10) and (5.11) in the 2E-PRP-MDM model but without including the vehicle index k in the variables r and H. In constraints (5.31), the parameter δ_{it} is initially defined as: $\delta_{it} = \max(0, d_{it} - \max(0, I_{i0} - \sum_{t'=1}^{t-1} d_{it})), \forall i \in \mathcal{N}_c$. This value is later updated based on the actual delivery quantities in the first echelon. Additionally, these constraints ensure compliance with production capacity limits.

5.4.1.2 *Phase IIa: Lot Sizing with Direct Shipment*

This phase focuses on determining optimal production quantities, scheduling warehouse and first-echelon customer visits, allocating delivery quantities, and assigning the required number of vehicles. The decision-making process adheres to production limits, inventory capacity, and vehicle constraints to ensure efficient and timely product distribution.

The primary goal is to minimize production, inventory holding, and transportation costs while ensuring that demand is effectively met. The mathematical model for this phase, PhIIa-MILP, captures these objectives.

$$[\mathbf{PhIIa-MILP}] \quad \min \sum_{t\in\mathcal{T}} px_t + \sum_{t\in\mathcal{T}} fy_t + \sum_{i\in\mathcal{N}_0\cup\mathcal{N}_c\cup\mathcal{N}_w} \sum_{t\in\mathcal{T}} h_i I_{it} + \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_c\cup\mathcal{N}_w} c_i v_{it} \tag{5.36}$$

Subject to Constraints (5.2), (5.3), (5.6), (5.8), (5.18) – (5.20), (5.23), and

$$x_t \leq y_t \min\left\{C, \sum_{t'\in\mathcal{T}} \sum_{i\in\mathcal{N}_c,\mathcal{N}_w} d_{i,t'}\right\}, \quad \forall t \in \mathcal{T} \tag{5.37}$$

In constraints (5.37), the total quantity dispatched from a warehouse to its assigned customers is treated as the warehouses effective demand, expressed as: $d_{it} = \sum_{i\in\mathcal{N}_{c'}} r_{ijt}, \forall i \in \mathcal{N}_w$

5.4.1.3 *Phase IIb: Routing Phase*

In Phase IIb, the decision variables r_{ijt} and H_{ijt}, determined in Phase I, are utilized to address a VRP and, when $K \geq 2$, a TSP at each time period. To enhance computational efficiency, a heuristic approach is employed for solving both problems. Specifically, we utilize the LKH-3 algorithm, as proposed by Helsgaun (2017).

For each warehouse $i \in \mathcal{N}_w$, if the LKH-3 heuristic fails to identify a feasible solution in period t, the parameter λ_{it} is decreased according to the formula: $\lambda_{it} = \lambda_{it} - \eta$, where η is a predefined constant. Conversely, when a feasible solution is found, λ_{it} is increased, provided it does not exceed 1, following: $\lambda_{it} = \min(\lambda_{it} + \eta, 1)$. As part of the diversification strategy, all λ_{it} values are reset to their initial state of 1 when necessary to explore alternative solutions.

5.4.1.4 *Update of visiting cost*

The visiting costs play a crucial role in linking Phase I and Phase IIb of the approach. Initially, these costs are derived from transportation expenses and are dynamically adjusted based on the solutions obtained in Phase IIb. This iterative update helps enhance retailer clustering in subsequent iterations.

To formalize the process, we define key parameters. Let S_{wt} represent the set of retailers served by warehouse w at time period t. The predecessor and successor of a given node i are denoted as i^- and i^+, respectively. For a given vehicle $k \in \mathcal{K}$, period $t \in \mathcal{T}$, and a customer $i \notin S_{wt}$, we define Δ_{kit} as the minimum additional cost of inserting i into vehicle k's route.

The visiting cost SC_{wit} is updated as follows, for each warehouse $w \in \mathcal{N}_w$, $t \in \mathcal{T}$ and $k \in \mathcal{K}$:

- A retailer i already in the assigned route ($i \in S_{wt}$), the cost is computed as: $SC_{wit} = c_{i^-i} + c_{ii^+} - c_{i^-i^+}$.
- A retailer i not currently in the route ($i \notin S_{wt}$), the cost is assigned based on the lowest insertion cost across all vehicles: $SC_{wit} = \min_{k \in \mathcal{K}} \Delta_{kit}$.

5.4.1.5 *Diversification mechanism*

The diversification strategy utilizes the best-known solution to adjust visiting costs, steering the heuristic search toward less-explored areas of the solution space. This is accomplished by modifying the visiting cost SC_{wit} for each retailer in period t based on the number of retailers served during that period. Specifically, the visiting cost is multiplied by the total number of served retailers plus one, incorporating insights from the current best solution.

This mechanism promotes a redistribution of retailer visits, encouraging a shift from periods with higher visit frequencies to those with fewer assigned deliveries, ultimately fostering a more balanced and diverse set of solutions.

To demonstrate how the diversification mechanism operates, consider a scenario with one warehouse, four retailers, and a three-period planning horizon. The initial visiting cost SC_{wit} and visit schedule H_{wit} are provided in Tables 5.5 and 5.6. By analyzing Table 5.6, we determine the number of retailers served in each period:

Table 5.5: Visiting Cost SC_{wit}

	t1	t2	t3
C1	10	10	10
C2	30	30	30
C3	5	5	5
C4	15	15	15

Table 5.6: Visiting periods H_{wit}

	t1	t2	t3
C1	0	1	1
C2	1	1	0
C3	0	1	0
C4	1	1	0

2 retailers in the first period, 4 in the second, and 1 in the third. The visiting cost for each retailer is then multiplied by the corresponding number of visits plus one, leading to an adjusted SC_{wit}, as shown in Table 5.7. This adjustment helps the heuristic redistribute retailer assignments, shifting visits from periods with high visit frequencies to those with fewer scheduled deliveries, thereby exploring alternative solutions. To illustrate, Tables 5.8 and 5.9 present two possible outcomes:

Table 5.7: New Visiting Cost SC_{wit}

	t1	t2	t3
C1	21	41	11
C2	61	121	31
C3	11	21	6
C4	31	61	16

Table 5.8: Solution 1

	t1	t2	t3
C1	1	0	1
C2	1	1	1
C3	0	1	1
C4	1	1	1

Table 5.9: Solution 2

	t1	t2	t3
C1	1	0	1
C2	1	0	0
C3	1	0	0
C4	1	0	0

- Solution 1 prioritizes visiting the first retailer in the first period, as the updated cost is lower.

- Solution 2 suggests moving all visits from the second period to the first since visiting costs in the first period are now lower for all retailers.

5.4.2 *Simulated Annealing with Path Relinking Procedure (SA-PR)*

The SA-PR algorithm is a heuristic optimization method structured into three key steps[1] The process begins with Step I, where an initial solution is generated. In Step II, simulated annealing is applied to refine this solution, after which the γ best solutions obtained are selected for further enhancement. Finally, Step III employs path relinking to improve the selected solutions.

5.4.2.1 *Step I: Initial Solution*

The initial solution is derived from the first iteration of the 3PI heuristic described in Section 5.4.1. This involves executing Phase I, Phase IIa, and Phase IIb once, resulting in the construction of the starting solution for the SA-PR algorithm.

5.4.2.2 *Step II: Simulated Annealing*

The SA algorithm consists of three key stages: the first stage generates new delivery plans for both echelons, the second focuses on production scheduling, and the third determines vehicle routing. Each of these stages is explained in detail below.

- **1. Delivery Neighboring Generation**: In this stage, modifications to delivery quantities (q_{it} and r_{ijt}) from the initial solution are introduced to explore alternative solutions. These adjustments involve shifting delivery quantities between periods while ensuring feasibility constraints are met.

  The first echelon, consisting of the manufacturer, warehouses, and first-level customers, is adjusted as follows:

  - A random subset of nodes (E, including first-level customers and warehouses) and two time periods (t, t') are selected.
  - If $t' < t$, the quantity to be transferred ($s_{it,t'}$) is determined by:

$$s_{it,t'} = \min\left\{q_{it}, \left(t' \times C - \sum_{i \in \mathcal{N}_c \cup \mathcal{N}_w} \sum_{t'' \in \mathcal{T}'} q_{i,t''}\right), \min_{t''} \left\{U_i - I_{i,t''}, I_{0,t''}\right\}\right\} \tag{5.38}$$

  - If $t' > t$, the quantity transfer follows:

$$s_{it,t'} = \min\left\{q_{it}, \min_{t''}\left\{I_{i,t''}\right\}\right\} \tag{5.39}$$

1 The term "Phase" was intentionally avoided to prevent confusion with the phases of the 3PI heuristic.

For the second echelon, adjustments are made to the delivery quantities r_{ijt} as follows:

- A random group of second-echelon customers (E), two warehouses (w_1 and w_2), and two time periods (t and t') are selected. The modified delivery quantity ($s_{i,w_1,w_2,t,t'}$) represents the amount shifted between warehouses.
- If $t' < t$, the quantity shift is determined by:

$$s_{i,w_1,w_2,t,t'} = \min\Big\{ r_{w_1,it}, \Big(t' \times C - \sum_{j \in \mathcal{N}_c \cup \mathcal{N}_w} \sum_{t'' \in \mathcal{T}'} q_{j,t''} \Big), \Big(Q_{w_2} \times K - \sum_{j \in \mathcal{N}_{c'}} r_{w_2,j,t'} \Big), \min_{t''} \left\{ U_i - I_{i,t''}, I_{w_2,t''} \right\} \Big\} \quad (5.40)$$

- If $t' > t$, the adjustment follows:

$$s_{i,w_1,w_2,t,t'} = \min\Big\{ r_{w_1,it}, \Big(Q_{w_2} \times K - \sum_{j \in \mathcal{N}_{c'}} r_{w_2,j,t'} \Big), \min_{t''} \{ I_{i,t''} \} \Big\} \quad (5.41)$$

Since split deliveries are not permitted, if the transfer from t to t' involves different warehouses ($w_1 \neq w_2$) and results in $s_{i,w_1,w_2,t,t'} < r_{w_1,it}$, then the transfer is set to zero. After each modification, the delivery quantities from the manufacturer to warehouses (w_1 and w_2) are updated.

This neighborhood exploration strategy is inspired by the approach introduced by Armentano et al. (2011). Readers interested in further details are encouraged to refer to that work.

- **2. Lot Sizing Model:** At this stage, production scheduling is adjusted to align with the revised first-echelon delivery plan (q_{it}) obtained in Phase I. The production planning problem corresponds to a LSP, which is formulated as a MILP model and solved using a commercial solver. The LSP model is presented below:

$$[\mathbf{LSP-M}] \quad \min \sum_{t \in \mathcal{T}} p x_t + \sum_{t \in \mathcal{T}} f y_t + \sum_{t \in \mathcal{T}} h_0 I_{0,t} \quad (5.42)$$

Subject to Constraints (5.2), (5.6), (5.7), (5.18), (5.23), and

$$x_t \leq y_t \sum_{t' \in \mathcal{T}} \sum_{i \in \mathcal{N}_c, N_w} q_{i,t'}, \quad \forall\, t \in \mathcal{T} \quad (5.43)$$

- **3. Routing Model**: In the final stage, vehicle routes are defined based on the updated delivery plans (q_{it} and r_{ijt}).

– For the first echelon, the required number of vehicles (v_{it}) for serving warehouses and first-echelon customers is determined using q_{it}. The transportation cost is then calculated as: $\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}_c \cup \mathcal{N}_w} c_i v_{it}$
– For the second echelon, the routing cost is computed using: $\sum_{(i,j) \in A_2} \sum_{w \in \mathcal{N}_w} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} c_{ij} Y_{wijkt}$

To solve the VRP and TSP for each time period, we utilize the LKH-3 heuristic, as proposed by Helsgaun (2017).

5.4.2.3 *Step III: Path Relinking*

The PR procedure is employed to enhance solution quality by exploring intermediate solutions between two feasible solutions Sol_1 and Sol_2. A bidirectional search is performed, alternating between the initial solution Sol_1 and the guiding solution Sol_2, effectively tracing two subpaths that converge at a common feasible solution. This process establishes a transition path between the two solutions, as illustrated in Figure 5.3. A set of elite solutions E is maintained, containing the γ best solutions obtained during the SA search. To ensure solution diversity, a minimum distance is enforced between solutions in E, where the distance is measured by the number of periods in which delivery quantities to customers differ. A new solution is added to E if it either:

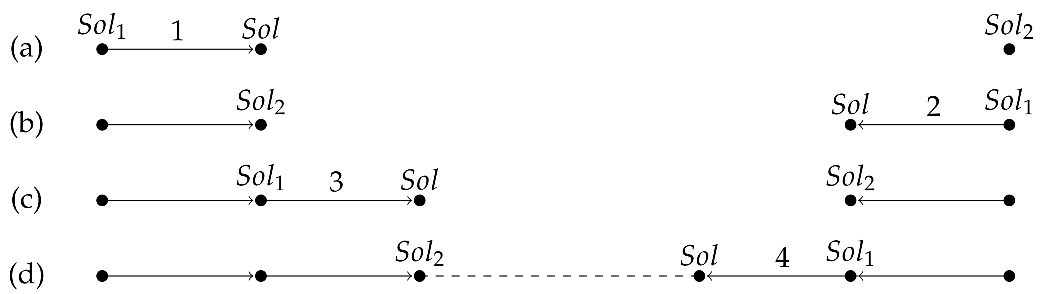


Figure 5.3: Mixed Path Relinking.

1. Has a lower cost than the worst solution in E, or
2. Improves solution diversity by increasing the minimum distance between solutions.

When a new solution enters E, the worst solution is removed. The initial solution Sol_1 is chosen as the best SA solution, while the guiding solution Sol_2 is selected from E as the one with maximum distance $D^{Sol_1}_{Sol_2}$ from Sol_1. This ensures that the elite set remains diverse and prevents the algorithm from getting trapped in local optima.

The Path Relinking algorithm for the first echelon follows the methodology of Armentano et al. (2011) and is detailed in Algorithm 5.1. The PR algorithm begins

Algorithm 5.1 Path Relinking

1: Input: $Sol_1,\ Sol_2,\ D_{Sol_2}^{Sol_1}$
2: **while** $Sol_1 \neq Sol_2$ **do**
3:     **for** $i \in \mathcal{N}_c \cup \mathcal{N}_w,\ t \in \mathcal{T}$ **do**
4:         **if** $q_{it}^{Sol_1} \neq q_{it}^{Sol_2}$ **then**
5:             $\Theta_{it} = q_{it}^{Sol_1} - q_{it}^{Sol_2}$
6:             **for** $t' \neq t$ **do**
7:                 **if** $t' \leq t$ **then**
8:                     $s_{it,t'} = \min(|\Theta_{it}|, (t' \times C - \sum_{i \in \mathcal{N}_c \cup \mathcal{N}_w} \sum_{t'' \in \mathcal{T}'} q_{i,t''}^{Sol_1}), \min_{t''}(U_i - I_{i,t''}, I_{0,t''}))$
9:                 **else**
10:                     $s_{it,t'} = \min\{|\Theta_{it}|\ , \min_{t''}\{I_{i,t''}\}\}$
11:                 **end if**
12:                 Determine the total cost after transferring $s_{it,t'}$
13:             **end for**
14:         **end if**
15:     **end for**
16:     Sol_3 is the best solution found after the transfer $s_{it,t'}$
17:     **if** $D_{Sol_3}^{Sol_2} \leq D_{Sol_2}^{Sol_1}$ **then**
18:         $Sol_1 = Sol_2$
19:         $Sol_2 = Sol_3$
20:     **end if**
21: **end while**

with two solutions, Sol_1 and Sol_2 (line 1). It then iteratively transfers delivery quantities between the two solutions while ensuring feasibility. The comparison between the delivery amounts $q_{it}^{Sol_1} \neq q_{it}^{Sol_2}$ for each warehouse, first-level customer, and period guides the process (lines 3-5).

