

الجمهورية الجزائرية الديمقراطية الشعبية

REPUBLIQUE ALGERIENNE DEMOCRATIQUE ET POPULAIRE

وزارة التعليم العالي والبحث العلمي

Ministère de l'Enseignement Supérieur et de la Recherche Scientifique

جامعة أبي بكر بلقايد – تلمسان

Université Aboubakr Belkaïd – Tlemcen –

Faculté de TECHNOLOGIE



THESIS

Submitted for the award of the **MASTER'S Degree**

In: Electrical Engineering

Specialty: Electric Drives

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Subject

**Intelligent Identification of Induction Machine Parameters for the
Synthesis and Experimental Validation of State Observers**

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Dedication 1

I dedicate this work first and foremost to my parents and family, whose unwavering love, sacrifices, prayers, and support have been the foundation of my academic journey. I would also like to dedicate this thesis to some remarkable friends who have been an important part of my life and studies in Algeria.

To Semmoud Fatima Zohra, my friend since our first year. Thank you for your friendship, encouragement, and the countless memories we have shared throughout these years. Being mentioned in your own thesis acknowledgements and being invited to your thesis defense are gestures I will always cherish.

To Kazi Tani Mouhssine, my friend since our second year. We have shared many challenges, experiences, and achievements throughout this journey, and by the grace of Allah, we are graduating together.

To Bencherif Djamaa, despite the language barrier, you have always been willing to help whenever I needed assistance. Your patience, kindness, and readiness to explain things have meant a great deal to me throughout my Masters studies.

A very special dedication goes to my partner and friend, Victor Godwin Ndun. We arrived in Algeria together, experienced both successes and setbacks together, and have walked this academic path side by side. Through every challenge and every milestone, your companionship and support have been invaluable. Alhamdulillah, we are reaching the finish line together.

Finally, I dedicate this work to all my friends, classmates, teachers, and everyone who contributed, directly or indirectly, to the completion of this thesis. May Allah reward you all abundantly for your kindness, support, and encouragement.

Isah Usman Abubakar
Tlemcen, 02 June 2026

Dedication 2

I dedicate this work to my beloved parents, whose endless love, sacrifices, and unwavering belief in me have been the driving force behind my academic journey. Their support and encouragement have shaped who I am today and made this achievement possible.

To my dear friends who have been an essential part of this journey: Imad Dich, who has been a loyal and supportive friend since my very first year, always standing by me through both challenges and successes; Kazi Tani Mousshine, whose generosity in sharing knowledge and assisting me with my academic work greatly contributed to my progress; and Isah Usman Abubakar, with whom I embarked on this journey in a new country, sharing the struggles, experiences, and determination that have now led us to completing this memoir together. Your companionship, support, and shared experiences have made this journey not only possible but truly memorable.

Finally I dedicate this work to my friends, classmates, and teachers whose guidance and support influenced the completion of this work.

Victor Godwin Ndun
Tlemcen, 02 June 2026

Acknowledgements

First and foremost, we would like to express our deepest gratitude to Allah, the Almighty, for granting us the strength, patience, and perseverance necessary to complete this work.

We would like to extend our sincere appreciation to my supervisor, **Mr. Mechernene Abdelkader**, for his valuable guidance, constructive advice, continuous support, and encouragement throughout the realization of this thesis. His expertise and dedication greatly contributed to the completion of this work.

We would also like to express our heartfelt thanks to our assistant supervisor, Mr. Loucif Mourad, for their availability, guidance, insightful remarks, and support throughout this research. Their contributions and encouragement were highly appreciated.

Our sincere thanks are extended to the members of the jury, Mr. BENHABIB Mohamed Choukri, President of the jury, and Mr. MELIANI Sidi Mohammed, Examiner, for accepting to evaluate this thesis and for the time and effort devoted to examining this work. Their valuable comments, constructive suggestions, and insightful observations were greatly appreciated.

We would also like to express our gratitude to the Department of Electrical Engineering and all its academic staff for the quality of education and training provided throughout our studies. We are particularly grateful to the department for granting us the opportunity to prepare and present this thesis in English, which greatly facilitated the communication of our work and allowed us to express our ideas more effectively.

A special word of thanks goes to our fellow students, friends, and colleagues who have shared this journey with us and contributed, directly or indirectly, to our academic and personal growth.

Finally, we would like to express our deepest gratitude to our families for their unconditional love, prayers, sacrifices, encouragement, and unwavering support throughout our academic journey. Their belief in us has always been a source of strength and motivation.

To everyone who contributed in one way or another to the completion of this thesis, we extend our sincere appreciation and gratitude.

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Tlemcen, 02 June 2026*

Abstract

This thesis deals with the modeling, parameter identification, and state observation of a three-phase induction machine. A mathematical model was developed in the two-axis reference frame to describe the machine dynamics. The machine parameters were initially determined using no-load and locked-rotor tests. To improve model accuracy, a Particle Swarm Optimization (PSO) algorithm was applied to refine the stator, rotor, and mutual inductances. The obtained parameters were validated experimentally using a dSPACE platform, and a good agreement was observed between simulated and measured current, flux, and speed responses. In addition, deterministic full-order and high-gain observers were designed to estimate non-measurable states, particularly the rotor flux. The results demonstrate the effectiveness of the proposed modeling, identification, and observation techniques for induction machine analysis and control.

Keywords: *Three-phase induction machine, modeling, parameter identification, PSO, state observer, rotor flux estimation.*

Résumé

Cette thèse porte sur la modélisation, l'identification des paramètres et l'observation d'état d'une machine asynchrone triphasée. Un modèle mathématique a été développé dans le repère biaxial pour décrire la dynamique de la machine. Les paramètres de la machine ont été initialement déterminés à l'aide d'essais à vide et à rotor bloqué. Afin d'améliorer la précision du modèle, un algorithme d'optimisation par essaims de particules (PSO) a été appliqué pour affiner les inductances statorique, rotorique et mutuelle. Les paramètres obtenus ont été validés expérimentalement à l'aide d'une plateforme dSPACE, et une bonne concordance a été observée entre les réponses simulées et mesurées du courant, du flux et de la vitesse. De plus, des observateurs déterministes d'ordre complet et à gain élevé ont été conçus pour estimer les états non mesurables, en particulier le flux rotorique. Les résultats démontrent l'efficacité des techniques de modélisation, d'identification et d'observation proposées pour l'analyse et la commande des machines asynchrones.

Mots-clés : Machine à induction triphasée, modélisation, identification des paramètres, PSO, observateur d'état, estimation du flux rotorique.

ملخص

تتناول هذه الأطروحة نمذجة وتحديد معالم ومراقبة حالة آلة حثية ثلاثية الأطوار. تم تطوير نموذج رياضي في إطار مرجعي ثنائي المحاور لوصف ديناميكيات الآلة. حُددت معالم الآلة مبدئيًا باستخدام اختبارات اللاحمل واختبارات الدوار لتحسين قيم الحث الذاتي للجزء الثابت (PSO) المقفل. ولتحسين دقة النموذج، طُبقت خوارزمية تحسين سرب الجسيمات ، ولوحظ توافق dSPACE والدوار والحث المتبادل. جرى التحقق تجريبيًا من صحة المعالم المُستخلصة باستخدام منصة جيد بين استجابات التيار والتدفق والسرعة المُحاكاة والمقاسة. بالإضافة إلى ذلك، صُممت مُراقبات حتمية كاملة الرتبة وعالية الكسب لتقدير الحالات غير القابلة للقياس، ولا سيما تدفق الدوار. تُظهر النتائج فعالية تقنيات النمذجة والتحديد والمراقبة المقترحة لتحليل آلة الحث والتحكم بها.

، مراقب الحالة، تقدير تدفق الدوار PSO الكلمات المفتاحية: آلة الحث ثلاثية الأطوار، النمذجة، تحديد المعالم،

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Nomenclature

The main notations and abbreviations used in this thesis are explained below, in their most commonly used form in the field of electrical engineering.

Nom	Symbole	Unité
Time	t	s
Phase Voltage	U	V
Line Voltage	U	V
Current	I	A
Power	P	W
Rotor Flux	ϕ	Wb
Stator Flux	ϕ	Wb
Power Factor	$\cos \varphi$	/
Frequence	f	Hz
rotor electrical angular speed	ω	rad/s
The slip angular speed	ω	rad/s
Electrical angular speed of the reference frame	Ω	rad/s
Instantaneous position of the rotor magnetic axis	θ	/
slip	s	%
number of pair of poles	p	/
Synchronous speed	n_g	<i>rpm</i>
Rotor speed	n_r	<i>rpm</i>
Mechanical rotor speed	Ω	<i>rpm</i>

Parameters

Name	Symbol	Unite
Impedance	Z	Ω
Reactance	X	Ω
Resistance	R	Ω
Inductance	L	H
Moment of Inertia	J	kg.m ²
Friction Coefficient	B	N.s/rad
Rotor Inductance	L_r	H
Stator Inductance	L_s	H
Magnetizing Inductance	L_m	H
Stator Resistance	R_s	Ω
Rotor Resistance	R_r	Ω
No load active power	P_0	W
No load reactive power	Q_0	VAR
Stator copper loss	P_{js0}	W
Short-Circuit active power	P_{cc}	W
Short-Circuit reactive power	Q_{cc}	VAR
Core loss	P_{fer}	W
Mechanical loss	P_{mec}	W
Rotor time constant	τ_r	s
Clarke reference frame	$\alpha\beta$	/
Park reference frame	dq	/
Continous time state-space model	ABC	/
Electromagnetic torque	T_e	Nm
Load torque	T_L	Nm
Viscous Friction	βv	Ns/m ²
Observation error	e	/
Observation gain	G	/
Observability matrix	ϑ	/
Cognitive Learning Factor	C_1	/
Social Learning Factor	C_2	/

Glossaries

Word	Signification
Asynchronous Machine (Induction Motor)	An electrical machine where the rotor speed is different from the synchronous speed of the stator magnetic field.
ABC Refer- ence Frame	A three-phase stationary coordinate system used to describe electrical quantities in their natural form.
Slip	Difference between synchronous speed and rotor speed.
D-q Model	Simplified model using two axes for analysis.
PSO	Particle Swarm Optimization
State-Space Model	Mathematical model using equations to describe the system.
Observer	A mathematical algorithm used to estimate internal states of a system that cannot be directly measured.