For each period $t' \neq t$, the algorithm adjusts delivery schedules, shifting quantities from period t to t' in Sol_1 while respecting production and inventory constraints (lines 611). After each update, the total cost of the new solution is computed (line 12), and the best solution is stored as Sol_3 (line 16). If the distance between Sol_2 and Sol_3 does not increase, the process is restarted by setting Sol_1 to Sol_2 and Sol_2 to Sol_3 (lines 17-19). The procedure continues until Sol_1 and Sol_2 converge to the same solution.

The PR algorithm for the second echelon follows the same structure as Algorithm 5.1, with modifications to account for warehouse-to-customer deliveries. The process begins with two solutions, Sol_1 and Sol_2 (line 1), and iteratively shifts delivery quantities between them. The comparison now involves second-level customer deliveries, ensuring that $r_{wit}^{Sol_1} \neq r_{wit}^{Sol_2}$ for each warehouse w, second-echelon customer i, and period t (lines 3-4). The deviation in delivery quantities is represented as:

$$\Theta_{wit} = r_{wit}^{Sol_1} - r_{wit}^{Sol_2} \tag{5.44}$$

For each period $t' \neq t$, the algorithm relocates deliveries both temporally (from t to t') and spatially (from warehouse w to warehouse w') while maintaining feasibility (lines 6-11). The quantity shift is determined by:

$$s_{i,w,w',t,t'} = \min\Big\{|\Theta_{wit}|, \Big(t' \times C - \sum_{j \in \mathcal{N}_c \cup \mathcal{N}_w} \sum_{t'' \in \mathcal{T}'} q_{j,t''}\Big), \Big(Q_{w'} \times K - \sum_{j \in \mathcal{N}_{c'}} r_{w',j,t'}\Big), \min_{t''}\{U_i - I_{i,t''}, I_{w',t''}\}\Big\} \quad (5.45)$$

For cases where $t' > t$, the shift follows:

$$s_{i,w,w',t,t'} = \min\Big\{|\Theta_{wit}|, \Big(Q_{w'} \times K - \sum_{j \in \mathcal{N}_{c'}} r_{w',j,t'}\Big), \min_{t''}\{I_{i,t''}\}\Big\} \quad (5.46)$$

If split deliveries are not allowed, then any transfer involving different warehouses ($w_1 \neq w_2$) and a partial shift ($s_{i,w_1,w_2,t,t'} < r_{w_1,it}$) is discarded. After adjusting the delivery schedule, the total cost of the new solution is computed (line 12), and the best solution is updated as Sol_3 (line 16). If no increase in distance between Sol_2 and Sol_3 is observed, the solutions are reassigned as $Sol_1 = Sol_2$ and $Sol_2 = Sol_3$ (line 17-19). This iterative process continues until both solutions converge.

5.4.2.4 *Pseudo-code of Simulated Annealing with Path Relinking*

This section describes the integration of SA and PR in our solution approach, as outlined in Algorithm 5.2.

The procedure starts by initializing an empty solution set E, which will store elite solutions. An initial solution Sol is generated following the method described in Section 5.4.2.1. The algorithm then iterates through SA-PR for a predefined number of iterations.

The SA procedure (line 6-24) operates in an iterative loop until the temperature τ falls below a predefined threshold τ_f. At each iteration:

1. A neighboring solution Sol' is generated by modifying the current solution Sol, as detailed in Section 5.4.2.2.
2. The cost difference Δ between Sol' and Sol is computed.
3. If Δ is negative (i.e., Sol' is an improvement), Sol' replaces Sol.
4. If Δ is positive, Sol' may still be accepted with a probability α, determined by the SA acceptance rule.
5. If Sol meets the criteria outlined in Section 5.4.2.3 and is not already present in E, it is added to the set.

6. If *Sol* has a lower cost than the best-known solution, it becomes the new best solution.

7. The temperature τ is reduced by a factor μ to gradually decrease the probability of accepting worse solutions over time.

Once the SA phase concludes, the set E is populated with diverse solutions. For Path Relinking Process (line 27-33) we conduct the following:

- The initial solution Sol_1 is selected as the best solution found during SA.
- The guiding solution Sol_2 is chosen as the solution in E that is farthest from Sol_1 in terms of decision-space distance.
- Path Relinking is then executed for both echelons, facilitating the exploration of new solutions.
- The resulting solution from Path Relinking is used as the starting solution for the next SA iteration.

5.5 NUMERICAL EXPERIMENTS

This section serves two primary objectives. First, it evaluates the computational performance of the proposed heuristics 3PI and SA-PR. Second, it provides an analysis of the results from a managerial standpoint. The MILP formulation for 2E-PRP-MDM was implemented and solved using CPLEX 12.8 with its default settings. The 3PI and SA-PR heuristics were developed in Python 3.8 and executed in Jupyter Notebook. All experiments were conducted on a 64-bit Windows 11 system equipped with an Intel(R) Xeon(R) CPU E5-2673 v4 @ 2.30GHz and 14 GB RAM. The 3PI heuristic incorporates a diversification mechanism every five iterations of the three-phase process and is set to terminate after 100 total iterations. The first and second phases are restricted to a 100-second execution time each. For the SA-PR heuristic, the parameters were set as follows: $\tau_0 = 140$, $\tau_f = 1$, and $\mu = 0.9$, following the approach of Shaabani and Kamalabadi (2016). The SA-PR heuristic was configured to run for three iterations, as additional iterations did not yield significant improvements in solution quality but led to a notable increase in CPU time. Finally, the lot sizing problem (Model **LSP-M**) was assigned a 100-second CPU time limit per execution.

5.5.1 *Generated data-sets*

The problem instances used in this study were derived by modifying existing benchmarks from Archetti et al. (2011) and incorporating data configurations from

Algorithm 5.2 Simulated Annealing with Path Relinking

set τ_0, τ_f, μ
$E = \varnothing$
Let Sol be the initial solution.
Set $Sol_{best} = Sol$
while a stopping criterion is not met **do**
  $\tau = \tau_0$
  while $\tau > \tau_f$ **do**
    Generate Neighbor solution Sol' for Sol as follow: use Phase I, Phase II and then Phase III
    $\Delta = cost(Sol') - Cost(Sol)$
    if $\Delta \leq 0$ **then**
      $Sol = Sol'$
    else
      $a = Uniform[0, 1]$
      $\alpha = e^{(-\Delta/\tau)}$
      if $a \leq \alpha$ **then**
        $Sol = Sol'$
      end if
    end if
    if Sol is not in E and respect conditions it enters E, $E = E \cup S$
    if cost(Sol) $\leq$ cost(Sol_{best}) **then**
      $Sol_{best} = Sol$
    end if
    $\tau = \tau \times \mu$
  end while
  $Sol_1 = Sol_{best}$
  Set Sol_2 as the solution with maximum distance D from Sol_{best} in E
  Sol = Path Relinking($Sol_1, Sol_2, D^{Sol_1}_{Sol_2}$) of the first echelon
  if cost(Sol) $\leq$ cost(Sol_{best}) **then**
    $Sol_{best} = Sol$
  end if
  Sol = Path Relinking $Sol, Sol_2, D^{Sol}_{Sol_2}$ of the second echelon
  if cost(Sol) $\leq$ cost(Sol_{best}) **then**
    $Sol_{best} = S$
  end if
end while

Adulyasak et al. (2014a). The instances introduced by Archetti et al. (2011) are publicly available on their website (http://orbrescia.unibs.it/instances).

For this study, we selected instances with 50 customers and structured them across two echelons. The first echelon (i) and second echelon (j) were configured as follows:

- (2,3), (4,6), (6,12), and (10,22) for each warehouse configuration $W = \{1, 2, 3\}$.
- Planning horizons of $T = 3$ and $T = 6$.

- The number of vehicles was set to $K = 1$ or 2 when $j \leq 12$, and $K = 2$ or 3 when $j \geq 22$.

A key assumption was that customer demand in a given period does not exceed vehicle capacity. This constraint ensures that each customer is assigned to one warehouse and one vehicle, maintaining consistency with the MILP model. Similar assumptions have been made in previous studies such as Senoussi et al. (2016) and Absi et al. (2018). Allowing split deliveries would require substantial modifications to the model, significantly increasing computational complexity.

We define testing scenarios by using the classes presented by Archetti et al. (2011):

- Base case instances were selected from the first class.
- High Production Cost cases were taken from the second class.
- High Transportation Cost cases were drawn from the third class.
- Low Inventory Cost cases were chosen from the first class but with inventory costs lower than the base case.

Combining these factors resulted in a total of 192 instances ($4 \times 3 \times 2 \times 2 \times 4$) for evaluating the algorithms performance.

5.5.2 *Performance Analysis*

This experiment evaluates the effectiveness of the proposed heuristics by comparing their results with the optimal solution when available or the best upper bound obtained by CPLEX within a 3600-second time limit. In order to analyze the outcomes, instances are categorized based on the total number of customers, where small instances include cases with 6 and 11 customers, medium instances contain 19 customers, and large instances consist of 33 customers.

The summary tables present instances organized according to scenario type, the number of warehouses, and the planning horizon. Each table is divided into three sections: the 3PI Heuristic, the SA-PR approach, and the computational time of CPLEX.

The heuristic performance is assessed using CPU time and solution quality. CPU time is recorded separately for both heuristics, where CPU^3 represents the computational time of the 3PI Heuristic and CPU^s corresponds to the computational time of the SA-PR approach. The quality of solutions is evaluated by calculating the percentage gap between heuristic results and the best upper bound from CPLEX, given by:

$$GAP^3 = 100 \times \frac{Sol^3 - UB^{CPLEX}}{UB^{CPLEX}} \tag{5.47}$$

$$GAP^s = 100 \times \frac{Sol^s - UB^{CPLEX}}{UB^{CPLEX}} \tag{5.48}$$

where Sol^3 and Sol^s denote the solutions obtained by the 3PI and the SA-PR, respectively, while UB^{CPLEX} represents the best upper bound determined by CPLEX. The last section of each table provides information on the computational time required by CPLEX to process each instance.

The results for small instances are presented in Table 5.10. The SA-PR approach consistently delivers high-quality solutions that closely approximate those obtained by CPLEX. On average, the difference between SA-PR and CPLEX solutions is just 0.87%, with SA-PR outperforming the 3PI in 87% of cases. The average solution gap (GAP^s) for SA-PR ranges between 0.08% and 3.14%. Although 3PI also finds solutions close to those of CPLEX, its best performance is observed in instances with high production costs, two warehouses, and six periods, where the gap is as low as 0.1%. Regarding computational efficiency, 3PI generally executes faster than SA-PR. However, both heuristics require more processing time than the CPLEX solver.

Table 5.10: Performance of the proposed heuristics against CPLEX for small instances

			3PI		SA-PR		CPLEX
Cases	N_w	T	**CPU**^{3}	**GAP**^{3}	**CPU**^{s}	**GAP**^{s}	**CPU**^{CPLEX}
Base Case	1	3	6.00	2.88	15.25	**1.55**	0.31
		6	11.00	**0.84**	20.00	0.89	1.01
	2	3	9.25	3.51	16.00	**1.95**	1.36
		6	16.25	0.75	20.75	**0.74**	8.39
	3	3	9.75	2.62	18.00	**1.63**	5.91
		6	19.50	1.53	22.00	**1.20**	10.25
High Production Cost	1	3	6.00	0.40	15.25	**0.20**	0.49
		6	12.00	**0.12**	20.75	**0.12**	1.82
	2	3	8.25	0.49	23.75	**0.23**	2.09
		6	17.00	0.10	20.50	**0.08**	5.72
	3	3	10.00	0.36	18.50	**0.23**	3.85
		6	20.00	**0.21**	21.00	**0.21**	10.43
High Transportation Cost	1	3	5.75	4.33	13.50	**1.51**	0.41
		6	16.00	3.07	43.75	1.21	3.83
	2	3	7.50	9.85	11.50	**1.13**	1.04
		6	15.00	4.12	38.50	**1.43**	38.30
	3	3	8.50	4.24	12.00	**3.14**	2.77
		6	19.50	5.14	17.00	**0.93**	37.23
Low Inventory Cost	1	3	6.50	1.37	14.75	**0.19**	0.35
		6	11.75	**0.31**	22.50	0.60	2.22
	2	3	9.25	1.64	18.50	**0.19**	1.04
		6	17.25	1.25	32.00	**0.49**	5.80
	3	3	9.00	2.52	12.25	**0.45**	2.50
		6	18.50	1.26	21.25	**0.64**	11.74
Average			**12.06**	2.20	20.39	**0.87**	6.62

For medium-sized instances, both heuristics exhibit strong performance, as detailed in Table 5.11. The solutions produced by the heuristics remain highly competitive, with average gaps of 2.64% for SA-PR and 3.75% for the 3PI when compared to CPLEX results. In terms of computational efficiency, both SA-PR and 3PI significantly surpass CPLEX. On average, SA-PR completes its computations in 61.48 seconds, making it approximately ten times faster than CPLEX, while 3PI requires only 31.31 seconds, achieving a twentyfold speed improvement over CPLEX. These results emphasize the heuristics' capability to efficiently generate high-quality solutions across different problem configurations. Table 5.12 displays

Table 5.11: Performance of the proposed heuristics against CPLEX for medium-sized instances

			3PI		SA-PR		CPLEX
Cases	N_w	T	**CPU^3**	**GAP^3**	**CPU^s**	**GAP^s**	**CPU^{CPLEX}**
Base Case	1	3	14.50	2.13	47.50	**1.22**	7.21
		6	25.50	**0.82**	52.00	1.26	107.4
	2	3	18.50	2.92	126.50	**2.40**	46.11
		6	32.50	1.41	50.00	**1.17**	1839.10
	3	3	22.00	**2.93**	43.00	3.01	170.98
		6	44.50	**3.18**	46.50	3.83	1936.55
High Production Cost	1	3	14.00	0.31	53.00	**0.14**	6.13
		6	25.50	**0.13**	64.50	0.14	312.97
	2	3	18.50	0.43	42.00	**0.41**	44.94
		6	36.00	0.22	51.00	**0.20**	1847.73
	3	3	21.00	0.38	53.00	**0.37**	231.42
		6	47.00	**0.45**	42.50	0.56	2121.13
High Transportation Cost	1	3	11.00	9.77	52.00	**1.99**	8.89
		6	50.50	7.85	32.00	**6.71**	318.41
	2	3	15.00	**8.29**	29.00	9.06	27.63
		6	49.50	**4.73**	29.00	5.64	1840.50
	3	3	19.50	11.41	27.00	**3.14**	244.91
		6	75.00	11.47	29.00	**6.92**	1979.00
Low Inventory Cost	1	3	13.50	2.56	129.00	**1.08**	7.19
		6	46.00	**1.39**	210.50	1.72	145.71
	2	3	60.50	2.86	87.00	**2.76**	1388.73
		6	18.50	4.48	61.00	**1.29**	37.98
	3	3	20.50	5.20	71.50	**3.56**	164.95
		6	52.50	**4.67**	47.00	**4.67**	1848.95
Average			**31.31**	3.75	61.48	**2.64**	695.19

the results for large-sized instances. Both heuristics produce high-quality solutions, with average gaps of 2.38% for the 3PI and 2.66% for SA-PR when compared to CPLEX. The smallest gap recorded is 0.06%, observed in scenarios with high production costs, three warehouses, and a planning horizon of three periods. In terms of computational efficiency, both heuristics significantly reduce CPU time compared to CPLEX. On average, 3PI requires 206.29 seconds, while SA-PR takes 212.33 seconds, making them approximately 16 times faster than CPLEX. These results highlight the heuristics' ability to rapidly generate high-quality solutions across various problem configurations.

Table 5.12: Performance of the proposed heuristics against CPLEX for large-sized instances

			3PI		SA-PR		CPLEX
Cases	N_w	T	**CPU**^{3}	**GAP**^{3}	**CPU**^{s}	**GAP**^{s}	**CPU**^{CPLEX}
Base Case	1	3	292.00	**1.57**	164.50	1.58	1828.28
		6	459.50	**0.96**	600.00	1.80	3600.00
	2	3	75.00	1.64	139.50	**1.47**	3600.00
		6	414.50	**0.40**	617.50	1.37	3600.00
	3	3	107.00	1.27	101.00	**1.23**	3600.00
		6	267.00	**1.58**	80.00	1.92	3600.00
High Production Cost	1	3	291.50	0.24	159.00	**0.23**	1499.54
		6	463.00	**0.15**	670.50	0.28	3600.00
	2	3	77.00	0.24	149.00	**0.21**	3283.21
		6	415.50	**0.10**	609.00	0.27	3600.00
	3	3	229.50	**0.06**	105.50	**0.06**	3600.00
		6	353.50	**0.23**	76.50	0.28	3600.00
High Transportation Cost	1	3	22.00	6.03	166.00	**5.63**	3600.00
		6	201.00	**3.30**	107.50	5.25	3600.00
	2	3	54.00	**5.29**	67.00	7.15	3600.00
		6	174.50	**3.49**	96.00	5.07	3600.00
	3	3	68.50	4.56	63.50	**2.47**	3600.00
		6	189.00	6.25	85.50	**6.01**	3600.00
Low Inventory Cost	1	3	42.50	3.64	208.00	**3.26**	2843.34
		6	245.00	**2.69**	149.50	3.24	3600.00
	2	3	82.50	4.54	192.50	**3.92**	2257.42
		6	267.50	**1.95**	118.50	3.03	3600.00
	3	3	82.50	**3.75**	151.50	4.66	3600.00
		6	222.00	**3.22**	73.50	3.39	3600.00
Average			212.33	**2.38**	**206.29**	2.66	3337.99

5.5.3 *Computational Performance of Heuristics on Medium and Large Sized Instances*

To conduct a fair performance comparison between CPLEX and the two proposed heuristics (SA-PR and 3PI), the CPLEX time limit was adjusted for medium and large-sized instances. This adjustment was based on the average and maximum CPU times recorded by the heuristics when solving instances in each category.

Tables 5.13 and 5.14 summarize the results of this analysis. The tables are structured into two sections according to the CPLEX time limits:

- Medium-sized instances: 45 seconds and 75 seconds
- Large-sized instances: 200 seconds and 450 seconds

Each section contains two key performance indicators: GAP^3 for the 3PI heuristic and GAP^s for SA-PR. These values are calculated using equations (5.47) and

(5.48). As seen in Table 5.13, both heuristics perform effectively under limited computation times.

- For the 75-second limit, the average gap is 0.27% for 3PI and -0.5% for SA-PR.
- When the time is restricted to 45 seconds, the gaps reduce to -0.14% for 3PI and -0.9% for SA-PR.
- Notably, both heuristics show performance improvements of up to 24%.

These results indicate that SA-PR consistently outperforms both 3PI and CPLEX under both time constraints. However, 3PI surpasses CPLEX only when the time limit is set to 45 seconds. Table 5.14 highlights the superior efficiency of both heuristics compared to CPLEX. When the time limit is set to 200 seconds, the average improvement is -5.23% for the 3PI and -4.97% for SA-PR. With an extended time limit of 450 seconds, the improvements remain significant at -4.47% for 3PI and -4.2% for SA-PR. These findings demonstrate that both heuristics can outperform CPLEX in approximately 60% of instances, achieving performance gains of up to 31%.

Table 5.13: Performance Comparison of Proposed Heuristics and CPLEX with 45 and 75 Second Time Limits for Medium-Sized Instances

			45 sec		75 sec	
Cases	N_w	T	**GAP^3**	**GAP^s**	**GAP^3**	**GAP^s**
Base Case	1	6	-24.60	-24.27	-23.41	-23.07
	2	3	2.58	2.06	2.92	2.40
		6	0.89	0.65	1.28	1.04
	3	3	1.20	1.28	1.86	1.94
		6	1.55	2.19	1.99	2.64
High Production Cost	1	6	-18.74	-18.73	-18.62	-18.61
	2	3	0.43	0.41	0.43	0.41
		6	0.14	0.13	0.19	0.17
	3	3	0.24	0.23	0.31	0.31
		6	0.10	0.21	0.28	0.39
High Transportation Cost	1	6	7.14	6.01	7.55	6.42
	2	6	0.57	1.50	1.11	2.05
	3	3	9.43	1.28	11.25	2.97
		6	5.64	1.32	5.42	1.12
Low Inventory Cost	1	6	0.99	1.32	1.03	1.36
	2	6	2.16	2.06	2.41	2.31
	3	3	5.06	3.42	5.14	3.51
		6	2.81	2.81	3.68	3.68
Average			-0.14	-0.90	0.27	-0.50

Table 5.14: Performance Comparison of Proposed Heuristics and CPLEX with 200 and 450 Second Time Limits for Large-Sized Instances

			200 sec		450 sec	
Cases	N_w	T	**GAP³**	**GAPˢ**	**GAP³**	**GAPˢ**
Base Case	1	3	-30.61	-30.60	-29.46	-29.45
		6	-31.19	-30.61	-28.59	-27.99
	2	3	-0.80	-0.97	0.14	-0.02
		6	-1.80	-0.85	-0.68	0.27
	3	3	-1.10	-1.14	-0.40	-0.44
		6	-1.39	-1.06	-0.19	0.15
High Production Cost	1	3	-29.20	-29.20	-29.20	-29.20
		6	-29.10	-29.01	-29.05	-28.96
	2	3	-0.14	-0.17	0.00	-0.03
		6	-0.24	-0.07	-0.08	0.09
	3	3	-0.10	-0.10	-0.07	-0.07
		6	-0.45	-0.40	-0.14	-0.09
High Transportation Cost	1	3	3.00	2.62	3.48	3.10
		6	-5.67	-3.85	-2.59	-0.72
	2	3	0.02	1.79	0.18	1.95
		6	-1.78	-0.27	-2.19	-0.68
	3	3	-6.06	-7.93	-5.11	-6.99
		6	-3.46	-3.67	1.46	1.23
Low Inventory Cost	1	3	3.19	2.81	3.21	2.83
		6	2.16	2.72	2.17	2.72
	2	3	3.78	3.16	3.89	3.28
		6	1.28	2.36	1.69	2.78
	3	3	2.41	3.30	2.45	3.35
		6	1.66	1.84	1.90	2.08
Average			-5.23	-4.97	-4.47	-4.20

5.5.4 *Results Analysis*

This study evaluates the performance of 3PI and SA-PR across four scenarios: Base Case, High Production Cost, High Transportation Cost, and Low Inventory Cost, as illustrated in Figures 5.4 and 5.5.

Figure 5.4 presents the average gap for both heuristics, demonstrating that SA-PR consistently outperforms 3PI, particularly in instances with high transportation costs. The smallest gap occurs in the high production cost scenario, where production levels remain stable and costs are elevated.

Conversely, Figure 5.5, which depicts CPU time, shows that SA-PR generally requires more computation time than 3PI, except in the high transportation cost case. Notably, the LKH-3 routing heuristic sometimes halts prematurely, even when the solution is far from optimal, highlighting the balance between computational effort and solution quality.

These results underscore the trade-off between solution quality and computational efficiency. While SA-PR delivers superior solutions, it does so at the expense of CPU time compared to 3PI.

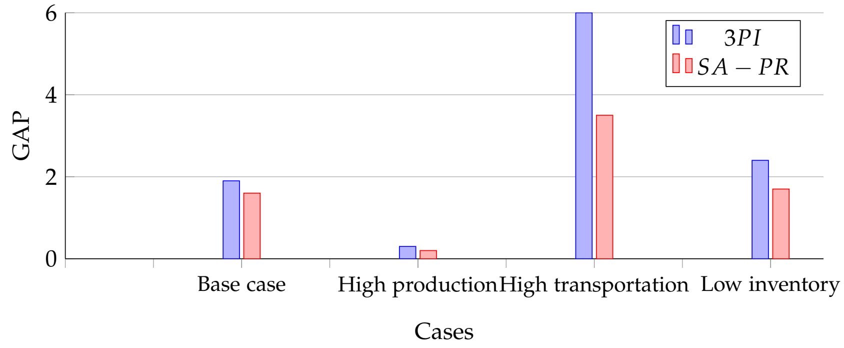


Figure 5.4: Comparison of Average Gap Between 3PI and SA-PR Heuristics for all Cases

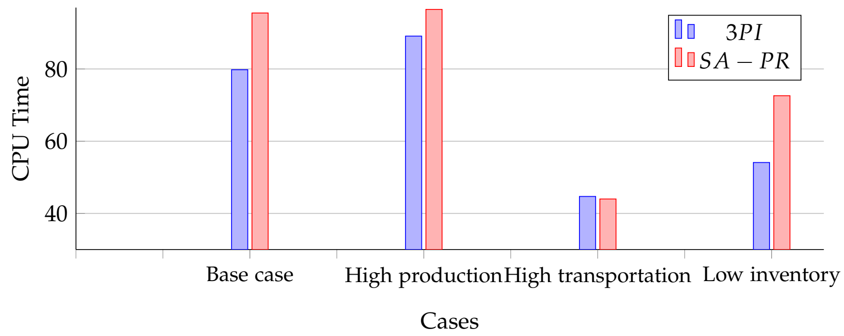


Figure 5.5: Comparison of Average CPU Time Between 3PI and SA-PR Heuristics for all Cases

5.5.5 *Managerial Insights*

This section explores key managerial insights regarding distribution strategies and the impact of warehouse count, expanding on the instances analyzed in Section 5.5. To provide a practical perspective, the current problem is compared with two real-world distribution models.

In the first scenario, each warehouse serves a predefined set of customers in the second echelon, ensuring that deliveries are assigned to a dedicated warehouse for each customer. In the second scenario, both the manufacturing plant and warehouses have the flexibility to deliver to all customers, optimizing delivery routes dynamically.

- **Nearest Warehouse Customers**: In this approach, customers place orders with the closest warehouse, meaning each warehouse is responsible for fulfilling orders within its designated area. To model this scenario, each customer is assigned to the nearest warehouse, forming predefined sets. The original formulation is modified by redefining the warehouse sets from $N_{c'}$ to $N_{w,c'}$, where $N_{w,c'}$ represents the set of customers served by warehouse w, as outlined in Section 5.3.

- **Flexible Dual Delivery**: This model introduces a more adaptable distribution strategy, where customers in both echelons can receive orders through two delivery modes:
  1. Direct shipment from the manufacturing plant.
  2. Warehouse delivery via vehicle routing.

  To accommodate this approach, modifications were made to the original formulation presented in Section 5.3 as follows:
  - The original sets N_c (first-level customers) and $N_{c'}$ (second-level customers) are merged into a single set $N_T = N_c \cup \mathcal{N}_{c'}$, representing the union of all customers in the system.
  - The previous constraints (5.3) and (5.5) are replaced to allow customers to receive their orders from either the manufacturing plant or warehouses:

$$I_{jt} = I_{j,t-1} + q_{jt} + \sum_{i \in \mathcal{N}_w} \sum_{k \in \mathcal{K}} r_{ijkt} - d_{jt}, \quad \forall j \in \mathcal{N}_T, \forall t \in \mathcal{T} - \{1\} \tag{5.49}$$

  - A binary variable u_{it} is introduced to indicate whether the manufacturing plant delivers directly to customer i in period t. This constraint ensures that the delivery quantity adheres to vehicle capacity limits:

$$v_{it} \leq Q_0 \times u_{it}, \quad \forall i \in \mathcal{N}_c \cup \mathcal{N}_w, \forall t \in \mathcal{T} \tag{5.50}$$

  - The original constraint (5.11) is modified to ensure that each customer receives their entire order from only one sourceeither directly from the plant or via a warehouse, but not both in the same period:

$$\sum_{i \in \mathcal{N}_w} \sum_{k \in \mathcal{K}} H_{ijkt} + u_{jt} \leq 1, \quad \forall j \in \mathcal{N}_T, \forall t \in \mathcal{T} \tag{5.51}$$

5.5.6 *Case Study*

This case study examines a soft drink manufacturing plant based in western Algeria. The company produces a variety of flavors (pineapple, strawberry, bitter, blackcurrant, orange, lemon, red apple, and green apple) and bottle sizes (33 CL, 1L, and 2L) through a two-stage production process: liquid flavor preparation (Stage I) and bottling (Stage II).

- Stage I (Flavor Preparation):
  The production process begins with tanks of different capacities preparing

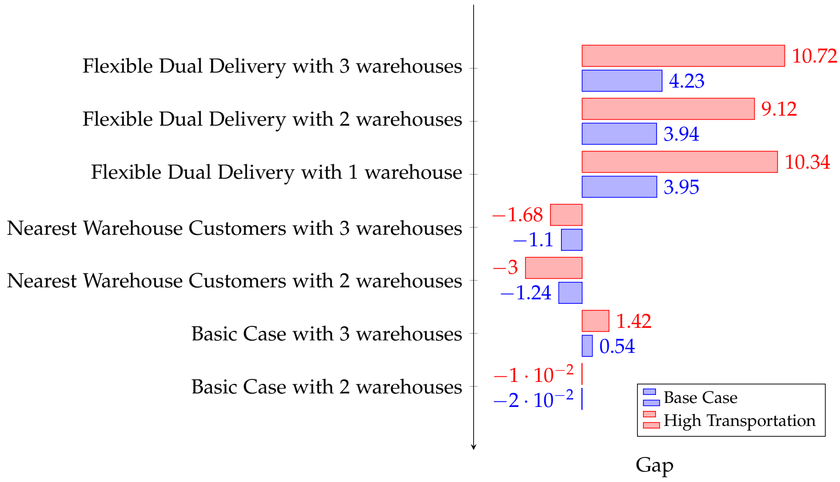
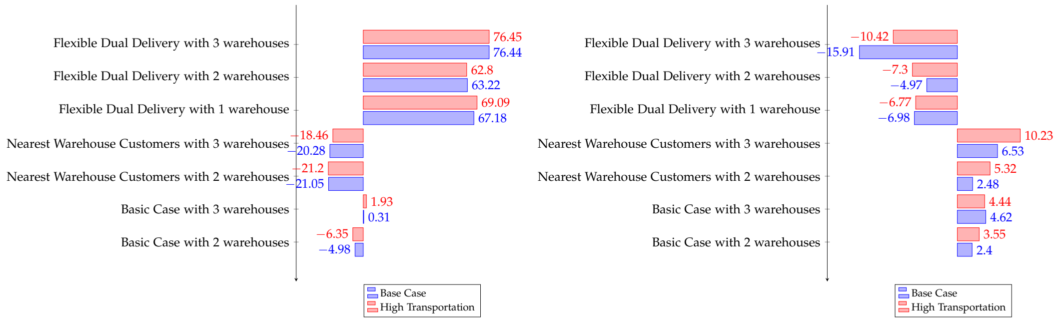


Figure 5.6: Comparing total cost results of the three situations



(a) Transportation cost of the first echelon (b) Transportation cost of the second echelon

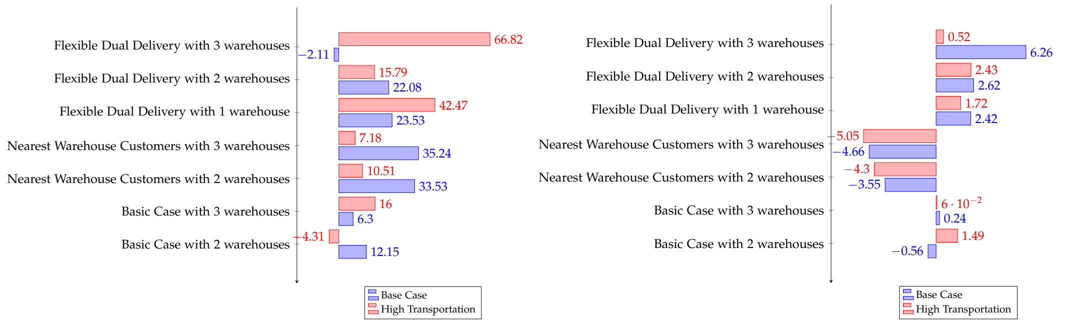
Figure 5.7: Transportation cost results of the three situations

the liquid flavor (syrup or concentrate). A single tank cannot process multiple flavors simultaneously, and it must be completely emptied before a new batch is prepared.