General Introduction

Electric motors play a fundamental role in modern industrial systems due to their reliability, robustness, and high efficiency. Among the various types of electrical machines, the three-phase induction machine is one of the most widely used because of its simple construction, low maintenance requirements, and ability to operate under different loading conditions. These characteristics have made it indispensable in industrial drives, transportation systems, renewable energy applications, and numerous automation processes.

The increasing demand for high-performance electric drives has highlighted the importance of accurate modeling, parameter identification, and state estimation techniques. The performance of advanced control strategies depends strongly on the precision of the machine model and its parameters. However, the electrical and mechanical parameters of induction machines may vary with operating conditions, temperature changes, magnetic saturation, and aging effects, leading to discrepancies between theoretical models and real systems.

To overcome these limitations, parameter identification methods are employed to determine machine parameters accurately from experimental measurements. Conventional identification techniques are often sensitive to measurement noise and may become computationally demanding when dealing with nonlinear systems. In this context, metaheuristic optimization methods such as Particle Swarm Optimization (PSO) have emerged as powerful tools capable of efficiently estimating machine parameters while minimizing modeling errors.

Furthermore, modern control systems increasingly rely on state observers to reconstruct variables that are difficult or impossible to measure directly. In induction machines, quantities such as rotor fluxes are essential for high-performance control but cannot be measured easily in practice. State observers provide an effective solution by estimating these variables from measurable inputs and outputs, thereby reducing sensor requirements and improving system reliability.

The objective of this thesis is to develop and validate a complete approach for the modeling, identification, and observation of a three-phase induction machine. First, a

mathematical model of the machine is established in the stationary reference frame. Then, the machine parameters are identified using experimental tests and optimization techniques based on Particle Swarm Optimization. Finally, deterministic and high-gain observer structures are implemented and evaluated through simulation in order to estimate the rotor fluxes and assess their convergence properties. The obtained simulation and experimental results demonstrate the effectiveness of the proposed methodology and confirm the relevance of combining accurate identification techniques with advanced state observation methods for improving the performance and reliability of induction motor drive.

Chapter I

Dynamic Modeling of the Induction Machine

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I.1 Introduction

Electrical machines are physical devices designed for the conversion of electrical energy into mechanical energy (motors) or vice versa (generators). These machines are found everywhere in modern life, providing the power for everything from home appliances and office equipment to complex industrial manufacturing processes and transportation systems [7].

Within the broad category of electric machines, the induction machine(induction machine) has established itself as the principal workhorse of modern industry. Its widespread adoption is due to several significant advantages: it possesses a simple and robust structure, requires virtually no maintenance, and offers low investment and running costs. A distinguishing feature of the asynchronous machine is that it requires no separate DC field current to operate. Instead, it functions as a rotating transformer, where rotor voltages and currents are induced by transformer action from the stator windings [7].

Historically, induction machines were primarily used in uncontrolled, constant-speed applications because they were more difficult to control efficiently than direct current (DC) machines. However, the introduction of vector control (field orientation) in the early 1970s revolutionized the field, proving that independent, decoupled control of flux and torque could be achieved in induction machines, matching the high-performance capabilities of DC drives [7].

The implementation of such advanced control strategies necessitates the use of dynamic mathematical models rather than traditional steady-state phasor models. These dynamic models must faithfully describe the machine's electromagnetic and mechanical behavior under arbitrary and often non-sinusoidal supply waveforms provided by modern power electronic inverters [7].

The objective of this chapter is to establish the mathematical foundation for the dynamic modeling of the induction machine. It begins by deriving the machine's fundamental equations in the three-phase domain. Due to the complexity of time-varying coefficients in this domain, a transformation procedure is employed to develop a Constant Parameter Model (CPM) in a two-phase (d-q) common reference frame. Finally, the chapter presents dynamic state-space representations and equivalent circuits, which serve as the essential tools for performance simulation and the design of high-performance vector controllers [7].

I.2 Overview of the Induction Machine

An induction machine, is an electrical device where the rotor voltage and current are induced in the rotor windings by transformer action from the stator. The defining feature of this machine is that no separate DC field current is required for operation [3].

The constitution of an induction machine consists of two primary electromagnetic parts separated by a uniform air gap: a stationary outer part called the stator and an inner rotating part called the rotor. Both components are built from three basic elements: the

core, the windings, and the insulation system [7], [3].

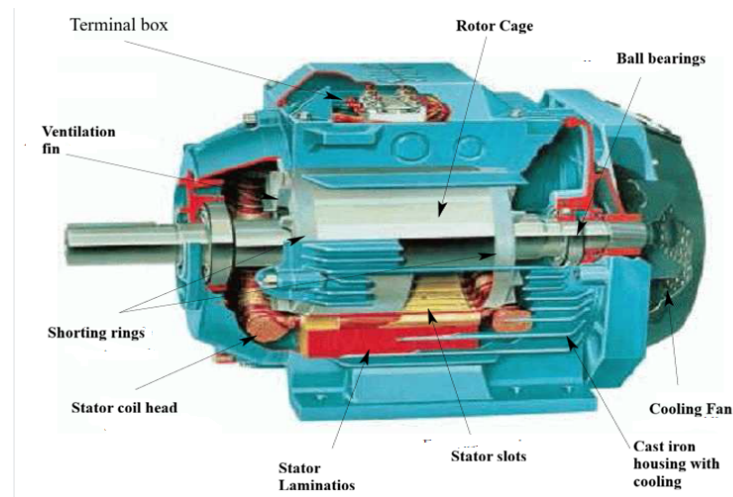


Figure I.1: Three-phase Induction Motor

- **Stator:** The stator construction is essentially identical to that of a synchronous machine. It consists of a stationary frame that provides physical support for a stack of ring-shaped, laminated iron cores. These cores have axial slots on their inner surface to house the stator windings, which are typically three-phase windings distributed to produce a uniform rotating magnetic field when energized [7], [3].

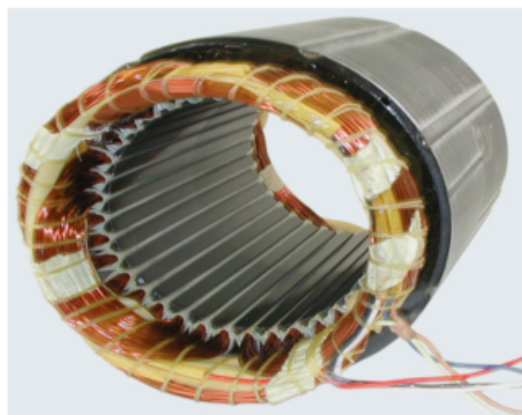


Figure I.2: Stator

- **Rotor:** The rotor is mounted on a steel shaft and serves as the field circuit of the machine. There are two primary types of rotor construction:
 - **Squirrel-Cage Rotor:** This design consists of a series of conducting bars laid into slots and shorted at both ends by large shorting rings. Because it has no brushes or slip rings, it is highly reliable and requires minimal maintenance [3].

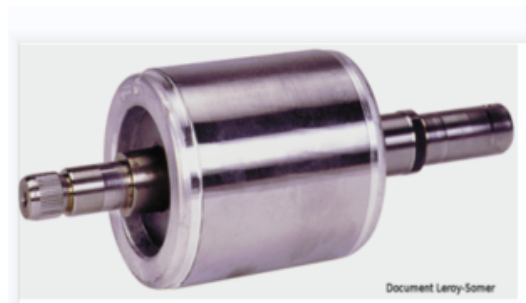


Figure I.3: Squirrel-Cage Rotor

- Wound Rotor: This type features a complete set of three-phase windings that mirror the stator windings. The ends of these rotor windings are connected to slip rings on the shaft, allowing external resistance to be inserted into the rotor circuit to modify the machine's torque-speed characteristics [3].

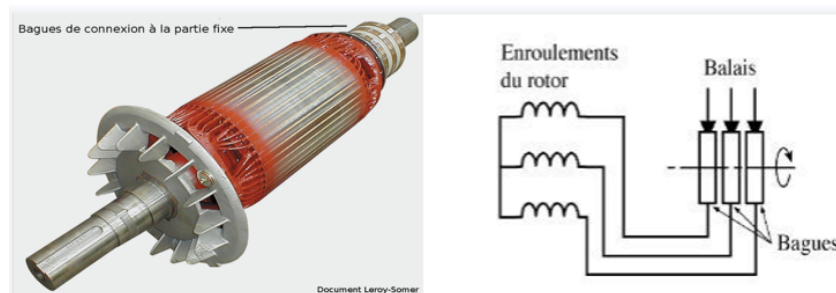


Figure I.4: Wound Rotor

Industrial Significance and Advantages

The induction machine is considered the principal workhorse of modern industry. Its widespread adoption is due to several key advantages:

- Simple and robust structure with high reliability,
- Low maintenance requirements and low cost,
- Capability to be built for an extremely wide range of power and speeds.

It is ubiquitous in industrial automation and variable-speed drives, commonly powering conveyors, robot arms, pumps, fans, and compressors. In high-performance applications, advanced techniques like vector control are used to achieve dynamic responses similar to those of DC machines.

I.3 Operating of the Induction Machine

The operating principle of an induction machine is based on the interaction of electrical and magnetic phenomena to convert energy. This process begins with the creation of a

rotating magnetic field and relies on the laws of electromagnetic induction to generate mechanical motion.

When a balanced three-phase set of currents each equal in magnitude and having a phase difference of 120° flows through a three-phase stator winding, it produces a rotating magnetic field of constant magnitude. This field is represented as a set of magnetic poles (North and South) that rotate around the inner surface of the stator [3].

The speed at which this magnetic field rotates is known as the synchronous speed n_g , which is directly proportional to the supply frequency (f) and inversely proportional to the number of pairs poles (p) in the machine [3]. The relationship is defined as:

$$n_g = \frac{60f}{p} \quad (\text{I.1})$$

As the stator magnetic field rotates, it "sweeps" across the stationary or slower-moving rotor conductors. According to Faraday's Law, a time-changing magnetic field passing through a coil induces a voltage within that coil. In the context of the induction machine, this induced voltage (E_{ind}) in the rotor bars is a result of the relative motion between the rotor and the rotating magnetic field. This physical action is why the machine is often called a rotating transformer, where the stator acts as the primary and the rotor as the short-circuited secondary [7] [3].

The difference between the synchronous speed (n_g) and the rotor speed (n_r) is used to define the state of the machine [3]. We use this to measure slip which is the relative speed expressed in percentage:

$$Slip(s) = \frac{n_g - n_r}{n_g} \quad (\text{I.2})$$

At standstill (locked rotor), the slip is 1, while at synchronous speed, it is 0. Normal motor operation occurs at a very low slip, typically where the rotor speed is near the synchronous speed but never exactly equal to it [3].