- Stage II (Bottling):
  The prepared liquid is transferred to the filling line, where it is bottled according to size and flavor. Each time the production switches to a different flavor or bottle type, a sequence-dependent setup time is required for cleaning and equipment adjustments.

For this study, the production system is modeled as a single production line, focusing on one flavor (pineapple) and one bottle size (1L). The cost per bottle is 59 Algerian Dinar (DA). The production line operates with:

- Setup cost:50,000 DA
- Production rate: 666 bottles per hour
- Working schedule: 8 hours per day, 5 days per week



(a) Inventory cost of the manufacturing (b) Inventory cost of the customers

Figure 5.8: Inventory cost results of the three situations

The manufacturer supplies products to one primary customer and a warehouse, which then distributes to 24 retailers, including grocery stores and soft drink distributors across three cities. The distances between key locations are:

- Manufacturer to Primary Customer: 3.3 km
- Manufacturer to Warehouse: 1.5 km

The manufacturing plant has a storage capacity of 10,000 bottles, and the inventory holding cost is 10% of the bottle price per month. The warehouse has a storage capacity of 20,000 bottles, and the inventory holding cost is 6% of the bottle price per month.

Transportation

- Manufacturers fleet capacity: 480 bottles per trip
- Warehouse fleet capacity: 6,300 bottles per trip
- Transportation cost: 10 DA per bottle per kilometer

Further details, including the distance matrix between the company, warehouse, and customers, are provided in the appendix.

5.5.6.1 *Integrated vs traditional supply chain*

This section evaluates the integrated supply chain approach developed in our study against the traditional supply chain model currently implemented by the Algerian soft drink manufacturer. Using real company data, we focus on the summer season, a period of peak demand, to analyze the differences in cost and efficiency. Figures 5.9, 5.10, 5.11, and 5.12 compare the performance of our integrated approach with the companys existing method.

Our results indicate that the integrated supply chain achieves an average cost reduction of 2.73%, primarily due to improved coordination between production, inventory, and transportation decisions.

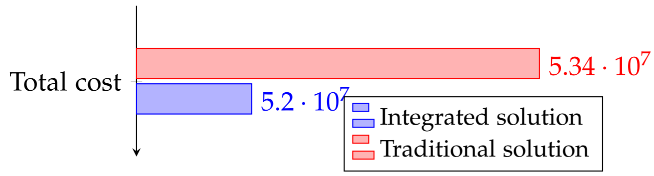


Figure 5.9: Comparing total costs

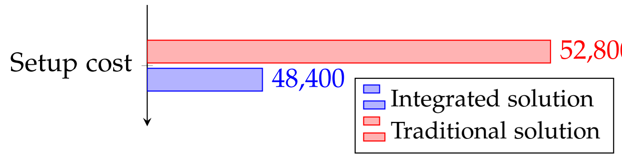


Figure 5.10: Comparing setup costs

- Setup Cost Reduction: The integrated model optimizes production scheduling, reducing unnecessary setup operations and consolidating processes. As a result, one production period is eliminated, leading to lower setup costs (Figure 5.10).
- Lower Transportation Costs: By minimizing the number of trips and optimizing delivery routes, transportation costs are reduced by 2.8% (Figure 5.12).
- Inventory Costs Adjustment: While total inventory holding costs increase, particularly at the customer level (Figure 5.11), these higher storage costs are offset by reductions in setup and transportation expenses, leading to overall cost savings.

5.5.6.2 *Evaluation of the Companys plan*

The company is considering expanding its distribution network by establishing a second warehouse in a new city, intended to serve six additional distributors. This section evaluates whether the new warehouse is necessary or if the existing warehouse can accommodate the increased demand. Additionally, we examine the effectiveness of assigning specific customers to each warehouse and aim to determine the most cost-efficient strategy for the company.

Figures 5.13, 5.14, 5.15, and 5.16 present the results of three different distribution strategies:

1. Companys Plan: Each warehouse is responsible for a specific set of customers the main warehouse serves customers in three cities, while the new warehouse exclusively serves those in the new city.
2. Single-Warehouse Strategy: Only the main warehouse is used to supply customers across all four cities, eliminating the need for the new warehouse.

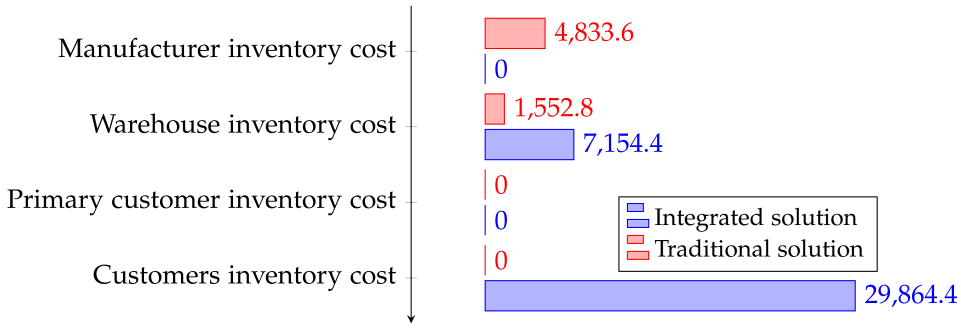


Figure 5.11: Comparing inventory costs

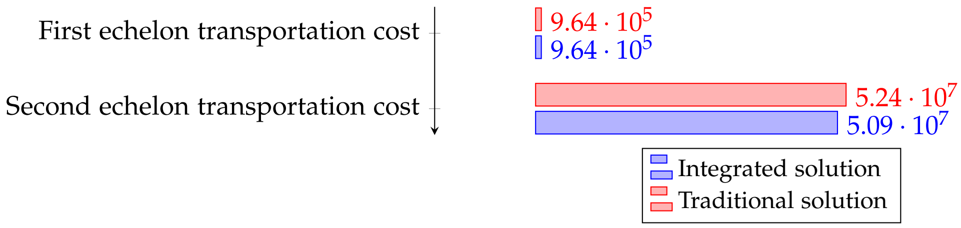


Figure 5.12: Comparing transportation costs

3. Flexible Warehouse Strategy: Both warehouses are allowed to serve all customers, without restrictions on assigned service areas.

The cost analysis reveals that the most cost-effective solution is to rely solely on the main warehouse, even when the model is given full flexibility. The companys current expansion plan results in higher costs compared to using just the main warehouse, although the cost difference remains relatively small (0.2% gap).

While the total cost savings do not strongly justify the second warehouse, other factors might support its establishment, such as:

- Improved service levels by reducing delivery lead times
- Closer proximity to customers for better market reach
- Increased operational flexibility in case of demand fluctuations

Therefore, although cost optimization does not strongly favor opening a second warehouse, the strategic benefits related to service quality and market accessibility could make expansion a viable option.

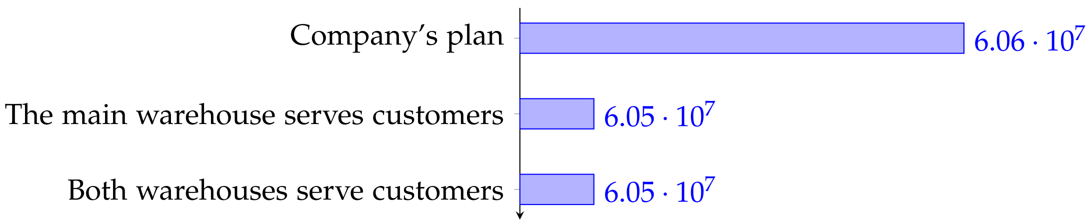
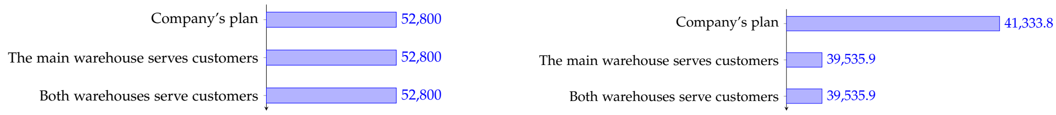
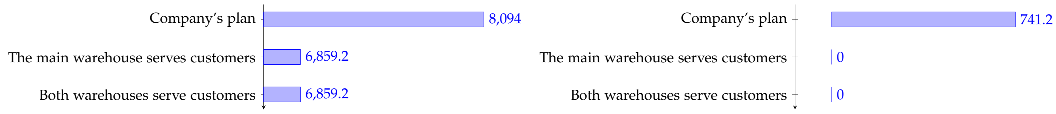


Figure 5.13: Total cost results of the three strategies



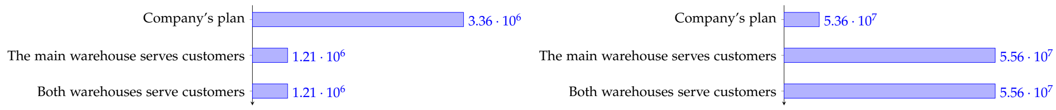
(a) Setup costs (b) Customers inventory costs

Figure 5.14: setup cost and customers inventory cost results of the three strategies



(a) Main warehouse inventory costs (b) Second warehouse inventory costs

Figure 5.15: Inventory cost results of the three strategies



(a) First echelon transportation costs (b) Second echelon transportation costs

Figure 5.16: Transportation cost results of the three strategies

5.6 CONCLUSION

This chapter explored the integration of production, inventory, and routing decisions within a two-echelon supply chain that incorporates multiple delivery modes. To solve this complex problem, we developed an enhanced version of the Two-Phase Iterative heuristic from Absi et al. (2015) and a metaheuristic approach inspired by Armentano et al. (2011), which integrates SA-PR. Computational results demonstrated that SA-PR consistently outperformed the extended heuristic in terms of solution quality while maintaining a similar computational time. Additionally, SA-PR identified superior solutions compared to CPLEX in certain instances and closely matched CPLEX solutions in most cases while reducing CPU time by a factor of ten.

From a managerial perspective, adopting the Flexible Dual Delivery strategy resulted in notable cost reductions ranging from 3.94% to 10.72%, with greater savings observed as the number of warehouses increased. This improvement stems from the ability to optimize resource allocation, reduce transportation distances, and lower logistics costs. Furthermore, findings indicate that the basic approach offers a balanced compromise between cost efficiency and operational complexity compared to both Flexible Dual Delivery and Nearest Warehouse Customers strategies.

A real-world case study involving an Algerian soft drink manufacturer validated the effectiveness of the proposed integrated model. When compared to the company's current supply chain approach, our model achieved an average cost reduction of 2.73%, demonstrating the potential operational and financial benefits of integrating production, inventory, and routing decisions.

The 2E-PRP can be further developed in multiple directions to enhance its practicality and efficiency. One potential improvement is the integration of uncertain demand and lead times, enabling better handling of fluctuating market conditions where exact demand and delivery schedules are unpredictable (Adulyasak et al., 2015a). Another extension involves expanding the model to multi-echelon supply chains, particularly those dealing with perishable goods, which require careful coordination of production, storage, and transportation to minimize waste and maintain product quality (Alvarez et al., 2022). Additionally, optimizing the SA-PR algorithm could enhance its adaptability to the Flexible Dual Delivery and Nearest Warehouse Customers strategies discussed in Section 5.5.5, while also improving computational efficiency and reducing CPU time. These enhancements would make the model more robust, allowing it to address a wider range of real-world supply chain challenges.

6 CONCLUSIONS AND FUTURE WORK

This concluding chapter summarizes the key contributions of this thesis and outlines future research directions that arise from our work. These perspectives are intended to guide future investigations and may also inspire further exploration by other researchers in the field.

Focusing on integrated production and distribution planning within a VMI framework, this thesis addresses three major research dimensionseach contributing to critical aspects of supply chain optimization. Our primary contribution lies in the joint optimization of production, inventory, and distribution decisions across different levels of complexity and coordination.

In the early part of this thesis, we developed decision-support tools to assist supply chain partners in selecting the most appropriate inventory management contract. Additionally, we proposed a contract ranking tool that operates under both deterministic and stochastic demand conditions, taking into account vendor risk aversion and cost-sharing mechanisms. Numerical experiments validated the effectiveness of this tool, demonstrating its practical relevance for real-world decision-making.

Once the VMI policy is adopted, we extended our work to address the operational implications through integrated production and distribution planning. We introduced and analyzed the "*lot-sizing problem with direct shipment and multiple trips*"a single-echelon problem formulated as a MILP model. This problem was solved using both a commercial solver and a HSA metaheuristic. Numerical experiments, including a real-world case study from the PPE industry, confirmed the superiority of our heuristic in terms of solution quality and computational efficiency.

To further enhance the realism and complexity of the modeling framework, we advanced our research to a two-echelon setting. This led to the formulation of the "*Two-Echelon Production Routing Problem (2E-PRP)*, integrating production, inventory, and routing decisions with mixed delivery modes. Due to the high computational complexity of the MILP formulation, commercial solvers struggled to find feasible solutions within acceptable time limits. To overcome this, we developed two advanced heuristics: SA-PR) and a 3PI heuristic. These approaches were rigorously evaluated through numerical experiments and a case study in the soft drink industry, where they demonstrated substantial cost savings and practical applicability.

Despite the significant contributions of this thesis, several limitations should be acknowledged.

First, the decision-making and ranking tools do not account for certain realistic constraints, such as production and vehicle capacity limitations. Moreover, these models could be further enriched by integrating additional practical considerations, including sustainability aspects (e.g., remanufacturing), and contractual constraints imposed by vendors and retailers, such as payment schedules and penalty costs. While the proposed MILP models are relatively straightforward to implement and can efficiently solve multiple instances within a reasonable computational time, the inclusion of such constraints would increase complexity, requiring the use of heuristics or metaheuristics combined with Machine Learning (ML) techniques to ensure computational tractability and timely solutions in practice.

Additionally, deterministic and stochastic models have yielded strong results in this study. Robust optimization methods may be more suitable for real-world scenarios, ensuring reliable solutions under uncertainty.

Future research could extend this framework to supply chains with multiple retailers, allowing for the simultaneous assessment of VMI adoption across different retailers. Additionally, the model could incorporate alternative inventory management strategies, such as Integrated Inventory Management (IIM) (Song and Dinwoodie 2008) and Customer-Inventory Management (CIM) (Chen et al. 2024). Furthermore, introducing an inventory cost-sharing contract between the vendor and retailer could provide a valuable addition to the set of contract options, promoting more balanced risk and cost distribution. It may also be beneficial to consider contract differentiation based on product characteristicsfor instance, applying a VMI-IVTR contract to product 1 and a VMI-IRTR contract to product 2allowing for more tailored and flexible coordination mechanisms.

Second, the integrated production and distribution planning model did not incorporate routing decisions. The model assumes direct deliveries rather than optimizing routes for multiple customers. As the customer base expands, route optimization could become a critical factor. Additionally, inter-site transfers were not considered, even though they could be valuable given the nature of PPE products. Another limitation is the assumption that all data is deterministic, particularly transportation costs, which are influenced by fluctuating fuel prices and uncertain travel times.

Future research will focus on addressing these limitations. A key priority is to develop models that account for uncertainty in transportation costs, as well as potential variations in processing times and production yields. To handle such uncertainties, advanced robust optimization techniques (Bertsimas et al. 2011) could be explored, ensuring that solutions remain effective under real-world variability.

Third, The 2E-PRP is limited by its assumption of deterministic demand and a fixed two-echelon supply chain structure. To enhance the models applicability, future research could explore this problem under stochastic or robust optimization frameworks, incorporating key uncertainties such as variable demand and lead times. This would enable more flexible and adaptive planning in response to fluctuating market conditions (Adulyasak et al. 2015a). Additionally, extending the model to multi-echelon supply chainsespecially those handling perishable goodswould address the complexities of coordinating production, storage, and transportation while minimizing waste and preserving product quality (Alvarez et al. 2022).

Furthermore, the current version of the SA-PR algorithm could be further optimized. Enhancing its adaptability to different delivery strategies, such as the Flexible Dual Delivery and Nearest Warehouse Customers approaches discussed in Section 5.5.5, could improve its robustness and versatility. Additionally, reducing its computational time would further enhance its usefulness for large-scale applications.

While this thesis has primarily focused on mathematical modeling and metaheuristic approaches to improve integrated production and distribution planning under the VMI framework, the rapid advancement of Generative Artificial Intelligence (GenAI) and AI agent technologies presents transformative opportunities for the future of supply chain management (L. Li et al., 2024). These technologies can greatly extend the relevance and scalability of the contributions developed in this work.

Traditional supply chain optimization modelssuch as those developed in this thesisoperate in structured environments with fixed input parameters and static decision rules. However, real-world logistics environments are increasingly dynamic, requiring systems that can learn, adapt, and optimize. Generative AI, powered by deep learning, enables systems to synthesize new data, simulate future scenarios, and generate real-time recommendations based on evolving operational contexts (Jackson et al. 2024).

Several major companies are beginning to integrate Generative AI (GAI) into their core operations. For example, Walmart is utilizing GAI technologies to automate negotiations and secure better pricing from vendors (Van Hoek et al. 2022).

In the maritime sector, Maersks Chief Technology and Information Officer has expressed the companys intent to significantly expand GAI applications across its business functions (Fan et al. 2024). The deployment of GAI is also gaining traction beyond retail and shipping. For instance, DHL is exploring using ChatGPT to automate and streamline logistics operations, including those within warehouses and transportation processes (Jackson et al. 2024). Similarly, Instacart, a prominent grocery delivery platform in the U.S., has partnered with OpenAI to embed ChatGPT into its service. This integration allows users to receive recipe suggestions, manage their shopping lists, and place delivery orders more efficiently (Jackson et al. 2024). Moreover, the potential of GAI to transform supply chain communication and decision-making continues to attract attention. Recent commentary in the Wall Street Journal highlights growing experimentation with GAI tools across various sectors (Young 2023), signaling a broader shift toward AI-enhanced supply chain ecosystems.

These use cases demonstrate how GenAI and AI agents can automate contract managementa topic closely aligned with the contract selection models proposed in this thesis.

One of the most immediate applications of GenAI in this domain is demand forecasting. Unlike traditional methods, which rely on historical sales data and static statistical models, future research could explore how AI agents leverage diverse data streamssuch as market trends, weather patterns, consumer sentiment, and real-time transactionsto improve forecast accuracy. These agents could also simulate negotiation outcomes under varying vendor and retailer constraints, or dynamically adapt contract terms in response to shifting market conditions, thereby enhancing the practicality and adaptability of VMI adoption tools.

Beyond strategic decision-making, Generative AI (GenAI) and AI agents hold transformative potential for operational planning. The integrated production-distribution models developed in this thesis could be enhanced through intelligent agents that employ reinforcement learning and dynamic scenario generation, enabling real-time optimization of production quantities and delivery routes in response to live demand signals. This represents a significant advancement over traditional static heuristic approaches. Industry applications already demonstrate this potential: FedEx and Amazon utilize AI-driven systems to continuously adapt logistics operations based on real-time variables including demand fluctuations, weather disruptions, and traffic conditions. In particular, Amazon has used GenAI to generate synthetic training data for warehouse robots, significantly improving their package sorting and inspection capabilities during high-volume periods such as Cyber Monday (Amazon Business 2021).

As part of future research, the integration of AI agents with digital twin technology presents a transformative opportunity for supply chain optimization. By continuously simulating and monitoring operational decisions, this combined approach could provide planners with real-time performance analytics, predictive risk alerts, and dynamically generated corrective strategies (Guo and and, 2025).

A
APPENDIX

Table A.1: Impact of high transportation costs on VMI contract rates.

	All or Nothing				Best VMI			
Range	**IRTR**	**IRTV**	**IVTR**	**IVTV**	**IRTR**	**IRTV**	**IVTR**	**IVTV**
1-400	0.00	2.58	0.00	0.58	0.67	10.40	22.38	0.58
401-800	0.00	0.00	0.00	0.00	0.63	0.00	25.63	0.00
801-1200	0.00	0.00	1.80	0.00	0.78	0.00	24.30	0.00
1201-1600	0.00	0.00	7.32	0.00	0.48	0.00	24.18	0.00
2001-3200	0.13	0.00	20.88	0.00	0.68	0.00	24.48	0.00

Table A.2: Impact of low transportation costs on VMI contract rates.

	All or Nothing				Best VMI			
Range	**IRTR**	**IRTV**	**IVTR**	**IVTV**	**IRTR**	**IRTV**	**IVTR**	**IVTV**
1-100	0.00	13.25	0.00	8.38	0.06	45.50	11.88	8.38
101-200	0.19	1.56	11.75	0.25	0.81	2.88	25.00	0.31
201-300	0.06	0.00	21.88	0.00	0.25	0.00	25.94	0.00
301-400	0.13	0.00	22.56	0.00	0.44	0.00	26.44	0.00
401-500	0.13	0.00	22.81	0.00	0.50	0.00	27.13	0.00

Table A.3: Impact of high inventory costs on VMI contract rates.

	All or Nothing				**Best VMI**			
Range	**IRTR**	**IRTV**	**IVTR**	**IVTV**	**IRTR**	**IRTV**	**IVTR**	**IVTV**
0-5	0.00	0.00	49.90	6.90	0.00	2.13	90.94	6.94
5-10	0.00	0.00	47.50	3.60	0.00	5.19	85.75	3.69
10-15	0.00	3.80	9.70	0.30	0.94	10.69	25.56	0.38
15-20	0.00	4.10	0.00	0.00	0.94	9.19	0.06	0.00
20-25	0.10	5.30	0.00	0.00	0.81	10.63	0.00	0.00

Table A.4: Impact of low inventory costs on VMI contract rates.

	All or Nothing				**Best VMI**			
Range	**IRTR**	**IRTV**	**IVTR**	**IVTV**	**IRTR**	**IRTV**	**IVTR**	**IVTV**
0-5	0.00	0.03	48.20	3.50	0.03	1.88	90.81	3.53
5-10	0.03	2.80	5.40	0.10	0.50	6.03	17.69	0.16
10-15	0.20	3.00	0.00	0.00	1.38	6.81	0.13	0.00

Table A.5: Detailed Average Cost Data and Results for selected VMI contracts under High Transportation costs.

C^v	1-400	1-400	801-1200	1201-1600	2001-2400	2001-2400	2401-2800	2801-3200	2801-3200
Contracts	IVTV	IRTV	IVTR	IVTR	IVTR	IRTR	IVTR	IVTR	IRTR
C^r	1305.14	1382.94	1083.57	1180.11	1509.39	1674.67	1502.82	1502.48	1868.00
$h^{r(v)}$	1.71	12.39	3.74	2.92	3.25	16.00	3.51	3.01	10.00
$h^{r(r)}$	8.43	3.42	12.87	12.20	11.89	11.33	12.01	11.30	9.00
S	3000.00	2548.39	2869.57	2842.11	2799.26	1000.00	2771.74	2851.56	3000.00
h^v	8.00	7.81	7.39	7.24	7.39	5.33	7.25	7.02	7.00
Setup cost (VMI)	6714.29	5612.90	5826.09	5500.00	5724.91	3000.00	5608.70	5648.44	6000.00
Setup cost (RMI)	6714.29	5774.19	5956.52	5618.42	5780.67	3000.00	5666.67	5703.13	6000.00
Vendor's inventory cost (VMI)	2969.14	3172.06	3634.17	4108.29	3703.64	1640.33	3725.26	3719.55	5054.00
Vendor's inventory cost (RMI)	4258.29	4003.03	4326.78	4684.45	4299.54	1648.33	4357.50	4293.63	5061.00
Retailer's inventory cost (VMI)	338.29	497.06	443.35	317.78	315.96	241.33	374.17	291.45	180.00
Retailer's inventory cost (RMI)	372.14	85.74	138.78	165.79	146.27	241.33	142.41	137.57	180.00
Transportation cost (VMI)	393.71	565.77	11966.87	12553.39	15951.97	16340.33	16233.61	16026.88	18680.00
Transportation cost (RMI)	12034.14	13898.35	11966.87	12553.39	15951.97	16340.33	16233.61	16026.88	18680.00

Table A.6: Detailed Average Cost Data and Results for selected VMI contracts under Low Transportation costs.

C^v	101–200				201–300				301–400		401–500	
Contracts	IVTV	IVTR	IRTV	IRTR	IVTV	IVTR	IRTV	IRTR	IVTR	IRTR	IVTR	IRTR
C^r	149.76	130.49	148.27	131.33	151.00	150.99	159.24	187.00	148.47	187.50	150.82	179.00
$h^{r(v)}$	2.78	3.38	11.88	12.33	2.25	3.44	12.60	18.00	3.49	12.00	3.48	19.00
$h^{r(r)}$	11.66	11.58	3.82	10.00	17.00	11.88	3.88	17.00	11.64	8.00	11.98	10.00
S	3007.46	3000.00	2976.42	2666.67	4000.00	2914.29	3120.00	4000.00	2864.27	2000.00	2912.33	4000.00
h^v	7.74	7.15	7.58	6.00	9.75	7.22	9.04	10.00	7.15	5.00	7.09	6.00
Setup cost (VMI)	5947.76	5611.70	5872.64	6666.67	6750.00	5725.71	6600.00	8000.00	5567.87	4000.00	5635.62	5500.00
Setup cost (RMI)	6104.48	5648.94	5919.81	6666.67	6750.00	5797.14	6600.00	8000.00	5612.19	4000.00	5698.63	5500.00
Vendor's inventory cost (VMI)	3810.92	4031.80	3784.17	2979.00	5206.75	3857.69	3895.12	8570.00	3860.01	3444.50	3867.44	5691.00
Vendor's inventory cost (RMI)	4747.19	4723.08	4658.47	3053.00	6886.25	4530.17	5319.76	8680.00	4527.54	3507.50	4544.16	5760.00
Retailer's inventory cost (VMI)	434.40	375.19	500.36	172.33	459.75	374.42	706.28	187.00	360.30	186.50	388.79	219.50
Retailer's inventory cost (RMI)	78.81	55.34	63.04	43.33	200.00	66.29	102.24	0.00	58.59	87.50	63.01	70.00
Transportation cost (VMI)	306.78	1384.39	400.28	1309.00	1082.00	1590.89	1195.68	2057.00	1580.96	1687.50	1594.27	1790.00
Transportation cost (RMI)	1606.34	1447.20	1518.58	1440.33	1515.50	1655.26	1536.00	2244.00	1643.88	1786.50	1656.79	1954.00

Table A.7: Detailed Average Cost Data and Results for selected VMI contracts under High Inventory costs.

$h^{r(v)}$	0.5–5		5–10		10–15				15–20		20–25	
Contracts	IVTV	IVTR	IVTV	IVTR	IVTV	IVTR	IRTV	IRTR	IRTV	IRTR	IRTV	IRTR
C^v	53.42	662.02	32.98	671.48	10.33	643.20	67.80	838.00	74.75	889.00	70.25	765.33
C^r	501.44	330.13	496.05	320.68	386.00	270.27	553.44	167.00	589.22	332.00	535.49	262.67
$h^{r(r)}$	12.47	12.60	12.51	12.60	13.17	13.48	11.00	13.00	11.54	12.00	11.34	12.67
S	3162.16	3021.28	3389.83	3034.21	2500.00	3403.85	3360.66	2000.00	3373.13	5000.00	3258.82	1666.67
h^v	12.93	12.48	13.02	12.58	14.17	13.71	13.08	12.00	13.55	11.00	13.46	10.67
Setup cost (VMI)	8009.01	7644.56	8525.42	7692.11	7833.33	8852.56	8262.30	4000.00	8597.01	10000.00	8458.82	4666.67
Setup cost (RMI)	8216.22	7782.23	8593.22	7767.11	7833.33	8858.97	8327.87	4000.00	8761.19	10000.00	8482.35	4666.67
Vendor's inventory cost (VMI)	4537.41	4677.32	4871.85	4686.58	3479.33	5255.93	4899.28	4452.00	5099.88	5973.00	4823.39	3940.67
Vendor's inventory cost (RMI)	5885.45	5719.89	6544.75	5788.45	5386.33	6576.38	6156.89	4596.00	6278.75	5973.00	6338.71	3984.67
Retailer's inventory cost (VMI)	324.76	283.69	1030.65	771.24	1517.17	1159.43	1273.69	312.00	1294.18	132.00	1428.59	231.33
Retailer's inventory cost (RMI)	163.35	104.29	127.92	95.73	15.67	54.44	174.77	156.00	179.69	132.00	164.61	230.33
Transportation cost (VMI)	576.23	3534.18	374.24	3444.91	114.00	3021.72	680.39	1336.00	765.25	3320.00	753.25	2524.33
Transportation cost (RMI)	5262.68	3549.76	5418.24	3465.04	4490.00	3053.95	5658.07	1503.00	6175.39	3320.00	5602.72	2529.33

Table A.8: Detailed Average Cost Data and Results for selected VMI contracts under Low Inventory costs.