If the rotor were to reach synchronous speed ($n_g = n_r$), the relative motion would become zero. Consequently, there would be no change in flux linkage, no induced voltage, no rotor current, and no torque. Therefore, in motor mode, the rotor always turns at a slower speed than the stator poles [3].

I.4 Steady-State Modeling of the Induction Motor

Steady-state modeling describes the performance of the induction machine when it is operating at a constant speed and supplied by balanced, sinusoidal three-phase voltages. While dynamic models are necessary for control design and transient analysis, static models are essential for general performance evaluation, power flow analysis, and initial parameter identification [10].

The Per-Phase Equivalent Circuit

The most common way to represent the induction machine in steady state is through a per-phase equivalent circuit, which is derived by viewing the machine as a rotating transformer [3]. Because the machine is excited (meaning power is supplied only to the stator), the model consists of the following components referred to a single phase:

- **Stator Circuit:** Represented by the stator resistance (R_1) and the stator leakage reactance (X_1). These account for the ohmic losses in the stator windings and the flux that does not link with the rotor [3].
- **Magnetizing Branch:** Unlike an ideal transformer, the induction machine has an air gap that requires a significant magnetizing current to establish the magnetic flux. This is modeled by a magnetizing reactance (X_m) in parallel with a resistance (R_c) that represents core losses (hysteresis and eddy currents) [3].
- **Rotor Circuit:** The rotor is represented by its resistance (R_r) and leakage reactance (X_r), both referred to the stator side. A critical feature of the static model is that the mechanical load is represented as a variable resistance dependent on slip (g), typically expressed as $\frac{R_r}{g}$ [3].

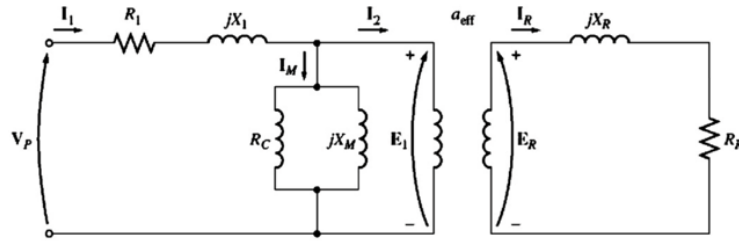


Figure I.5: Transformer model [3].

Steady-State Mathematical Equations

In steady state, all time derivatives of flux linkages and currents in the dynamic equations are set to zero. The machine's behavior is then described using phasor variables or space vectors rotating at a constant angular frequency (ω_g) [10].

The relationship between the stator voltage (V_s), current (i_s), and flux linkage (ψ_s) is simplified to [4]:

$$V_s = R_s i_s + j\omega_g \psi_s \quad (\text{I.3})$$

Torque-Speed Characteristics

The static model is used to derive the torque-speed characteristic curve, which typically shows three regions of operation [3]:

- **Low-Slip Region:** Normal operating range where torque is roughly proportional to slip.

- **Moderate-Slip Region:** The region approaching pullout torque (maximum torque), where rotor reactance becomes significant.
- **High-Slip Region:** The region near standstill (locked rotor), where torque actually decreases with increased slip.

The torque-speed characteristic curve of an induction motor provides critical information regarding its operational limits, stability, and design trade-offs. These are the observations regarding this curve:

- **Zero Torque at Synchronous Speed:** The induced torque of the motor is exactly zero at synchronous speed (n_g). This occurs because there is no relative motion between the magnetic field and the rotor, resulting in no induced voltage or current.
- **Linear Operating Region:** The curve is nearly linear between no-load and full-load conditions. In this low-slip range, the rotor resistance is much larger than the rotor reactance, causing the rotor current and induced torque to increase linearly with slip.
- **Pullout (Breakdown) Torque:** There is a maximum possible torque that the motor can supply, known as pullout or breakdown torque. This typically occurs at a point where the increase in rotor current is exactly balanced by the decrease in the rotor power factor. For most motors, this peak is 2 to 3 times the rated full-load torque.
- **Starting Torque:** For a typical induction motor, the starting torque (torque at zero speed) is slightly larger than its full-load torque, allowing it to start under load.

I.5 Dynamic Modeling of the Asynchronous Machine

In order to analyse the dynamic behaviour of the induction machine, a mathematical model is required. For this purpose, several simplifying assumptions are introduced. The stator phase windings are considered identical and uniformly distributed in space, with a displacement of 120 electrical degrees between phases. The magnetomotive force created by each winding is assumed to vary sinusoidally along the air-gap periphery. Parasitic capacitances are neglected and the air gap is regarded as uniform, which allows the influence of stator and rotor slotting to be ignored. The squirrel-cage rotor is replaced by an equivalent balanced three-phase winding [3].

In addition, the electrical parameters of the machine, namely the stator and rotor resistances and their leakage inductances, are assumed constant. Core losses are neglected and the magnetisation characteristic is taken to be linear; therefore, magnetic saturation effects are not considered.

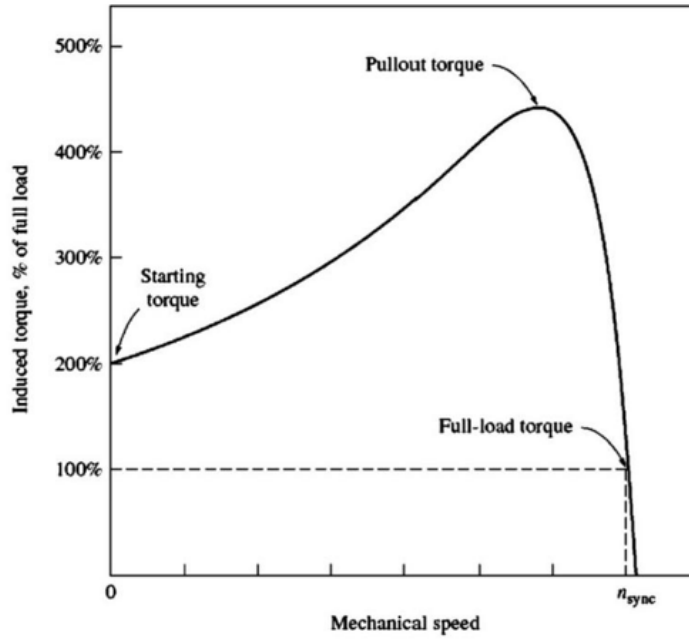


Figure I.6: Induction motor torque-speed characteristic curve [3].

Based on these assumptions, the induction machine can be described in the three-phase (abc) reference frame using instantaneous quantities, leading to the set of equations presented below.

$$V_{sABC} = R_s i_{sABC} + \frac{d\psi_{sABC}}{dt} \quad (\text{I.4})$$

$$V_{rABC} = 0 = R_r i_{rABC} + \frac{d\psi_{rABC}}{dt} \quad (\text{I.5})$$

$$\psi_{sABC} = L_s i_{sABC} + L_m i_{rABC} \quad (\text{I.6})$$

$$\psi_{rABC} = L_r i_{rABC} + L_m i_{sABC} \quad (\text{I.7})$$

L_s and L_r are the stator and rotor self-inductances and L_m is the mutual inductance.

I.5.1 Space-vector representation of the induction machine.

$$V_{sABC} = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}; i_{sABC} = \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}; \psi_{sABC} = \begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \end{bmatrix} \quad (\text{I.7})$$

$$V_{rABC} = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}; i_{rABC} = \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}; \psi_{rABC} = \begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \end{bmatrix} \quad (\text{I.8})$$

$$i = \begin{bmatrix} i_{sABC} \\ i_{rABC} \end{bmatrix}; L = \begin{bmatrix} L_s & L_{sr}(\theta) \\ L_{rs}(\theta) & L_r \end{bmatrix} \quad (\text{I.9})$$

$$L_s = \begin{bmatrix} L_{AA} & L_{AB} & L_{AC} \\ L_{BA} & L_{BB} & L_{BC} \\ L_{CA} & L_{CB} & L_{CC} \end{bmatrix}; L_s = L_s \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} \quad (\text{I.10})$$

$$L_r = \begin{bmatrix} L_{AA} & L_{AB} & L_{AC} \\ L_{BA} & L_{BB} & L_{BC} \\ L_{CA} & L_{CB} & L_{CC} \end{bmatrix}; L_r = L_r \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} \quad (\text{I.11})$$

$$L_{rs}(\theta) = L_m(\theta) = \begin{bmatrix} \cos(\theta) & \cos(\theta + \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) \\ \cos(\theta - \frac{2\pi}{3}) & \cos(\theta) & \cos(\theta + \frac{2\pi}{3}) \\ \cos(\theta + \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta) \end{bmatrix} \quad (\text{I.12})$$

The variable θ defines the instantaneous position of the rotor magnetic axis relative to the stationary magnetic axis of the stator. It is obtained by integrating the rotor electrical angular velocity over time:

$$\theta(t) = \theta(0) + \int_0^t \omega(\tau) d\tau \quad (\text{I.13})$$

I.5.2 Simplifying Assumptions

To establish the mathematical model of the induction machine, the following idealized assumptions are adopted:

- The stator windings are identical and evenly distributed in space with an electrical offset of (120°),
- The magnetomotive forces produced by the windings are assumed to be sinusoidal along the air gap,
- The air gap is uniform and constant; the effect of stator and rotor slots is neglected,
- The parasitic capacitances of the windings are neglected,
- The rotor cage is considered a balanced three-phase winding,
- The leakage resistances and inductances of the windings are considered constant,
- Iron losses (hysteresis and eddy currents) are neglected,
- The magnetization characteristic is assumed to be linear, meaning that magnetic saturation is neglected.

Under these assumptions, the asynchronous machine can be described by a linear model in the instantaneous three-phase domain [7].

I.5.3 Electrical and magnetic equations of the unsaturated induction motor

The model is expressed in the synchronous rotating reference frame (d-q) rotating at angular speed (ω_g).