$h^{r(v)}$	0.5–5			5–10				10–15	
Contracts	IVTV	IVTR	IRTV	IVTV	IVTR	IRTV	IRTR	IRTV	IRTR
C^v	29.17	661.48	46.00	9.40	704.02	40.32	715.00	40.46	536.00
C^r	532.33	326.74	629.00	552.80	335.54	534.88	404.00	483.50	232.75
$h^{r(r)}$	6.54	6.51	5.00	7.00	7.24	5.60	7.00	5.96	6.75
S	3285.71	2884.07	4000.00	3600.00	2982.86	2988.89	1000.00	3000.00	2000.00
h^v	6.88	6.59	8.00	7.40	7.44	7.19	5.00	7.31	5.75
Setup cost (VMI)	6062.50	5502.59	8000.00	7600.00	5942.86	5955.56	3000.00	6000.00	4750.00
Setup cost (RMI)	6080.36	5544.69	8000.00	7600.00	5977.14	5955.56	3000.00	6020.83	4750.00
Vendor's inventory cost (VMI)	3841.68	3749.22	5040.00	3947.20	3908.07	3683.72	1980.00	3774.81	2943.00
Vendor's inventory cost (RMI)	4734.53	4376.53	6000.00	4998.00	4691.91	4543.08	1990.00	4594.29	2955.75
Retailer's inventory cost (VMI)	296.54	281.86	600.00	843.20	694.98	740.28	105.00	750.52	120.25
Retailer's inventory cost (RMI)	97.17	65.01	0.00	2.40	49.33	82.69	105.00	89.23	112.75
Transportation cost (VMI)	317.52	3508.38	552.00	112.40	3665.58	428.41	4040.00	421.06	2453.25
Transportation cost (RMI)	5542.29	3513.66	7548.00	6562.80	3672.55	5644.20	4040.00	4990.98	2466.25

Table A.9: Distances between warehouses of the Soft Drink company and customers

Nodes	W1	W2	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22	C23	C24	C25	C26	C27	C28	C29	C30
W1	0	81.9	6.2	7.8	8.5	9	10.2	6.5	6.3	8.4	172	170	168	166	169	166	166	168	165	167	167	169	77.7	73	70.2	72.4	85.8	83.9	79.7	84	79	79.4
W2	81.9	0	88.8	87.2	89.2	89.7	90.9	85.8	85.6	91.3	97.7	95.9	94.3	92.3	94.8	91.9	92.1	94	90.8	93.1	92.6	94.9	63.4	61.4	60.6	61.5	5.2	2.2	3.9	2.2	3.3	3.7
C1	6.2	88.8	0	2.2	3.9	4.4	5.6	6.7	6.7	6	177	176	174	172	174	171	172	174	170	173	172	174	72.1	74.3	71.4	73.1	91.4	89.5	85.3	89.6	84.6	85
C2	7.8	87.2	2.2	0	4.6	5.2	5.8	7.5	7.5	6.8	176	175	173	171	174	171	171	173	170	172	171	174	71.3	73.5	70.7	72.3	90.6	88.8	84.6	86.9	83.8	84.2
C3	8.5	89.2	3.9	4.6	0	0.55	3.2	3.4	3.4	2.7	179	178	176	174	176	173	174	175	172	175	174	176	74	76.2	73.3	75	93.3	91.4	87.2	91.5	86.5	86.9
C4	9	89.7	4.4	5.2	0.55	0	3.1	3.8	3.8	2.2	180	178	176	174	177	174	174	176	173	175	175	177	74.6	76.8	73.9	75.6	93.9	92	87.8	92.1	87.1	87.5
C5	10.2	90.9	5.6	5.8	3.2	3.1	0	6.1	6.7	5	182	180	178	176	179	176	176	178	175	177	176	179	76.4	78.6	75.7	77.4	95.7	93.8	89.6	93.9	88.9	89.3
C6	6.5	85.8	6.7	7.5	3.4	3.8	6.1	0	0.65	1.5	175	173	171	169	172	169	169	171	168	170	170	172	76.9	79.1	76.2	77.9	96.2	94.3	90.1	94.4	89.4	89.8
C7	6.3	85.6	6.7	7.5	3.4	3.8	6.7	0.65	0	2.1	182	181	178	177	179	176	176	178	175	177	177	179	76.9	79.1	76.3	77.9	96.1	94.2	90	94.3	89.3	89.7
C8	8.4	91.3	6	6.8	2.7	2.2	5	1.5	2.1	0	181	180	178	176	178	176	176	178	174	177	176	179	76.2	78.4	75.6	77.2	95.5	93.6	95.5	93.7	94.8	89.2
C9	172	97.7	177	176	179	180	182	175	182	181	0	2.1	4	8.3	11.3	9.7	8.6	10.6	8	5	16.1	13.5	86.8	86.7	86.8	85.5	92.6	99.7	102	99.8	101	101
C10	170	95.9	176	175	178	178	180	173	181	180	2.1	0	2.4	4.5	11.8	5.9	7	9.1	6.4	3.4	10.8	12	88.5	88.4	88.5	87.2	90.5	97.6	99.5	97.6	98.7	99.1
C11	168	94.3	174	173	176	176	178	171	178	178	4	2.4	0	3.1	8.8	4.5	6.1	8.2	5	2	9.4	11.4	87.5	87.5	87.6	86.3	89	96.1	98	96.2	97.3	97.7
C12	166	92.3	172	171	174	174	176	169	177	176	8.3	4.5	3.1	0	7.9	2.1	4.9	7	2.6	1.8	7	8.1	81	81	81.1	79.8	86.6	93.7	95.6	93.8	94.9	95.3
C13	169	94.8	174	174	176	177	179	172	179	178	11.3	11.8	8.8	7.9	0	8	3.3	1.7	4.7	9.1	3.4	0.35	72.7	72.6	72.7	71.4	88.8	95.9	97.8	96	97.1	97.5
C14	166	91.9	171	171	173	174	176	169	176	176	9.7	5.9	4.5	2.1	8	0	2.8	5.3	1.1	3	6.4	7.5	80.4	80.4	80.5	79.2	86	93.1	95	93.2	94.3	94.7
C15	166	92.1	172	171	174	174	176	169	176	176	8.6	7	6.1	4.9	3.3	2.8	0	2.3	2.4	6.1	5.3	3.3	80.5	80.5	80.6	79.3	86.1	93.2	95.1	93.3	94.4	94.8
C16	168	94	174	173	175	176	178	171	178	178	10.6	9.1	8.2	7	1.7	5.3	2.3	0	4.7	8.1	4.1	2.1	72.3	72.3	72.4	71.1	88.2	95.3	97.2	95.3	96.4	96.8
C17	165	90.8	170	170	172	173	175	168	175	174	8	6.4	5	2.6	4.7	1.1	2.4	4.7	0	3.5	5.6	4.8	79.6	79.6	79.7	78.4	85.2	92.3	94.2	92.4	93.5	93.9
C18	167	93.1	173	172	175	175	177	170	177	177	5	3.4	2	1.8	9.1	3	6.1	8.1	3.5	0	8.4	9.5	82.5	82.4	82.5	81.2	88	95.1	97	95.2	96.3	96.7
C19	167	92.6	172	171	174	175	176	170	177	176	16.1	10.8	9.4	7	3.4	6.4	5.3	4.1	5.6	8.4	0	3.7	72.4	72.4	72.5	71.2	86	93.1	95	93.2	94.3	94.7
C20	169	94.9	174	174	176	177	179	172	179	179	13.5	12	11.4	8.1	0.35	7.5	3.3	2.1	4.8	9.5	3.7	0	72.5	72.5	72.6	71.3	89.1	96.2	98.1	96.3	97.4	97.8
C21	77.7	63.4	72.1	71.3	74	74.6	76.4	76.9	76.9	76.2	86.8	88.5	87.5	81	72.7	80.4	80.5	72.3	79.6	82.5	72.4	72.5	0	4.2	2.8	2.2	66.9	65.1	63.5	63.2	62.8	63.2
C22	73	61.4	74.3	73.5	76.2	76.8	78.6	79.1	79.1	78.4	86.7	88.4	87.5	81	72.6	80.4	80.5	72.3	79.6	82.4	72.4	72.5	4.2	0	3	2.3	68.3	66.5	64.9	64.6	64.1	64.6
C23	70.2	60.6	71.4	70.7	73.3	73.9	75.7	76.2	76.3	75.6	86.8	88.5	87.6	81.1	72.7	80.5	80.6	72.4	79.7	82.5	72.5	72.6	2.8	3	0	2.1	64.5	62.6	61	60.7	60.3	60.7
C24	72.4	61.5	73.1	72.3	75	75.6	77.4	77.9	77.9	77.2	85.5	87.2	86.3	79.8	71.4	79.2	79.3	71.1	78.4	81.2	71.2	71.3	2.2	2.3	2.1	0	66.1	62.5	62.7	62.4	61.9	62.4
C25	85.8	5.2	91.4	90.6	93.3	93.9	95.7	96.2	96.1	95.5	92.6	90.5	89	86.6	88.8	86	86.1	88.2	85.2	88	86	89.1	66.9	68.3	64.5	66.1	0	4.7	7.5	5.2	8.6	7.7
C26	83.9	2.2	89.5	88.8	91.4	92	93.8	94.3	94.2	93.6	99.7	97.6	96.1	93.7	95.9	93.1	93.2	95.3	92.3	95.1	93.1	96.2	65.1	66.5	62.6	62.5	4.7	0	2.8	0.19	2.9	2.6
C27	79.7	3.9	85.3	84.6	87.2	87.8	89.6	90.1	90	95.5	102	99.5	98	95.6	97.8	95	95.1	97.2	94.2	97	95	98.1	63.5	64.9	61	62.7	7.5	2.8	0	2.7	1.1	0.072
C28	84	2.2	89.6	86.9	91.5	92.1	93.9	94.4	94.3	93.7	99.8	97.6	96.2	93.8	96	93.2	93.3	95.3	92.4	95.2	93.2	96.3	63.2	64.6	60.7	62.4	5.2	0.19	2.7	0	2.8	2.5
C29	79	3.3	84.6	83.8	86.5	87.1	88.9	89.4	89.3	94.8	101	98.7	97.3	94.9	97.1	94.3	94.4	96.4	93.5	96.3	94.3	97.4	62.8	64.1	60.3	61.9	8.6	2.9	1.1	2.8	0	1
C30	79.4	3.7	85	84.2	86.9	87.5	89.3	89.8	89.7	89.2	101	99.1	97.7	95.3	97.5	94.7	94.8	96.8	93.9	96.7	94.7	97.8	63.2	64.6	60.7	62.4	7.7	2.6	0.072	2.5	1	0

BIBLIOGRAPHY

Absi, N., Archetti, C., Dauzère-Pérès, S., & Feillet, D. (2015). A two-phase iterative heuristic approach for the production routing problem. *Transportation Science*, *49*(4), 784–795.

Absi, N., Archetti, C., Dauzère-Pérès, S., Feillet, D., & Speranza, M. G. (2018). Comparing sequential and integrated approaches for the production routing problem. *European Journal of Operational Research*, *269*(2), 633–646.

Adulyasak, Y., Cordeau, J.-F., & Jans, R. (2014a). Formulations and branch-and-cut algorithms for multivehicle production and inventory routing problems. *INFORMS Journal on Computing*, *26*(1), 103–120.

Adulyasak, Y., Cordeau, J.-F., & Jans, R. (2014b). Optimization-based adaptive large neighborhood search for the production routing problem. *Transportation science*, *48*(1), 20–45.

Adulyasak, Y., Cordeau, J.-F., & Jans, R. (2015a). Benders decomposition for production routing under demand uncertainty. *Operations Research*, *63*(4), 851–867.

Adulyasak, Y., Cordeau, J.-F., & Jans, R. (2015b). The production routing problem: A review of formulations and solution algorithms. *Computers & Operations Research*, *55*, 141–152.

Aggarwal, A., & Park, J. K. (1993). Improved algorithms for economic lot size problems. *Operations research*, *41*(3), 549–571.

Akbalik, A. (2006). *Optimisation de la gestion intégrée des flux physiques dans une chaîne logistique: Extensions du problème de dimensionnement de lot* [Doctoral dissertation, l'Institut National de Polytechnique de Grenoble].

Aksen, D., Altnkemer, K., & Chand, S. (2003). The single-item lot-sizing problem with immediate lost sales. *European Journal of Operational Research*, *147*(3), 558–566.

Almehdawe, E., & Mantin, B. (2010). Vendor managed inventory with a capacitated manufacturer and multiple retailers: Retailer versus manufacturer leadership. *International journal of production economics*, *128*(1), 292–302.

Alp, O., Erkip, N. K., & Güllü, R. (2003). Optimal lot-sizing/vehicle-dispatching policies under stochastic lead times and stepwise fixed costs. *Operations Research*, *51*(1), 160–166.

Alvarez, A., Miranda, P., & Rohmer, S. U. (2022). Production routing for perishable products. *Omega*, *111*, 102667.

Amazon Business. (2021). *How ai and analytics are transforming procurement* [Accessed July 23, 2023]. https://business.amazon.com/en/discover-more/blog/how-ai-and-ml-are-transforming-procurement

Andel, T. (1996). Manage inventory, own information. *Transportation & Distribution*.

Andersson, H., Hoff, A., Christiansen, M., Hasle, G., & Løkketangen, A. (2010). Industrial aspects and literature survey: Combined inventory management and routing. *Computers & operations research*, *37*(9), 1515–1536.

Archetti, C., Bertazzi, L., Paletta, G., & Speranza, M. G. (2011). Analysis of the maximum level policy in a production-distribution system. *Computers & Operations Research*, *38*(12), 1731–1746.

Archetti, C., & Speranza, M. G. (2016). The inventory routing problem: The value of integration. *International Transactions in Operational Research*, *23*(3), 393–407.

Arkin, E., Joneja, D., & Roundy, R. (1989). Computational complexity of uncapacitated multi-echelon production planning problems. *Operations research letters*, *8*(2), 61–66.

Armentano, V. A., Shiguemoto, A. L., & Løkketangen, A. (2011). Tabu search with path relinking for an integrated production–distribution problem. *Computers & Operations Research*, *38*(8), 1199–1209.

Bard, J. F., & Nananukul, N. (2009). The integrated production–inventory–distribution–routing problem. *Journal of Scheduling*, *12*(3), 257–280.

Bard, J. F., & Nananukul, N. (2010). A branch-and-price algorithm for an integrated production and inventory routing problem. *Computers & Operations Research*, *37*(12), 2202–2217.

Beamon, B. M. (1998). Supply chain design and analysis:: Models and methods. *International journal of production economics*, *55*(3), 281–294.

Bellman, R. (1957). Dynamic programming, princeton univ. *Press Princeton, New Jersey*, *39*.

Benfedel, R., Belkaid, F., & Brahimi, N. (2025a). Production and distribution planning with direct shipment and multi-trips. *Journal of Industrial and Management Optimization*, *21*(5), 3960–3984.

Benfedel, R., Belkaid, F., & Brahimi, N. (2025b). Production routing decisions in a two-echelon supply chain with multiple delivery modes. *International Transactions in Operational Research*.

Berghman, L., Kergosien, Y., & Billaut, J.-C. (2023). A review on integrated scheduling and outbound vehicle routing problems. *European Journal of Operational Research*.

Bertsimas, D., Brown, D. B., & Caramanis, C. (2011). Theory and applications of robust optimization. *SIAM review*, *53*(3), 464–501.

Bertsimas, D., & Tsitsiklis, J. (1993). Simulated annealing. *Statistical science*, *8*(1), 10–15.

Bettayeb, B., Brahimi, N., & Lemoine, D. (2018). Integrated dynamic single item lot-sizing and quality inspection planning. *International Journal of Production Research*, *56*(7), 2611–2627.

Bichescu, B. C., & Fry, M. J. (2009). Vendor-managed inventory and the effect of channel power. *OR spectrum*, *31*, 195–228.

Birge, J. R., & Louveaux, F. (2011). *Introduction to stochastic programming* (2nd). Springer Science & Business Media.

Bitran, G. R., & Yanasse, H. H. (1982). Computational complexity of the capacitated lot size problem. *Management Science*, *28*(10), 1174–1186.

Blumenfeld, D. E., Burns, L. D., Diltz, J. D., & Daganzo, C. F. (1985). Analyzing trade-offs between transportation, inventory and production costs on freight networks. *Transportation Research Part B: Methodological*, *19*(5), 361–380.

Bookbinder, J. H., Gümü, M., & Jewkes, E. M. (2010). Calculating the benefits of vendor managed inventory in a manufacturer-retailer system. *International Journal of Production Research*, *48*(19), 5549–5571.

Boudia, M., Louly, M. A. O., & Prins, C. (2007). A reactive grasp and path relinking for a combined production–distribution problem. *Computers & Operations Research*, *34*(11), 3402–3419.

Boudia, M., Louly, M. A. O., & Prins, C. (2008). Fast heuristics for a combined production planning and vehicle routing problem. *Production Planning and Control*, *19*(2), 85–96.

Boudia, M., & Prins, C. (2009). A memetic algorithm with dynamic population management for an integrated production–distribution problem. *European journal of operational research*, *195*(3), 703–715.

Bourland, K. E., Powell, S. G., & Pyke, D. F. (1996). Exploiting timely demand information to reduce inventories. *European journal of operational research*, *92*(2), 239–253.

Boutarfa, Y., Senoussi, A., Brahimi, N., & Aouam, T. (2024). Integration and substitution in hybrid manufacturing and refurbishing systems. *International Journal of Production Economics*, *274*, 109308. https://doi.org/https://doi.org/10.1016/j.ijpe.2024.109308

Brahimi, N. (2004). *Planification de la production : Modèles et algorithmes pour les problèmes de dimensionnement de lots.* [Doctoral dissertation, Ecole des mines de Nantes].

Brahimi, N., Absi, N., Dauzère-Pérès, S., & Nordli, A. (2017). Single-item dynamic lot-sizing problems: An updated survey. *European Journal of Operational Research*, *263*(3), 838–863.

Brahimi, N., & Aouam, T. (2016). Multi-item production routing problem with backordering: A milp approach. *International Journal of Production Research*, *54*(4), 1076–1093.

Brahimi, N., Dauzere-Peres, S., Najid, N. M., & Nordli, A. (2006). Single item lot sizing problems. *European Journal of Operational Research*, *168*(1), 1–16.

Brahimi, N., Dauzère-Pérès, S., & Najid, N. M. (2006). Capacitated multi-item lot-sizing problems with time windows. *Operations Research*, *54*(5), 951–967.

Brandão, J. C. S., & Mercer, A. (1998). The multi-trip vehicle routing problem. *Journal of the Operational research society*, *49*, 799–805.

Brekkå, I., Randøy, S., Fagerholt, K., Thun, K., & Vadseth, S. T. (2022). The fish feed production routing problem. *Computers & Operations Research*, *144*, 105806.

C., W. (2008). *Approche intégrée en planification et ordonnancement de la production* [Doctoral dissertation, Ecole Nationale Supt'erieure des Mines de Saint-Etienne].

Cachon, G., & Terwiesch, C. (2008). *Matching supply with demand* (Vol. 20012). McGraw-Hill Publishing New York.

Cachon, G. P., & Fisher, M. (2000). Supply chain inventory management and the value of shared information. *Management science*, *46*(8), 1032–1048.

Cachon, G. P., & Lariviere, M. A. (2005). Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management science*, *51*(1), 30–44.

Cattaruzza, D., Absi, N., & Feillet, D. (2016). Vehicle routing problems with multiple trips. *4or*, *14*, 223–259.

Cattaruzza, D., Absi, N., & Feillet, D. (2018). Vehicle routing problems with multiple trips. *Annals of Operations Research*, *271*(1), 127–159.

Cattaruzza, D., Absi, N., Feillet, D., & Vigo, D. (2014). An iterated local search for the multi-commodity multi-trip vehicle routing problem with time windows. *Computers & Operations Research*, *51*, 257–267.

Ceschia, S., Di Gaspero, L., & Schaerf, A. (2017). Solving discrete lot-sizing and scheduling by simulated annealing and mixed integer programming. *Computers & Industrial Engineering*, *114*, 235–243.

Chand, S., Hsu, V. N., Sethi, S., & Deshpande, V. (2007). A dynamic lot sizing problem with multiple customers: Customer-specific shipping and backlogging costs. *IIE Transactions*, *39*(11), 1059–1069.

Chandra, P. (1993). A dynamic distribution model with warehouse and customer replenishment requirements. *Journal of the Operational Research Society*, *44*(7), 681–692.

Chandra, P., & Fisher, M. L. (1994). Coordination of production and distribution planning. *European journal of operational research*, *72*(3), 503–517.

Chekoubi, Z., Trabelsi, W., Sauer, N., & Majdouline, I. (2022). The integrated production-inventory-routing problem with reverse logistics and remanufacturing: A two-phase decomposition heuristic. *Sustainability*, *14*(20), 13563.

Chen, S., Cohen, M. A., & Lee, H. (2024). Enhancing customer–supplier coordination through customer-managed inventory. *Management Science*, *70*(12), 9073–9088.

Chopra, S., & Meindl, P. (2013). *Supply chain management by pearson* (5th ed.). Pearson Education.

Christopher, D., Towill. (2000). Supply chain migration from lean and functional to agile and customised. *Supply Chain Management: An International Journal*.

Christopher, M. (2022). *Logistics and supply chain management*. Pearson Uk.

Coelho, L. C., Cordeau, J.-F., & Laporte, G. (2012). The inventory-routing problem with transshipment. *Computers & Operations Research*, *39*(11), 2537–2548.

Crainic, T. G., Gajpal, Y., & Gendreau, M. (2015). Multi-zone multi-trip vehicle routing problem with time windows. *INFOR: Information Systems and Operational Research*, *53*(2), 49–67.

Crowder, H., Johnson, E. L., & Padberg, M. (1983). Solving large-scale zero-one linear programming problems. *Operations Research*, *31*(5), 803–834.

Cunha, P., Raupp, F., & Oliveira, F. (2017). A two-stage stochastic programming model for periodic replenishment control system under demand uncertainty. *Computers & Industrial Engineering*, *107*, 313–326.

Dantzig, G. B., & Ramser, J. H. (1959). The truck dispatching problem. *Management science*, *6*(1), 80–91.

Darwish, M. A., & Odah, O. (2010). Vendor managed inventory model for single-vendor multi-retailer supply chains. *European Journal of Operational Research*, *204*(3), 473–484.

Desgrippes, E. (2005). *Coordination entre la production et la distribution dans une chaînelogistique* [Doctoral dissertation, l'Institut National de Polytechnique de Grenoble].

Díaz-Madroñero, M., Mula, J., & Peidro, D. (2014). A review of discrete-time optimization models for tactical production planning. *International Journal of Production Research*, *52*(17), 5171–5205.

Dong, Y., & Xu, K. (2002). A supply chain model of vendor managed inventory. *Transportation research part E: logistics and transportation review*, *38*(2), 75–95.

Dorling, K., Scott, J., & Deakins, E. (2006). Determinants of successful vendor managed inventory relationships in oligopoly industries. *International Journal of Physical Distribution & Logistics Management*, *36*(3), 176–191.

Drexl, A., & Kimms, A. (1997). Lot sizing and schedulingsurvey and extensions. *European Journal of operational research*, *99*(2), 221–235.

Fan, S., Ilk, N., Kumar, A., Xu, R., & Zhao, J. L. (2024). Blockchain as a trust machine: From disillusionment to enlightenment in the era of generative ai.

Fattahi, P., Hosseini, S. M. H., Jolai, F., & Tavakkoli-Moghaddam, R. (2014). A branch and bound algorithm for hybrid flow shop scheduling problem with setup time and assembly operations. *Applied Mathematical Modelling*, *38*(1), 119–134.

Federgruen, A., & Tzur, M. (1999). Time-partitioning heuristics: Application to one warehouse, multiitem, multiretailer lot-sizing problems. *Naval Research Logistics (NRL)*, *46*(5), 463–486.

Fleischmann, B. (1990). The vehicle routing problem with multiple use of vehicles. *Fachbereich Wirtschaftswissenschaften, Universität Hamburg*.

Florian, M., & Klein, M. (1971). Deterministic production planning with concave costs and capacity constraints. *Management science*, *18*(1), 12–20.

Fry, M. J., Kapuscinski, R., & Olsen, T. L. (2001). Coordinating production and delivery under a (z, Z)-type vendor-managed inventory contract. *Manufacturing & Service Operations Management*, *3*(2), 151–173.

Fumero, F., & Vercellis, C. (1999). Synchronized development of production, inventory, and distribution schedules. *Transportation science*, *33*(3), 330–340.

Galasso, F. (2007). *Aide à la planification dans les chaines logistiques en présence de demande flexible* [Doctoral dissertation, Laboratoire dAnalyse et dArchitecture des Systemes - LAAS (Toulouse, France)].

Gavirneni, S. (2002). Information flows in capacitated supply chains with fixed ordering costs. *Management science*, *48*(5), 644–651.

Gavirneni, S., Kapuscinski, R., & Tayur, S. (1999). Value of information in capacitated supply chains. *Management science*, *45*(1), 16–24.

Gerchak, Y., Khmelnitsky, E., & Robinson, L. W. (2007). Untruthful probabilistic demand forecasts in vendor-managed revenue-sharing contracts: Coordinating the chain. *Naval Research Logistics (NRL)*, *54*(7), 742–749.

Gerchak, Y., & Wang, Y. (2004). Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand. *Production and operations Management*, *13*(1), 23–33.

Glover, F. (1989). Tabu searchpart i. *ORSA Journal on computing*, *1*(3), 190–206.

Goldberg, D. E. (1989). *Genetic algorithms in search, optimization, and machine learning*. Addison-Wesley.

Gomez Urrutia, E. D., Aggoune, R., & Dauzère-Pérès, S. (2014). Solving the integrated lot-sizing and job-shop scheduling problem. *International Journal of Production Research*, *52*(17), 5236–5254.

Gomory, R. (1958). Outline of an algorithm for integer solutions to linear programs. *Bulletin of American Mathematical Society*, *64:275278*.

Gümü, M., Jewkes, E. M., & Bookbinder, J. H. (2008). Impact of consignment inventory and vendor-managed inventory for a two-party supply chain. *International Journal of Production Economics*, *113*(2), 502–517.

Guo, D., & and, S. M. (2025). The role of digital twins in lean supply chain management: Review and research directions. *International Journal of Production Research*, *63*(5), 1851–1872. https://doi.org/10.1080/00207543.2024.2372655

Helsgaun, K. (2017). An extension of the lin-kernighan-helsgaun tsp solver for constrained traveling salesman and vehicle routing problems. *Roskilde: Roskilde University*, *12*.

Herer, Y. T., & Tzur, M. (2001). The dynamic transshipment problem. *Naval Research Logistics (NRL)*, *48*(5), 386–408.

Hernandez, F., Feillet, D., Giroudeau, R., & Naud, O. (2014). A new exact algorithm to solve the multi-trip vehicle routing problem with time windows and limited duration. *4or*, *12*, 235–259.

Holland, J. H. (1992). *Adaptation in natural and artificial systems: An introductory analysis with applications to biology, control, and artificial intelligence*. MIT press.

Hrabec, D., Hvattum, L. M., & Hoff, A. (2022). The value of integrated planning for production, inventory, and routing decisions: A systematic review and meta-analysis. *International Journal of Production Economics*, *248*, 108468.

Hsu, V. N., Li, C.-L., & Xiao, W.-Q. (2005). Dynamic lot size problems with one-way product substitution. *IIE transactions*, *37*(3), 201–215.

Huang, G. Q., Lau, J. S., & Mak, K. (2003). The impacts of sharing production information on supply chain dynamics: A review of the literature. *International journal of production research*, *41*(7), 1483–1517.

Jackson, I., Ivanov, D., Dolgui, A., & and, J. N. (2024). Generative artificial intelligence in supply chain and operations management: A capability-based framework for analysis and implementation. *International Journal of Production Research*, *62*(17), 6120–6145. https://doi.org/10.1080/00207543.2024.2309309

Jans, R., & Degraeve, Z. (2007). Meta-heuristics for dynamic lot sizing: A review and comparison of solution approaches. *European journal of operational research*, *177*(3), 1855–1875.

Jaruphongsa, W., Çetinkaya, S., & Lee, C.-Y. (2007). Outbound shipment mode considerations for integrated inventory and delivery lot-sizing decisions. *Operations Research Letters*, *35*(6), 813–822.

Jaruphongsa, W., & Lee, C.-Y. (2008). Dynamic lot-sizing problem with demand time windows and container-based transportation cost. *Optimization Letters*, *2*(1), 39–51.

Karimi, B., Ghomi, S. F., & Wilson, J. (2003). The capacitated lot sizing problem: A review of models and algorithms. *Omega*, *31*(5), 365–378.

Karimi, N., & Davoudpour, H. (2015). A branch and bound method for solving multi-factory supply chain scheduling with batch delivery. *Expert Systems with Applications*, *42*(1), 238–245.

Kirkpatrick, S. (1984). Optimization by simulated annealing: Quantitative studies. *Journal of statistical physics*, *34*, 975–986.

Kirkpatrick, S., Gelatt Jr, C. D., & Vecchi, M. P. (1983). Optimization by simulated annealing. *science*, *220*(4598), 671–680.

Koç, Ç., & Karaoglan, I. (2016). The green vehicle routing problem: A heuristic based exact solution approach. *Applied Soft Computing*, *39*, 154–164.

Krajewski, L. J., & Malhotra, M. K. (2022). *Operations management: Processes and supply chains*. Pearson.

Kuik, R., & Salomon, M. (1990). Multi-level lot-sizing problem: Evaluation of a simulated-annealing heuristic. *European Journal of Operational Research*, *45*(1), 25–37.

Lai, M.-Y., Liu, C.-S., & Tong, X.-J. (2010). A two-stage hybrid meta-heuristic for pickup and delivery vehicle routing problem with time windows. *Journal of Industrial and Management Optimization*, *6*(2), 435–451.

Lee, C. C., & Chu, W. H. J. (2005). Who should control inventory in a supply chain? *European Journal of Operational Research*, *164*(1), 158–172.

Lee, H. L., & Billington, C. (1995). The evolution of supply-chain-management models and practice at hewlett-packard. *Interfaces*, *25*(5), 42–63.

Lee, H. L., So, K. C., & Tang, C. S. (2000). The value of information sharing in a two-level supply chain. *Management science*, *46*(5), 626–643.

Lee, J.-Y., & Cho, R. K. (2014). Contracting for vendor-managed inventory with consignment stock and stockout-cost sharing. *International Journal of Production Economics*, *151*, 158–173.

Lee, J.-Y., & Johnson, G. (2020). Contracting for vendor-managed inventory with a time-dependent stockout penalty. *International Transactions in Operational Research*, *27*(3), 1573–1599.

Lemoine, D. (2008). *Modèles génériques et méthodes de résolution pour la planification tactique mono-site et multi-site.* [Doctoral dissertation, Université Blaise Pascal - Clermont-Ferrand II].

Li, C.-L., Hsu, V. N., & Xiao, W.-Q. (2004). Dynamic lot sizing with batch ordering and truckload discounts. *Operations Research*, *52*(4), 639–654.

Li, L., Liu, Y., Jin, Y., Cheng, T. E., & Zhang, Q. (2024). Generative ai-enabled supply chain management: The critical role of coordination and dynamism. *International Journal of Production Economics*, *277*, 109388. https://doi.org/https://doi.org/10.1016/j.ijpe.2024.109388

Loparic, M., Marchand, H., & Wolsey, L. A. (2003). Dynamic knapsack sets and capacitated lot-sizing. *Mathematical Programming*, *95*, 53–69.

Lotfi, V., & Yoon, Y.-S. (1994). An algorithm for the single-item capacitated lot-sizing problem with concave production and holding costs. *Journal of the Operational Research Society*, *45*(8), 934–941.

Macedo, P. B., Alem, D., Santos, M., Junior, M. L., & Moreno, A. (2016). Hybrid manufacturing and remanufacturing lot-sizing problem with stochastic demand, return, and setup costs. *The International Journal of Advanced Manufacturing Technology*, *82*, 1241–1257.