(ω_g): electrical angular speed of the reference frame (synchronous speed)

(ω_r): rotor electrical angular speed

The slip angular speed is therefore:

$$\omega_{sl} = \omega_g - \omega_r \quad (\text{I.14})$$

Electrical equations

$$V_{ds} = R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_g \psi_{qs} \quad (\text{I.15})$$

$$V_{qs} = R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_g \psi_{ds} \quad (\text{I.16})$$

$$V_{dr} = 0 = R_r i_{dr} + \frac{d\psi_{dr}}{dt} - (\omega_g - \omega_r) \psi_{qr} \quad (\text{I.17})$$

$$V_{qr} = 0 = R_r i_{qr} + \frac{d\psi_{qr}}{dt} + (\omega_g - \omega_r) \psi_{dr} \quad (\text{I.18})$$

Magnetic equations

$$\psi_{ds} = L_s i_{ds} + L_m i_{dr} \quad (\text{I.19})$$

$$\psi_{qs} = L_s i_{qs} + L_m i_{qr} \quad (\text{I.20})$$

$$\psi_{dr} = L_r i_{dr} + L_m i_{ds} \quad (\text{I.21})$$

$$\psi_{qr} = L_r i_{qr} + L_m i_{qs} \quad (\text{I.22})$$

I.5.3-a Electrical equations in the ABC reference frame

In the natural three-phase stationary reference frame (ABC), the stator and rotor electrical equations of the induction machine expressed with instantaneous quantities are: **Stator voltages**

$$V_{sA} = R_s i_{sA} + \frac{d\psi_{sA}}{dt} \quad (\text{I.23})$$

$$V_{sB} = R_s i_{sB} + \frac{d\psi_{sB}}{dt} \quad (\text{I.24})$$

$$V_{sC} = R_s i_{sC} + \frac{d\psi_{sC}}{dt} \quad (\text{I.25})$$

Rotor voltages

$$V_{rA} = 0 = R_r i_{rA} + \frac{d\psi_{rA}}{dt} \quad (\text{I.26})$$

$$V_{rB} = 0 = R_r i_{rB} + \frac{d\psi_{rB}}{dt} \quad (\text{I.27})$$

$$V_{rC} = 0 = R_r i_{rC} + \frac{d\psi_{rC}}{dt} \quad (\text{I.28})$$

I.5.3-b Magnetic equations in the ABC reference frame

Stator fluxes

$$\psi_{sA} = L_s i_{sA} + L_m i_{rA} \quad (\text{I.29})$$

$$\psi_{sB} = L_s i_{sB} + L_m i_{rB} \quad (\text{I.30})$$

$$\psi_{sC} = L_s i_{sC} + L_m i_{rC} \quad (\text{I.31})$$

Rotor fluxes

$$\psi_{rA} = L_r i_{rA} + L_m i_{sA} \quad (\text{I.32})$$

$$\psi_{rB} = L_r i_{rB} + L_m i_{sB} \quad (\text{I.33})$$

$$\psi_{rC} = L_r i_{rC} + L_m i_{sC} \quad (\text{I.34})$$

I.5.4 Park Transformation and d-q Model Axis Equations of the Induction Machine

In order to simplify the mathematical model of the induction motor, the three-phase variables expressed in the abc reference frame are transformed into a two-axis rotating reference frame (d-q) using the Park transformation. The d-q frame rotates at the synchronous electrical angular speed ω_g . The Park transformation is defined by:

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \quad (\text{I.35})$$

Where, $\theta(t) = \int \omega_g(t) dt$

In the synchronous rotating reference frame, the electrical equations of the unsaturated induction motor are:

$$V_{ds} = R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_g \psi_{qs} \quad (\text{I.36})$$

$$V_{qs} = R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_g \psi_{ds} \quad (\text{I.37})$$

$$V_{dr} = 0 = R_r i_{dr} + \frac{d\psi_{dr}}{dt} - (\omega_g - \omega_r) \psi_{qr} \quad (\text{I.38})$$

$$V_{qr} = 0 = R_r i_{qr} + \frac{d\psi_{qr}}{dt} + (\omega_g - \omega_r) \psi_{dr} \quad (\text{I.39})$$

Where ω_r is the rotor electrical speed.

The flux-current relations in the d-q reference frame are:

$$\psi_{ds} = L_s i_{ds} + L_m i_{dr} \quad (\text{I.40})$$

$$\psi_{qs} = L_s i_{qs} + L_m i_{qr} \quad (\text{I.41})$$

$$\psi_{dr} = L_r i_{dr} + L_m i_{ds} \quad (\text{I.42})$$

$$\psi_{qr} = L_r i_{qr} + L_m i_{qs} \quad (\text{I.43})$$

I.5.5 Expression of Electromagnetic Power and Torque

The electromechanical energy conversion in an induction machine is characterized by the electromagnetic torque developed by the interaction between the stator currents and the rotor magnetic flux.

In the synchronous rotating reference frame d-q, the electromagnetic torque is expressed by:

$$T_e = \frac{3}{2}p(\psi_{dr}i_{qs} - \psi_{qr}i_{ds}) \quad (\text{I.44})$$

where p is the number of pole pairs, i_{ds} and i_{qs} are the stator current components, and ψ_{dr} , ψ_{qr} are the rotor flux components. When the reference frame is aligned with the rotor flux (field-oriented control), the quadrature rotor flux component becomes zero ($\psi_{qr} = 0$). The torque expression then simplifies to:

$$T_e = \frac{3}{2}p(\psi_{dr}i_{qs}) \quad (\text{I.45})$$

Electromagnetic power transferred across the air gap is expressed as:

$$P_e = T_e\omega_g \quad (\text{I.46})$$

where ω_g is the synchronous electrical angular speed.

I.5.6 Mechanical Equations

The mechanical behavior of the induction motor is governed by the rotational dynamics equation. The electromagnetic torque developed by the machine accelerates the rotor and balances the load torque and mechanical losses.

The mechanical equation of motion is:

$$J\frac{d\omega_m}{dt} = T_e - T_L - B\omega_m \quad (\text{I.47})$$

Where, $\omega_r = p\omega_m$

J : total moment of inertia of the rotor and load

T_e : electromagnetic torque

T_L : load torque

B : viscous friction coefficient

I.5.7 Choice of Reference Frames

The original model of the asynchronous machine in the natural three-phase (abc) coordinate system is characterized by differential equations with time-varying coefficients. This complexity arises from the mutual inductances between the stator and rotor, which are periodic functions of the rotor angular position θ .

To simplify the control and analysis of the machine, we apply Park's Transformation. This mathematical tool projects the three-phase variables onto a two-dimensional orthogonal system (d-q). The primary advantage of this transformation is that it can eliminate the position-dependency of the inductances, provided we choose an appropriate rotational speed ω_g for the d-q frame.

The selection of the reference frame speed is not arbitrary; it is a strategic choice based on the specific objectives of the modeling.

I.5.7-a The Stator-Fixed Reference Frame

In this frame, the axes are fixed to the stator ($\omega_g = 0$), and the machine variables are expressed in the stationary $\alpha - \beta$ coordinates.

Voltage Equations

$$V_{\alpha s} = R_s i_{\alpha s} + \frac{d\psi_{\alpha s}}{dt} \quad (\text{I.48})$$

$$V_{\beta s} = R_s i_{\beta s} + \frac{d\psi_{\beta s}}{dt} \quad (\text{I.49})$$

$$V_{\alpha r} = 0 = R_r i_{\alpha r} + \frac{d\psi_{\alpha r}}{dt} + \omega_r \psi_{\beta r} \quad (\text{I.50})$$

$$V_{\beta r} = 0 = R_r i_{\beta r} + \frac{d\psi_{\beta r}}{dt} - \omega_r \psi_{\alpha r} \quad (\text{I.51})$$

I.5.7-b Rotor Reference Frame

Here, $\omega_g = \omega_r$. The observer sits on the rotor bar. The d-q axes rotate at the mechanical speed ω_r .

Voltage Equations

$$V_{ds} = R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_r \psi_{qs} \quad (\text{I.52})$$

$$V_{qs} = R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_r \psi_{ds} \quad (\text{I.53})$$

$$V_{dr} = 0 = R_r i_{dr} + \frac{d\psi_{dr}}{dt} \quad (\text{I.54})$$

$$V_{qr} = 0 = R_r i_{qr} + \frac{d\psi_{qr}}{dt} \quad (\text{I.55})$$

I.5.7-c Synchronous Reference Frame

The axes rotate at the synchronous electrical speed of the stator magnetic field: $\omega_g = \omega_r$. The difference between the synchronous speed and the rotor speed defines the slip speed ($\omega_{sl} = \omega_g - \omega_r$).

Voltage Equations

$$V_{ds} = R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_g \psi_{qs} \quad (\text{I.56})$$

$$V_{qs} = R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_g \psi_{ds} \quad (\text{I.57})$$

$$V_{dr} = 0 = R_r i_{dr} + \frac{d\psi_{dr}}{dt} - (\omega_g - \omega_r) \psi_{qr} \quad (\text{I.58})$$

$$V_{qr} = 0 = R_r i_{qr} + \frac{d\psi_{qr}}{dt} + (\omega_g - \omega_r) \psi_{dr} \quad (\text{I.59})$$

The flux-current equations remain the same regardless of the reference frame

$$\psi_{ds} = L_s i_{ds} + L_m i_{dr} \quad (\text{I.60})$$

$$\psi_{qs} = L_s i_{qs} + L_m i_{qr} \quad (\text{I.61})$$

$$\psi_{dr} = L_r i_{dr} + L_m i_{ds} \quad (\text{I.62})$$

$$\psi_{qr} = L_r i_{qr} + L_m i_{qs} \quad (\text{I.63})$$

I.5.8 Modeling of the voltage-fed induction machine in the rotor reference frame

In this case, the d-q reference frame is chosen to rotate with the rotor. The angular speed of the reference frame is therefore equal to the rotor electrical speed:

$$\omega_g = \omega_r \quad (\text{I.64})$$

By substituting this condition into the general electrical equations of the induction machine in the synchronous (d-q) reference frame, the rotor coupling terms $(\omega_g - \omega_r)$ become zero. The rotor voltage equations are then simplified to:

$$V_{dr} = 0 = R_r i_{dr} + \frac{d\psi_{dr}}{dt} \quad (\text{I.65})$$

$$V_{qr} = 0 = R_r i_{qr} + \frac{d\psi_{qr}}{dt} \quad (\text{I.66})$$

The stator equations remain:

$$V_{ds} = R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_r \psi_{qs} \quad (\text{I.67})$$

$$V_{qs} = R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_r \psi_{ds} \quad (\text{I.68})$$

I.5.9 Modeling of the Voltage-Fed Induction Machine in the Stationary Stator Reference Frame

In the stationary stator reference frame, the axes are fixed to the stator windings. The angular speed of the reference frame is therefore zero:

$$\omega_g = 0 \quad (\text{I.69})$$

Substituting this condition into the general equations gives the following electrical model:

$$V_{\alpha s} = R_s i_{\alpha s} + \frac{d\psi_{\alpha s}}{dt} \quad (\text{I.70})$$

$$V_{\beta s} = R_s i_{\beta s} + \frac{d\psi_{\beta s}}{dt} \quad (\text{I.71})$$

$$V_{\alpha r} = 0 = R_r i_{\alpha r} + \frac{d\psi_{\alpha r}}{dt} + \omega_r \psi_{\beta r} \quad (\text{I.72})$$

$$V_{\beta r} = 0 = R_r i_{\beta r} + \frac{d\psi_{\beta r}}{dt} - \omega_r \psi_{\alpha r} \quad (\text{I.73})$$

I.6 Different models and state-space representations of the induction machine

I.6.1 Representation I

The first representation is based on stator currents and rotor fluxes as state variables. This formulation is widely used in the analysis and control of induction machines because stator currents are directly measurable quantities, while rotor fluxes contain essential information about the electromagnetic state of the machine. The model provides a good compromise between physical interpretation and suitability for control applications.