Magee, J. (1985). *Production planning and inventory control*. McGraw-Hill Book Company.

Malicki, S., & Minner, S. (2021). Cyclic inventory routing with dynamic safety stocks under recurring non-stationary interdependent demands. *Computers & Operations Research*, *131*, 105247.

Manne, A. S. (1958). Programming of economic lot sizes. *Management science*, *4*(2), 115–135.

Melega, G. M., de Araujo, S. A., & Jans, R. (2018). Classification and literature review of integrated lot-sizing and cutting stock problems. *European Journal of Operational Research*, *271*(1), 1–19.

Mentzer, J. T., DeWitt, W., Keebler, J. S., Min, S., Nix, N. W., Smith, C. D., & Zacharia, Z. G. (2001). Defining supply chain management. *Journal of Business logistics*, *22*(2), 1–25.

Mishra, B. K., & Raghunathan, S. (2004). Retailer-vs. vendor-managed inventory and brand competition. *Management Science*, *50*(4), 445–457.

Mladenovi, N., & Hansen, P. (1997). Variable neighborhood search. *Computers & operations research*, *24*(11), 1097–1100.

Mulvey, J. M., Vanderbei, R. J., & Zenios, S. A. (1995). Robust optimization of large-scale systems. *Operations research*, *43*(2), 264–281.

Neves-Moreira, F., Almada-Lobo, B., Cordeau, J.-F., Guimarães, L., & Jans, R. (2019). Solving a large multi-product production-routing problem with delivery time windows. *Omega*, *86*, 154–172.

Özdamar, L., & Barbarosoglu, G. (2000). An integrated lagrangean relaxation-simulated annealing approach to the multi-level multi-item capacitated lot sizing problem. *International Journal of production economics*, *68*(3), 319–331.

Özer, Ö., Zheng, Y., & Ren, Y. (2014). Trust, trustworthiness, and information sharing in supply chains bridging china and the united states. *Management Science*, *60*(10), 2435–2460.

Pasandideh, S. H. R., Niaki, S. T. A., & Roozbeh Nia, A. (2010). An investigation of vendor-managed inventory application in supply chain: The eoq model with shortage. *The International Journal of Advanced Manufacturing Technology*, *49*, 329–339.

Pasternack, B. A. (1985). Optimal pricing and return policies for perishable commodities. *Marketing science*, *4*(2), 166–176.

Pochet, Y. (1988). Valid inequalities and separation for capacitated economic lot sizing. *Operations Research Letters*, *7*(3), 109–115.

Pochet, Y., & Wolsey, L. A. (2010). Single item lot-sizing with non-decreasing capacities. *Mathematical Programming*, *121*, 123–143.

Qiu, Y., Ni, M., Wang, L., Li, Q., Fang, X., & Pardalos, P. M. (2018). Production routing problems with reverse logistics and remanufacturing. *Transportation Research Part E: Logistics and Transportation Review*, *111*, 87–100.

Qiu, Y., Zhou, D., Du, Y., Liu, J., Pardalos, P. M., & Qiao, J. (2021). The two-echelon production routing problem with cross-docking satellites. *Transportation Research Part E: Logistics and Transportation Review*, *147*, 102210.

Rad, R. H., Razmi, J., Sangari, M. S., & Ebrahimi, Z. F. (2014). Optimizing an integrated vendor-managed inventory system for a single-vendor two-buyer supply chain with determining weighting factor for vendor s ordering cost. *International Journal of Production Economics*, *153*, 295–308.

Radzuan, K., Omar, M. F., Nawi, M. N. M., Rahim, M., & Yaakob, M. (2018). Vendor managed inventory practices: A case in manufacturing companies. *International Journal of Supply Chain Management*, *7*(4), 196–201.

Ramos, B., Alves, C., & Valerio de Carvalho, J. (2022). An arc flow formulation to the multitrip production, inventory, distribution, and routing problem with time windows. *International Transactions in Operational Research*, *29*(1), 526–553.

Rizk, N., Martel, A., & Ramudhin, A. (2006). A lagrangean relaxation algorithm for multi-item lot-sizing problems with joint piecewise linear resource costs. *International Journal of Production Economics*, *102*(2), 344–357.

Robinson, P., Narayanan, A., & Sahin, F. (2009). Coordinated deterministic dynamic demand lot-sizing problem: A review of models and algorithms. *Omega*, *37*(1), 3–15.

Ru, J., Shi, R., & Zhang, J. (2018). When does a supply chain member benefit from vendor-managed inventory? *Production and operations management*, *27*(5), 807–821.

Ru, J., & Wang, Y. (2010). Consignment contracting: Who should control inventory in the supply chain? *European Journal of Operational Research*, *201*(3), 760–769.

Ruokokoski, M., Solyali, O., Cordeau, J.-F., Jans, R., & Süral, H. (2010). *Efficient formulations and a branch-and-cut algorithm for a production-routing problem*. Citeseer.

Sainathan, A., & Groenevelt, H. (2019). Vendor managed inventory contracts–coordinating the supply chain while looking from the vendors perspective. *European Journal of Operational Research*, *272*(1), 249–260.

Schenekemberg, C. M., Scarpin, C. T., Pecora Jr, J. E., Guimarães, T. A., & Coelho, L. C. (2021). The two-echelon production-routing problem. *European Journal of Operational Research*, *288*(2), 436–449.

SENOUSSI, A. (2016). *Optimisation intégrée des fonctions production, stockage et distribution dans une chaine logistique* [Doctoral dissertation, Laboratoire dAutomatique et Productique, Departement de Génie Industriel, Batna].

Senoussi, A., Mouss, N. K., Penz, B., Brahimi, N., & Dauzère-Pérès, S. (2016). Modeling and solving a one-supplier multi-vehicle production-inventory-distribution problem with clustered retailers. *The International Journal of Advanced Manufacturing Technology*, *85*, 971–989.

Shaabani, H., & Kamalabadi, I. N. (2016). An efficient population-based simulated annealing algorithm for the multi-product multi-retailer perishable inventory routing problem. *Computers & Industrial Engineering*, *99*, 189–201.

Shaw, D. X., & Wagelmans, A. P. (1998). An algorithm for single-item capacitated economic lot sizing with piecewise linear production costs and general holding costs. *Management Science*, *44*(6), 831–838.

Shiguemoto, A. L., & Armentano, V. A. (2010). A tabu search procedure for coordinating production, inventory and distribution routing problems. *International Transactions in Operational Research*, *17*(2), 179–195.

Simchi-Levi, D., Kaminsky, P., & Simchi-Levi, E. (2003). *Designing and managing the supply chain: Concepts, strategies, and cases*. McGraw-hill New York.
Simchi-Levi, D., Kaminsky, P., & Simchi-Levi, E. (2009). Designing and managing the supply chain 3rd.
Song, D.-P., & Dinwoodie, J. (2008). Quantifying the effectiveness of VMI and integrated inventory management in a supply chain with uncertain lead-times and uncertain demands. *Production Planning & Control*, *19*(6), 590–600.
Stadtler, H. (2005). Supply chain management and advanced planning—-basics, overview and challenges. *European journal of operational research*, *163*(3), 575–588.
Staggemeier, A. T., & Clark, A. (2001). A survey of lot-sizing and scheduling models.
Ståhl Elvander, M., Sarpola, S., & Mattsson, S.-A. (2007). Framework for characterizing the design of VMI systems. *International Journal of Physical Distribution & Logistics Management*, *37*(10), 782–798.
Stapleton, D., Hanna, J. B., & Ross, J. R. (2006). Enhancing supply chain solutions with the application of chaos theory. *Supply Chain Management: An International Journal*, *11*(2), 108–114.
Talbi, E.-G., Yalaoui, F., & Amodeo, L. (2016). *Metaheuristics for production systems* (Vol. 60). Springer.
Taylor, T. A. (2002). Supply chain coordination under channel rebates with sales effort effects. *Management science*, *48*(8), 992–1007.
Theurich, F., Fischer, A., & Scheithauer, G. (2021). A branch-and-bound approach for a vehicle routing problem with customer costs. *EURO Journal on Computational Optimization*, *9*, 100003.
Thomas, D. J., & Griffin, P. M. (1996). Coordinated supply chain management. *European journal of operational research*, *94*(1), 1–15.
Tian, X., Wang, H., & Erjiang, E. (2021). Forecasting intermittent demand for inventory management by retailers: A new approach. *Journal of Retailing and Consumer Services*, *62*, 102662.
Toth, P., & Vigo, D. (2014). *Vehicle routing: Problems, methods, and applications*. SIAM.
Tsay, A. A. (1999). The quantity flexibility contract and supplier-customer incentives. *Management science*, *45*(10), 1339–1358.
Tyan, J., & Wee, H.-M. (2003). Vendor managed inventory: A survey of the taiwanese grocery industry. *Journal of Purchasing and Supply Management*, *9*(1), 11–18.
van Norden, L., & van de Velde, S. (2005). Multi-product lot-sizing with a transportation capacity reservation contract. *European Journal of Operational Research*, *165*(1), 127–138.
Van Hoek, R., DeWitt, M., Lacity, M., & Johnson, T. (2022). How walmart automated supplier negotiations. *Harvard Business Review*, *8*(11), 2022.
Van Hoesel, C., & Wagelmans, A. P. M. (1996). An o (t 3) algorithm for the economic lot-sizing problem with constant capacities. *Management science*, *42*(1), 142–150.
Vasant, P. (2010). Hybrid simulated annealing and genetic algorithms for industrial production management problems. *International Journal of Computational Methods*, *7*(02), 279–297.
Vincent, F. Y., Susanto, H., Jodiawan, P., Ho, T.-W., Lin, S.-W., & Huang, Y.-T. (2022). A simulated annealing algorithm for the vehicle routing problem with parcel lockers. *IEEE Access*, *10*, 20764–20782.
Wagelmans, A., Van Hoesel, S., & Kolen, A. (1992). Economic lot sizing: An o (n log n) algorithm that runs in linear time in the wagner-whitin case. *Operations Research*, *40*(1-supplement-1), S145–S156.
Wagner, H. M., & Whitin, T. M. (1958). Dynamic version of the economic lot size model. *Management science*, *5*(1), 89–96.
Wallin, C., Johnny Rungtusanatham, M., & Rabinovich, E. (2006). What is the right inventory management approach for a purchased item? *International Journal of Operations & Production Management*, *26*(1), 50–68.
Wei, Q., Zhang, J., Zhu, G., Dai, R., & Zhang, S. (2020). Retailer vs. vendor managed inventory with considering stochastic learning effect. *Journal of the Operational Research Society*, *71*(4), 628–646.
Wolsey, L. A. (2002). Solving multi-item lot-sizing problems with an mip solver using classification and reformulation. *Management Science*, *48*(12), 1587–1602.
Yamur, E., & Kesen, S. E. (2021). Multi-trip heterogeneous vehicle routing problem coordinated with production scheduling: Memetic algorithm and simulated annealing approaches. *Computers & Industrial Engineering*, *161*, 107649.
Young, L. (2023). *Chatbots are stepping toward supply chains* [Accessed July 23, 2023]. https://www.wsj.com/articles/chatbots-are-stepping-toward-supply-chains-5661039a
Yu, H., Tang, L., Xu, Y., & Wang, Y. (2015). How much does VMI better than RMI in a global environment? *International Journal of Production Economics*, *170*, 268–274.

Zangwill, W. I. (1969). A backlogging model and a multi-echelon model of a dynamic economic lot size production systema network approach. *Management Science*, *15*(9), 506–527.

Zeddam, B. (2024). *Planification intégrée des services de production et de transport sous des contraintes énergétiques et environnementales* [Doctoral dissertation, Université de Tlemcen].

Zheng, Y.-S., & Zipkin, P. (1990). A queueing model to analyze the value of centralized inventory information. *Operations Research*, *38*(2), 296–307.

Abstract In today's competitive global market, companies face pressures to enhance efficiency, reduce costs, and improve customer satisfaction by optimizing their supply chain processes. Despite advancements, supply chains suffer from systemic inefficiencies like the bullwhip effect, which increases operational costs and decreases service levels. Addressing this issue requires robust coordination mechanisms, notably Vendor Managed Inventory (VMI), where manufacturers manage retailer inventories using shared demand data. Implementing VMI integrates key supply chain functions. This thesis addresses practical and strategic supply chain challenges by focusing on VMI and integrated production-distribution planning. It aims to develop advanced decision-support tools to efficiently integrate production, inventory, and distribution planning, optimizing resource management and operational performance while minimizing overall costs.
Keywords Vendor-Managed Inventory, Integrated Planning, Decision-making, Lot Sizing Problem, Production Routing Problem, Direct Shipment, Metaheuristic, Heuristic
الملخص في ظل المنافسة المتزايدة في السوق العالمية، تواجه الشركات ضغوطًا مستمرة لتحسين الكفاءة وخفض التكاليف وزيادة رضا العملاء من خلال تحسين عمليات سلاسل التوريد الخاصة بها. رغم التقدم المحرز، تعاني سلاسل التوريد من اختلالات نظامية، من أبرزها ظاهرة تأثير السوط (Bullwhip Effect)، التي تؤدي إلى زيادة التكاليف التشغيلية وتراجع مستوى الخدمة. تتطلب معالجة هذه المشكلة آليات تنسيق فعّالة، ومن أهمها نظام إدارة المخزون من قِبل المورد (VMI)، حيث يتولى المصنعون إدارة مخزونات تجار التجزئة اعتمادًا على بيانات الطلب المشتركة. يسهم تطبيق نظام VMI في دمج الوظائف الأساسية لسلسلة التوريد. تتناول هذه الأطروحة التحديات العملية والاستراتيجية في سلاسل التوريد من خلال التركيز على نظام VMI والتخطيط المتكامل للإنتاج والتوزيع. كما تهدف إلى تطوير أدوات متقدمة لدعم القرار لدمج تخطيط الإنتاج والمخزون والتوزيع بكفاءة، وتحسين إدارة الموارد والأداء التشغيلي مع تقليل التكاليف الإجمالية.
الكلمات المفتاحية إدارة المخزون بواسطة المورد، التخطيط المتكامل، صنع القرار، مشكلة تحديد حجم الدفعة، مشكلة توجيه الإنتاج، الشحن المباشر، الطرق الشبه ابتكارية، الطرق الاستدلالية.
Résumé Dans un marché mondial fortement concurrentiel, les entreprises font face à des pressions constantes pour accroître leur efficacité, réduire leurs coûts et améliorer la satisfaction des clients en optimisant les processus de leur chaîne logistique. Malgré les progrès réalisés, les chaînes logistiques continuent de souffrir dinefficacités systémiques telles que l'effet coup de fouet (bullwhip effect), entraînant une augmentation des coûts opérationnels et une diminution des niveaux de service. Pour pallier ce problème, il est nécessaire d'instaurer des mécanismes solides de coordination, notamment la Gestion Partagée des Approvisionnements (Vendor Managed Inventory - VMI), où les fabricants gèrent les stocks des détaillants en utilisant les données partagées de la demande. L'implémentation du VMI permet d'intégrer les fonctions clés de la chaîne logistique. Cette thèse traite des défis pratiques et stratégiques des chaînes logistiques en mettant l'accent sur le VMI et la planification intégrée de la production et de la distribution. Elle vise à développer des outils avancés d'aide à la décision afin d'intégrer efficacement la production, la gestion des stocks et la distribution, en optimisant la gestion des ressources et la performance opérationnelle tout en minimisant les coûts globaux.
Mots-clés Gestion partagée des approvisionnements, planification intégrée, prise de décision, problème de dimensionnement des lots, problème de tournées et production, livraison directe, métaheuristique, heuristique