I.6.1-a Model and representation of order 4 in the d-q and $\alpha - \beta$ frames of reference:

$$V_{ds} = R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_g \psi_{qs} \quad (\text{I.74})$$

$$V_{qs} = R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_g \psi_{ds} \quad (\text{I.75})$$

$$V_{dr} = 0 = R_r i_{dr} + \frac{d\psi_{dr}}{dt} - (\omega_g - \omega_r) \psi_{qr} \quad (\text{I.76})$$

$$V_{qr} = 0 = R_r i_{qr} + \frac{d\psi_{qr}}{dt} + (\omega_g - \omega_r) \psi_{dr} \quad (\text{I.77})$$

$$\dot{\psi}_{ds} = L_s \dot{i}_{ds} + L_m \dot{i}_{dr} \quad (\text{I.78})$$

$$\dot{\psi}_{qs} = L_s \dot{i}_{qs} + L_m \dot{i}_{qr} \quad (\text{I.79})$$

$$\dot{\psi}_{dr} = L_r \dot{i}_{dr} + L_m \dot{i}_{ds} \quad (\text{I.80})$$

$$\dot{\psi}_{qr} = L_r \dot{i}_{qr} + L_m \dot{i}_{qs} \quad (\text{I.81})$$

From (I.80)

$$i_{dr} = \frac{\psi_{dr}}{L_r} - \frac{L_m i_{ds}}{L_r} \quad (\text{I.82})$$

From (I.81)

$$i_{qr} = \frac{\psi_{qr}}{L_r} - \frac{L_m i_{qs}}{L_r} \quad (\text{I.83})$$

Substitute equations (I.82) and (I.83) in equations (I.78) and (I.79) respectively:

$$\psi_{ds} = L_s i_{ds} + L_m \left(\frac{\psi_{dr} - L_m i_{ds}}{L_r} \right) \quad (\text{I.84})$$

$$\psi_{ds} = \left(L_s - \frac{L_m^2}{L_r} \right) i_{ds} + \frac{L_m}{L_r} \psi_{dr} \quad (\text{I.85})$$

Taking $\sigma L_s L_r = L_s L_r - L_m^2$:

$$\sigma L_s = L_s - \frac{L_m^2}{L_r} \quad (\text{I.86})$$

$$\psi_{ds} = \sigma L_s i_{ds} + \frac{L_m}{L_r} \psi_{dr} \quad (\text{I.87})$$

$$\psi_{qs} = L_s i_{qs} + L_m \left(\frac{\psi_{qr} - L_m i_{qs}}{L_r} \right) \quad (\text{I.88})$$

$$\psi_{qs} = \left(L_s - \frac{L_m^2}{L_r} \right) i_{qs} + \frac{L_m}{L_r} \psi_{qr} \quad (\text{I.89})$$

$$\psi_{qs} = \sigma L_s i_{qs} + \frac{L_m}{L_r} \psi_{qr} \quad (\text{I.90})$$

Differentiating (I.87) and (I.90):

$$\dot{\psi}_{ds} = \sigma L_s \dot{i}_{ds} + \frac{L_m}{L_r} \dot{\psi}_{dr} \quad (\text{I.91})$$

$$\dot{\psi}_{qs} = \sigma L_s \dot{i}_{qs} + \frac{L_m}{L_r} \dot{\psi}_{qr} \quad (\text{I.92})$$

Substitute into equations (I.74) and (I.75) respectively:

$$V_{ds} = R_s i_{ds} + \sigma L_s \dot{i}_{ds} + \frac{L_m}{L_r} \dot{\psi}_{dr} - \omega_g \left(\sigma L_s i_{qs} + \frac{L_m}{L_r} \psi_{qr} \right)$$

$$\dot{i}_{ds} = \frac{1}{\sigma L_s} V_{ds} - \frac{R_s}{\sigma L_s} i_{ds} + \omega_g i_{qs} - \frac{L_m}{\sigma L_s L_r} \dot{\psi}_{dr} + \frac{\omega_g L_m}{\sigma L_s L_r} \psi_{qr} \quad (\text{I.93})$$

$$V_{qs} = R_s i_{qs} + \sigma L_s \dot{i}_{qs} + \frac{L_m}{L_r} \dot{\psi}_{qr} + \omega_g \left(\sigma L_s i_{ds} + \frac{L_m}{L_r} \psi_{dr} \right)$$

$$\dot{i}_{qs} = \frac{1}{\sigma L_s} V_{qs} - \frac{R_s}{\sigma L_s} i_{qs} - \omega_g i_{ds} - \frac{L_m}{\sigma L_s L_r} \dot{\psi}_{qr} - \frac{\omega_g L_m}{\sigma L_s L_r} \psi_{dr} \quad (\text{I.94})$$

From equation (I.76),

$$\dot{\psi}_{dr} = -R_r i_{dr} + (\omega_g - \omega_r) \psi_{qr} \quad (\text{I.95})$$

Substituting the value of i_{dr} from equation (I.82):

$$\dot{\psi}_{dr} = -\frac{R_r}{L_r} \psi_{dr} + \frac{R_r L_m}{L_r} i_{ds} + (\omega_g - \omega_r) \psi_{qr} \quad (\text{I.96})$$

From equation (I.77),

$$\dot{\psi}_{qr} = -R_r i_{qr} - (\omega_g - \omega_r) \psi_{dr} \quad (\text{I.97})$$

Substituting the value of i_{qr} from equation (I.83):

$$\dot{\psi}_{qr} = -\frac{R_r}{L_r} \psi_{qr} + \frac{R_r L_m}{L_r} i_{qs} - (\omega_g - \omega_r) \psi_{dr} \quad (\text{I.98})$$

Substitute the values $\dot{\psi}_{dr}$ and $\dot{\psi}_{qr}$ of into equations (I.93) and (I.94):

$$\dot{i}_{ds} = -\left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2}\right) i_{ds} + \omega_g i_{qs} + \frac{R_r L_m}{\sigma L_s L_r^2} \psi_{dr} + \frac{L_m \omega_r}{\sigma L_s L_r} \psi_{qr} + \frac{1}{\sigma L_s} V_{ds} \quad (\text{I.99})$$

$$\dot{i}_{qs} = -\left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2}\right) i_{qs} - \omega_g i_{ds} + \frac{R_r L_m}{\sigma L_s L_r^2} \psi_{qr} - \frac{L_m \omega_r}{\sigma L_s L_r} \psi_{dr} + \frac{1}{\sigma L_s} V_{qs} \quad (\text{I.100})$$

$$\dot{x} = Ax + Bu \quad (\text{I.101})$$

$$\begin{bmatrix} \dot{i}_{ds} \\ \dot{i}_{qs} \\ \dot{\psi}_{dr} \\ \dot{\psi}_{qr} \end{bmatrix} = \begin{bmatrix} -\left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2}\right) & \omega_g & \frac{R_r L_m}{\sigma L_s L_r^2} & \frac{L_m \omega_r}{\sigma L_s L_r} \\ -\omega_g & -\left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2}\right) & -\frac{L_m \omega_r}{\sigma L_s L_r} & \frac{R_r L_m}{\sigma L_s L_r^2} \\ \frac{R_r L_m}{L_r} & 0 & -\frac{R_r}{L_r} & (\omega_g - \omega_r) \\ 0 & \frac{R_r L_m}{L_r} & -(\omega_g - \omega_r) & -\frac{R_r}{L_r} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ \psi_{dr} \\ \psi_{qr} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} \quad (\text{I.102})$$

For the $\alpha\beta$ frame $\omega_g = 0$

$$\begin{bmatrix} \dot{i}_{\alpha s} \\ \dot{i}_{\beta s} \\ \dot{\psi}_{\alpha r} \\ \dot{\psi}_{\beta r} \end{bmatrix} = \begin{bmatrix} -\left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2}\right) & 0 & \frac{R_r L_m}{\sigma L_s L_r^2} & \frac{L_m \omega_r}{\sigma L_s L_r} \\ 0 & -\left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2}\right) & -\frac{L_m \omega_r}{\sigma L_s L_r} & \frac{R_r L_m}{\sigma L_s L_r^2} \\ \frac{R_r L_m}{L_r} & 0 & -\frac{R_r}{L_r} & -\omega_r \\ 0 & \frac{R_r L_m}{L_r} & \omega_r & -\frac{R_r}{L_r} \end{bmatrix} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ \psi_{\alpha r} \\ \psi_{\beta r} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{\alpha s} \\ V_{\beta s} \end{bmatrix} \quad (\text{I.103})$$

I.6.1-b Model and representation of order 5 in the $d-q$ and $\alpha-\beta$ frames of reference

$$X = \begin{bmatrix} i_{ds} & i_{qs} & \psi_{dr} & \psi_{qr} & \omega_r \end{bmatrix}^T$$

$$J \frac{d\omega_m}{dt} = T_e - T_L - B\omega_m \quad (\text{I.104})$$

With $\omega_r = p\omega_m$

$$\frac{d\omega_r}{dt} = \frac{p}{J} (T_e - T_L) - \frac{B}{J} \omega_r \quad (\text{I.105})$$

$$T_e = \frac{3}{2} p (\psi_{ds} i_{qs} - \psi_{qs} i_{ds}) \quad (\text{I.106})$$

$$\psi_s = \sigma L_s i_s + \frac{L_m}{L_r} \psi_r \quad (\text{I.107})$$

$$T_e = \frac{3}{2}p \left[\left(\sigma L_s i_{ds} + \frac{L_m}{L_r} \psi_{dr} \right) i_{qs} - \left(\sigma L_s i_{qs} + \frac{L_m}{L_r} \psi_{qr} \right) i_{ds} \right] \quad (\text{I.108})$$

$$T_e = \frac{3}{2}p \frac{L_m}{L_r} (\psi_{dr} i_{qs} - \psi_{qr} i_{ds}) \quad (\text{I.109})$$

$$\frac{d\omega_r}{dt} = \frac{p}{J} \left[\frac{3}{2}p \frac{L_m}{L_r} (\psi_{dr} i_{qs} - \psi_{qr} i_{ds}) - T_L \right] - \frac{B}{J} \omega_r \quad (\text{I.110})$$

$$X = f(x) = \begin{cases} \dot{i}_{ds} = - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right) i_{ds} + \omega_g i_{qs} + \frac{R_r L_m}{\sigma L_s L_r^2} \psi_{dr} + \frac{L_m \omega_r}{\sigma L_s L_r} \psi_{qr} + \frac{1}{\sigma L_s} V_{ds} \\ \dot{i}_{qs} = - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right) i_{qs} - \omega_g i_{ds} + \frac{R_r L_m}{\sigma L_s L_r^2} \psi_{qr} - \frac{L_m \omega_r}{\sigma L_s L_r} \psi_{dr} + \frac{1}{\sigma L_s} V_{qs} \\ \dot{\psi}_{dr} = - \frac{R_r}{L_r} \psi_{dr} + \frac{R_r L_m}{L_r} i_{ds} + (\omega_g - \omega_r) \psi_{qr} \\ \dot{\psi}_{qr} = - \frac{R_r}{L_r} \psi_{qr} + \frac{R_r L_m}{L_r} i_{qs} - (\omega_g - \omega_r) \psi_{dr} \\ \dot{\omega}_r = \frac{p}{J} \left[\frac{3}{2}p \frac{L_m}{L_r} (\psi_{dr} i_{qs} - \psi_{qr} i_{ds}) - T_L \right] - \frac{B}{J} \omega_r \end{cases} \quad (\text{I.111})$$

For the $\alpha\beta$ frame $\omega_g = 0$

$$X = f(x) = \begin{cases} \dot{i}_{\alpha s} = - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right) i_{\alpha s} + \frac{R_r L_m}{\sigma L_s L_r^2} \psi_{\alpha r} + \frac{L_m \omega_r}{\sigma L_s L_r} \psi_{\beta r} + \frac{1}{\sigma L_s} V_{\alpha s} \\ \dot{i}_{\beta s} = - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right) i_{\beta s} + \frac{R_r L_m}{\sigma L_s L_r^2} \psi_{\beta r} - \frac{L_m \omega_r}{\sigma L_s L_r} \psi_{\alpha r} + \frac{1}{\sigma L_s} V_{\beta s} \\ \dot{\psi}_{\alpha r} = - \frac{R_r}{L_r} \psi_{\alpha r} + \frac{R_r L_m}{L_r} i_{\alpha s} - \omega_r \psi_{\beta r} \\ \dot{\psi}_{\beta r} = - \frac{R_r}{L_r} \psi_{\beta r} + \frac{R_r L_m}{L_r} i_{\beta s} + \omega_r \psi_{\alpha r} \\ \dot{\omega}_r = \frac{p}{J} \left[\frac{3}{2}p \frac{L_m}{L_r} (\psi_{\alpha r} i_{\beta s} - \psi_{\beta r} i_{\alpha s}) - T_L \right] - \frac{B}{J} \omega_r \end{cases}$$

I.6.2 Representation II

In this representation, all stator and rotor flux components are selected as state variables. Using only flux variables leads to a more compact electromagnetic description and highlights the magnetic coupling between the stator and rotor circuits. This formulation is particularly useful for observer design and theoretical analysis of the machine dynamics.

I.6.2-a Model and representation of order 4 in the $d-q$ and $\alpha-\beta$ frames of reference

$$\psi_{ds} = \sigma L_s i_{ds} + \frac{L_m}{L_r} \psi_{dr} \quad (\text{I.87})$$

$$\psi_{qs} = \sigma L_s i_{qs} + \frac{L_m}{L_r} \psi_{qr} \quad (\text{I.90})$$

From equations (I.87) and (I.90)

$$i_{ds} = \frac{1}{\sigma L_s} \psi_{ds} - \frac{L_m}{\sigma L_s L_r} \psi_{dr} \quad (\text{I.112})$$

$$i_{qs} = \frac{1}{\sigma L_s} \psi_{qs} - \frac{L_m}{\sigma L_s L_r} \psi_{qr} \quad (\text{I.113})$$

From equation (I.81),

$$i_{qr} = \frac{\psi_{qr} - L_m i_{qs}}{L_r} \quad (\text{I.114})$$

Substitute the value of i_{qr} into equation (I.79):

$$\psi_{qs} = L_s i_{qs} + \frac{\psi_{qr} - L_m i_{qs}}{L_r} L_m \quad (\text{I.115})$$

$$\psi_{qs} = \left(L_s - \frac{L_m^2}{L_r} \right) i_{qs} + \frac{L_m}{L_r} \psi_{qr} \quad (\text{I.116})$$

Taking $\sigma L_s L_r = L_s L_r - L_m^2$

$$\psi_{qs} = \sigma L_s i_{qs} + \frac{L_m}{L_r} \psi_{qr} \quad (\text{I.117})$$

$$i_{qs} = \frac{1}{\sigma L_s} \psi_{qs} - \frac{L_m}{\sigma L_s L_r} \psi_{qr} \quad (\text{I.118})$$

$$i_{qr} = \frac{\psi_{qr} - L_m i_{qs}}{L_r} \quad (\text{I.119})$$

Substitute i_{qs} into i_{qr}

$$i_{qr} = -\frac{L_m}{\sigma L_s L_r} \psi_{qs} + \frac{1}{\sigma L_r} \psi_{qr} \quad (\text{I.120})$$

Substitute equations (I.112) and (I.113) into equations (I.74) and (I.75) respectively:

$$\dot{\psi}_{ds} = V_{ds} - R_s \left(\frac{1}{\sigma L_s} \psi_{ds} - \frac{L_m}{\sigma L_s L_r} \psi_{dr} \right) + \omega_g \psi_{qs} \quad (\text{I.121})$$

$$\dot{\psi}_{ds} = V_{ds} - \frac{R_s}{\sigma L_s} \psi_{ds} + \frac{R_s L_m}{\sigma L_s L_r} \psi_{dr} + \omega_g \psi_{qs} \quad (\text{I.122})$$

$$\dot{\psi}_{qs} = V_{ds} - R_s \left(-\frac{L_m}{\sigma L_s L_r} \psi_{qr} + \frac{1}{\sigma L_s} \psi_{qs} \right) - \omega_g \psi_{ds} \quad (\text{I.123})$$

$$\dot{\psi}_{qs} = V_{qs} - \frac{R_s}{\sigma L_s} \psi_{qs} + \frac{R_s L_m}{\sigma L_s L_r} \psi_{qr} - \omega_g \psi_{ds} \quad (\text{I.124})$$

From equation (I.80)

$$i_{ds} = \frac{\psi_{dr} - L_r i_{dr}}{L_m} \quad (\text{I.125})$$

Substitute the value of i_{ds} into equation (I.78):

$$\psi_{ds} = L_m i_{dr} + \frac{\psi_{dr} - L_r i_{dr}}{L_m} L_s \quad (\text{I.126})$$

$$\psi_{ds} = \left(L_m - \frac{L_s L_r}{L_m} \right) i_{dr} + \frac{L_s}{L_m} \psi_{dr} \quad (\text{I.127})$$

$$\psi_{ds} = \left(\frac{L_m^2 - L_s L_r}{L_m} \right) i_{dr} + \frac{L_s}{L_m} \psi_{dr} \quad (\text{I.128})$$

$$\left(\frac{L_m^2 - L_s L_r}{L_m} \right) i_{dr} = \psi_{ds} - \frac{L_s}{L_m} \psi_{dr} \quad (\text{I.129})$$

$$i_{dr} = \frac{L_m \psi_{ds} - L_s \psi_{dr}}{L_m^2 - L_s L_r} \quad (\text{I.130})$$

Taking $\sigma L_s L_r = L_s L_r - L_m^2$

$$i_{dr} = \frac{L_m \psi_{ds} - L_s \psi_{dr}}{-\sigma L_s L_r} \quad (\text{I.131})$$

$$i_{dr} = -\frac{L_m}{\sigma L_s L_r} \psi_{ds} + \frac{1}{\sigma L_r} \psi_{dr} \quad (\text{I.132})$$

Substitute equations (I.120) and (I.132) into equations (I.76) and (I.77) respectively:

$$\dot{\psi}_{dr} = \frac{R_r L_m}{\sigma L_s L_r} \psi_{ds} - \frac{R_r}{\sigma L_r} \psi_{dr} + (\omega_g - \omega_r) \psi_{qr} \quad (\text{I.133})$$

$$\dot{\psi}_{qr} = \frac{R_r L_m}{\sigma L_s L_r} \psi_{qs} - \frac{R_r}{\sigma L_r} \psi_{qr} - (\omega_g - \omega_r) \psi_{dr} \quad (\text{I.134})$$

$$\dot{x} = Ax + Bu \quad (\text{I.135})$$

$$\begin{bmatrix} \dot{\psi}_{ds} \\ \dot{\psi}_{qs} \\ \dot{\psi}_{dr} \\ \dot{\psi}_{qr} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{\sigma L_s} & \omega_g & \frac{R_s L_m}{\sigma L_s L_r} & 0 \\ -\omega_g & -\frac{R_s}{\sigma L_s} & 0 & \frac{R_s L_m}{\sigma L_s L_r} \\ \frac{R_r L_m}{\sigma L_s L_r} & 0 & -\frac{R_r}{\sigma L_r} & (\omega_g - \omega_r) \\ 0 & \frac{R_r L_m}{\sigma L_s L_r} & -(\omega_g - \omega_r) & -\frac{R_r}{\sigma L_r} \end{bmatrix} \begin{bmatrix} \psi_{ds} \\ \psi_{qs} \\ \psi_{dr} \\ \psi_{qr} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} \quad (\text{I.136})$$

For the $\alpha\beta$ frame $\omega_g = 0$

$$\begin{bmatrix} \dot{\psi}_{\alpha s} \\ \dot{\psi}_{\beta s} \\ \dot{\psi}_{\alpha r} \\ \dot{\psi}_{\beta r} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{\sigma L_s} & 0 & \frac{R_s L_m}{\sigma L_s L_r} & 0 \\ 0 & -\frac{R_s}{\sigma L_s} & 0 & \frac{R_s L_m}{\sigma L_s L_r} \\ \frac{R_r L_m}{\sigma L_s L_r} & 0 & -\frac{R_r}{\sigma L_r} & -\omega_r \\ 0 & \frac{R_r L_m}{\sigma L_s L_r} & \omega_r & -\frac{R_r}{\sigma L_r} \end{bmatrix} \begin{bmatrix} \psi_{\alpha s} \\ \psi_{\beta s} \\ \psi_{\alpha r} \\ \psi_{\beta r} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{\alpha s} \\ V_{\beta s} \end{bmatrix}$$

I.6.2-b Model and representation of order 5 in the $d-q$ and $\alpha-\beta$ frames of reference

$$T_e = \frac{3}{2} p \left[\frac{(L_r \psi_{qs} - L_m \psi_{qr})}{\sigma L_s L_r} \psi_{ds} - \frac{(L_r \psi_{ds} - L_m \psi_{dr})}{\sigma L_s L_r} \psi_{qs} \right] \quad (\text{I.137})$$

$$T_e = \frac{3}{2} p \frac{L_m}{\sigma L_s L_r} (\psi_{qs} \psi_{dr} - \psi_{ds} \psi_{qr}) \quad (\text{I.138})$$

$$\frac{d\omega_r}{dt} = \frac{p}{J} \left[\frac{3}{2} p \frac{L_m}{\sigma L_s L_r} (\psi_{qs} \psi_{dr} - \psi_{ds} \psi_{qr}) - T_L \right] - \frac{B}{J} \omega_r \quad (\text{I.139})$$

$$X = f(x) = \begin{cases} \dot{\psi}_{ds} = V_{ds} - \frac{R_s}{\sigma L_s} \psi_{ds} + \frac{R_s L_m}{\sigma L_s L_r} \psi_{dr} + \omega_g \psi_{qs} \\ \dot{\psi}_{qs} = V_{qs} - \frac{R_s}{\sigma L_s} \psi_{qs} + \frac{R_s L_m}{\sigma L_s L_r} \psi_{qr} - \omega_g \psi_{ds} \\ \dot{\psi}_{dr} = \frac{R_r L_m}{\sigma L_s L_r} \psi_{ds} - \frac{R_r}{\sigma L_r} \psi_{dr} + (\omega_g - \omega_r) \psi_{qr} \\ \dot{\psi}_{qr} = \frac{R_r L_m}{\sigma L_s L_r} \psi_{qs} - \frac{R_r}{\sigma L_r} \psi_{qr} - (\omega_g - \omega_r) \psi_{dr} \\ \dot{\omega}_r = \frac{p}{J} \left[\frac{3}{2} p \frac{L_m}{\sigma L_s L_r} (\psi_{qs} \psi_{dr} - \psi_{ds} \psi_{qr}) - T_L \right] - \frac{B}{J} \omega_r \end{cases}$$

For the $\alpha\beta$ frame $\omega_g = 0$

$$\dot{\psi}_{\alpha s} = V_{\alpha s} - \frac{R_s}{\sigma L_s} \psi_{\alpha s} + \frac{R_s L_m}{\sigma L_s L_r} \psi_{\alpha r} \quad (\text{I.140})$$

$$\dot{\psi}_{\beta s} = V_{\beta s} - \frac{R_s}{\sigma L_s} \psi_{\beta s} + \frac{R_s L_m}{\sigma L_s L_r} \psi_{\beta r} \quad (\text{I.141})$$

$$\dot{\psi}_{\alpha r} = \frac{R_r L_m}{\sigma L_s L_r} \psi_{\alpha s} - \frac{R_r}{\sigma L_r} \psi_{\alpha r} - \omega_r \psi_{\beta r} \quad (\text{I.142})$$

$$\dot{\psi}_{\beta r} = \frac{R_r L_m}{\sigma L_s L_r} \psi_{\beta s} - \frac{R_r}{\sigma L_r} \psi_{\beta r} + \omega_r \psi_{\alpha r} \quad (\text{I.143})$$

$$\dot{\omega}_r = \frac{p}{J} \left[\frac{3}{2} p \frac{L_m}{\sigma L_s L_r} (\psi_{\beta s} \psi_{\alpha r} - \psi_{\alpha s} \psi_{\beta r}) - T_L \right] - \frac{B}{J} \omega_r \quad (\text{I.144})$$

$$X = f(x) = \begin{cases} \dot{\psi}_{\alpha s} = V_{\alpha s} - \frac{R_s}{\sigma L_s} \psi_{\alpha s} + \frac{R_s L_m}{\sigma L_s L_r} \psi_{\alpha r} \\ \dot{\psi}_{\beta s} = V_{\beta s} - \frac{R_s}{\sigma L_s} \psi_{\beta s} + \frac{R_s L_m}{\sigma L_s L_r} \psi_{\beta r} \\ \dot{\psi}_{\alpha r} = \frac{R_r L_m}{\sigma L_s L_r} \psi_{\alpha s} - \frac{R_r}{\sigma L_r} \psi_{\alpha r} - \omega_r \psi_{\beta r} \\ \dot{\psi}_{\beta r} = \frac{R_r L_m}{\sigma L_s L_r} \psi_{\beta s} - \frac{R_r}{\sigma L_r} \psi_{\beta r} + \omega_r \psi_{\alpha r} \\ \dot{\omega}_r = \frac{p}{J} \left[\frac{3}{2} p \frac{L_m}{\sigma L_s L_r} (\psi_{\beta s} \psi_{\alpha r} - \psi_{\alpha s} \psi_{\beta r}) - T_L \right] - \frac{B}{J} \omega_r \end{cases}$$

I.6.3 Representation III

The previous equations can be grouped into a compact state-space representation. This formulation simplifies the mathematical description of the machine and facilitates the application of modern control, estimation, and observer design techniques. Moreover, it provides a unified framework for simulation and numerical implementation.

I.6.3-a Model and representation of order 2 in the $d-q$ and $\alpha-\beta$ frames of reference

$$[X = [i_s, \psi_r]^T$$

$$i_s = i_{ds} + J i_{qs} \quad (\text{I.145})$$

$$\psi_r = \psi_{dr} + J \psi_{qr} \quad (\text{I.146})$$

$$V_s = V_{ds} + J V_{qs} \quad (\text{I.147})$$

Stator dynamics

$$\dot{i}_s = - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} + j\omega_g \right) i_s + \frac{L_m}{\sigma L_s L_r} \left(\frac{R_r}{L_r} - j\omega_r \right) \psi_r + \frac{1}{\sigma L_s} V_s \quad (\text{I.148})$$

Rotor dynamics

$$\dot{\psi}_r = \frac{R_r L_m}{L_r} i_s - \left(\frac{R_r}{L_r} - j(\omega_g - \omega_r) \right) \psi_r \quad (\text{I.149})$$

$$\begin{bmatrix} \dot{i}_s \\ \dot{\psi}_r \end{bmatrix} = \begin{bmatrix} - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} + j\omega_g \right) & \frac{L_m}{\sigma L_s L_r} \left(\frac{R_r}{L_r} - j\omega_r \right) \\ \frac{R_r L_m}{L_r} & - \left(\frac{R_r}{L_r} - j(\omega_g - \omega_r) \right) \end{bmatrix} \begin{bmatrix} i_s \\ \psi_r \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma L_s} \\ 0 \end{bmatrix} V_s \quad (\text{I.150})$$

For the $\alpha\beta$ frame $\omega_g = 0$ **Stator dynamics**

$$\dot{i}_s = - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right) i_s + \frac{L_m}{\sigma L_s L_r} \left(\frac{R_r}{L_r} - j\omega_r \right) \psi_r + \frac{1}{\sigma L_s} V_s \quad (\text{I.151})$$

Rotor dynamics

$$\dot{\psi}_r = \frac{R_r L_m}{L_r} i_s - \left(\frac{R_r}{L_r} + j\omega_r \right) \psi_r \quad (\text{I.152})$$

$$\begin{bmatrix} \dot{i}_s \\ \dot{\psi}_r \end{bmatrix} = \begin{bmatrix} - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right) & \frac{L_m}{\sigma L_s L_r} \left(\frac{R_r}{L_r} - j\omega_r \right) \\ \frac{R_r L_m}{L_r} & - \left(\frac{R_r}{L_r} + j\omega_r \right) \end{bmatrix} \begin{bmatrix} i_s \\ \psi_r \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma L_s} \\ 0 \end{bmatrix} V_s$$

I.6.3-b Model and representation of order 3 in the $d-q$ and $\alpha-\beta$ frames of reference

$$X = [i_{ds}, i_{qs}, \psi_r]^T$$

$$f(x) = \begin{cases} \dot{i}_{ds} = - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right) i_{ds} + \omega_g i_{qs} + \frac{R_r L_m}{\sigma L_s L_r} \psi_r + \frac{1}{\sigma L_s} V_{ds} \\ \dot{i}_{qs} = - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right) i_{qs} - \omega_g i_{ds} - \frac{L_m \omega_r}{\sigma L_s L_r} \psi_r + \frac{1}{\sigma L_s} V_{qs} \\ \dot{\psi}_r = \frac{R_r L_m}{L_r} i_{ds} - \frac{R_r}{L_r} \psi_r \end{cases}$$

For the $\alpha\beta$ frame $\omega_g = 0$

$$X = [i_{\alpha s}, i_{\beta s}, \psi_r]^T$$

$$f(x) = \begin{cases} \dot{i}_{\alpha s} = - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right) i_{\alpha s} + \frac{R_r L_m}{\sigma L_s L_r} \psi_r + \frac{1}{\sigma L_s} V_{\alpha s} \\ \dot{i}_{\beta s} = - \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right) i_{\beta s} - \frac{L_m \omega_r}{\sigma L_s L_r} \psi_r + \frac{1}{\sigma L_s} V_{\beta s} \\ \dot{\psi}_r = \frac{R_r L_m}{L_r} i_{\alpha s} - \frac{R_r}{L_r} \psi_r \end{cases}$$

The different state-space representations presented above describe the same physical system but using different state variables. Each formulation offers specific advantages depending on the intended application, whether for simulation, parameter identification, control design, or state observation. In this work, the selected representation serves as the basis for the development of the induction machine model and the subsequent observer design.

I.7 Conclusion

This chapter presented the dynamic modeling of the induction machine as a basis for analysis, control, and performance evaluation. After a brief overview of its structure and importance, both steady-state and dynamic models were discussed, highlighting the limitations of steady-state analysis for transient behavior.

To simplify the model, the Park transformation was introduced to convert the system into the dq reference frame with constant parameters, making it more suitable for control and simulation. State-space representations were then derived to support system analysis and controller design, including advanced methods such as vector control and observer-based techniques.

In conclusion, this chapter established the mathematical foundation of the induction machine required for parameter identification and advanced control development in the following chapters.

Chapter II

Classical Identification of the Induction Motor

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II.1 Introduction

Accurate knowledge of the electrical and mechanical parameters of asynchronous machines is essential for their analysis, modeling, and control. This chapter builds upon the modeling framework established in Chapter 1 by focusing on the identification of the parameters involved. Although nameplate data and manufacturer specifications provide initial information, they are generally insufficient for determining all the parameters required for an accurate representation of the machine. Consequently, experimental tests are necessary to identify these parameters with greater precision [6].

Prior to performing these tests, it is essential to establish a mathematical model of the machine, expressed as a system of equations involving the parameters to be identified [6].

This chapter first presents the development of various equivalent circuit models of the asynchronous machine. It then introduces a classical experimental approach for the identification of both electrical and mechanical parameters. Finally, simulation results are presented and analyzed in order to validate the proposed models and assess their suitability for subsequent control applications [6].

II.2 Equivalent Circuits of the Induction Machine

The induction motor is commonly represented by a per-phase equivalent circuit based on the transformer analogy. This representation is justified by the magnetic coupling between the stator and rotor through the air gap. The model uses resistances and inductances to describe the machine's electrical behavior, where the stator and rotor resistances account for copper losses, and the inductances represent leakage and magnetizing effects. These parameters directly influence the voltage drops, power factor, and overall dynamic behavior of the machine, and are essential for parameter identification and model validation [6].

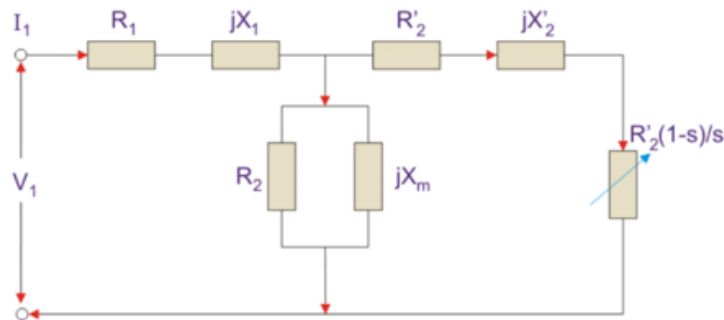


Figure II.1: Equivalent Circuits of the Induction Machine [6]

Here all the other parameters are same except-
 R_2 is the rotor winding resistance with referred to stator winding.
 X_2 is the rotor winding inductance with referred to stator winding.
 $\frac{R_2(1-s)}{s}$ is the resistance which shows the power which is converted to mechanical power

output or useful power. The power dissipated in that resistor is the useful power output or shaft power.

II.3 Generalized Equivalent Circuit of the Induction Motor in Dynamic Operation

The dynamic behavior of the three-phase induction motor can be described using the generalized model in the Park reference frame, as established in Chapter 1. This model is based on a system of differential equations that represent the electrical and magnetic interactions between the stator and rotor circuits. By introducing the stator and rotor leakage inductances, defined respectively as:

$$l_s = L_s - L_m \quad (\text{II.1})$$

$$l_r = L_r - L_m \quad (\text{II.2})$$

the model can be expressed in a more compact and physically meaningful form.

The magnetizing current, which is responsible for establishing the air-gap flux, is given by:

$$i_m = i_s + i_r \quad (\text{II.3})$$

Using these definitions, the voltage and flux equations of the machine can be rewritten as:

$$V_s = R_s i_s + l_s \frac{di_s}{dt} + L_m \frac{di_m}{dt} + j\omega_g \psi_s \quad (\text{II.4})$$

$$V_r = 0 = R_r i_r + l_r \frac{di_r}{dt} + L_m \frac{di_m}{dt} + j\omega_{sl} \psi_r \quad (\text{II.5})$$

$$\psi_s = l_s i_s + L_m i_m \quad (\text{II.6})$$

$$\psi_r = l_r i_r + L_m i_m \quad (\text{II.7})$$

The figure below illustrates the generalized equivalent circuit of the induction motor in dynamic operation:

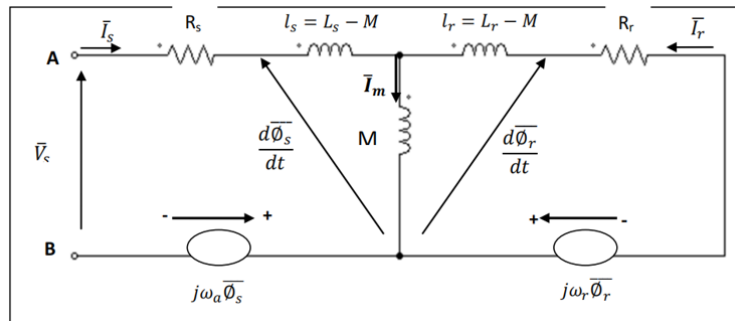


Figure II.2: Generalized Equivalent Circuit [6]

The generalized equivalent circuit above illustrates the electromagnetic coupling between the stator and rotor in the Park frame. It integrates ohmic losses, transformer effects through leakage and mutual inductances, and rotational emfs $j\omega\psi$. By modeling these dynamic interactions, the circuit captures the machine's behavior during transient states, making it the fundamental basis for advanced motor control and simulation [6].

II.4 Dynamic Equivalent Circuit of the Induction Motor in the Stator Reference Frame

The generalized dynamic model presented previously can be simplified by expressing it in the stator reference frame. In this case, the reference frame is fixed to the stator, and therefore its angular speed is zero: $\omega_g = 0$

As a result, the stator voltage equations no longer include rotational terms, while the rotor equations remain dependent on the rotor speed through the slip [6]. Under these conditions, the voltage equations become:

$$V_s = R_s i_s + l_s \frac{di_s}{dt} + L_m \frac{di_m}{dt} \quad (\text{II.8})$$

$$V_r = 0 = R_r i_r + l_r \frac{di_r}{dt} + L_m \frac{di_m}{dt} + j\omega_r \psi_r \quad (\text{II.9})$$

The flux equations remain unchanged:

$$\psi_s = l_s i_s + L_m i_m \quad (\text{II.10})$$

$$\psi_r = l_r i_r + L_m i_m \quad (\text{II.11})$$

The figure below illustrates the dynamic equivalent circuit of the induction motor in the stator reference frame:

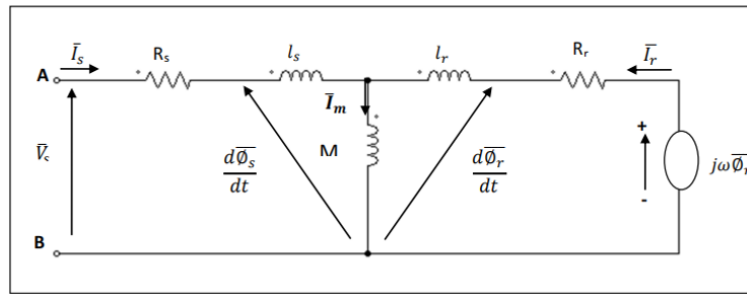


Figure II.3: Dynamic Equivalent Circuit [6]

In the stator-fixed reference frame ($\alpha\beta$), the frame speed is zero ($\omega_g = 0$), which eliminates the stator's rotational emf source. While this simplifies the circuit, it highlights a major practical drawback: stator and rotor leakage inductances (l_s and l_r) cannot be measured independently through standard tests. Consequently, the model is often simplified by mathematically 'referring' all magnetic leakage to either the stator or the rotor side to create a more usable equivalent circuit [6].

II.5 Steady-State Equivalent Circuit of the Induction Machine

Under steady-state sinusoidal operation, the equations of the induction machine can be expressed in the frequency domain by replacing the time derivative operator with $j\omega_g$, where ω_g is the stator angular frequency [6]. In the stator reference frame, the electrical equations are written as:

$$V_s = (R_s + j\omega_g L_s) i_s + j\omega_g L_m i_r \quad (\text{II.12})$$

$$V_r = 0 = (R_r + j\omega_{sl} L_r) i_r + j\omega_{sl} L_m i_s \quad (\text{II.13})$$

Since the rotor is short-circuited, its voltage is equal to zero.

To account for the mechanical operation of the machine, the rotor pulsation is expressed as a function of the slip g :

$$\omega_{sl} = \omega_g - \omega_r = g\omega_g \quad (\text{II.14})$$

Substituting this relation into the rotor equation and dividing by g , one obtains:

$$V_r = 0 = \left(\frac{R_r}{g} + j\omega_g L_r \right) i_r + j\omega_g L_m i_s \quad (\text{II.15})$$

This formulation highlights the term $\frac{R_r}{g}$, which reflects the influence of slip on the electrical behavior of the machine and plays a key role in the electromechanical energy conversion process.

In order to obtain a unified representation, rotor quantities are referred to the stator side using a change of variable equivalent to a transformation ratio:

$$i_r = a i_r' \quad (\text{II.16})$$

This leads to the definition of the referred parameters:

$$R_r' = a^2 R_r, L_r' = a^2 L_r, L_m' = a L_m$$

The equations then becomes:

$$\begin{aligned} V_s &= (R_s + j\omega_g L_s) i_s + j\omega_g L_m' i_r' \\ V_r &= 0 = \left(\frac{R_r'}{g} + j\omega_g L_r' \right) i_r' + j\omega_g L_m' i_s \end{aligned} \quad (\text{II.17})$$

To better represent the physical phenomena, the stator and rotor leakage inductances are introduced as:

$$l_s = L_s - L_m' \quad (\text{II.19})$$

$$l_r' = L_r' - L_m' \quad (\text{II.20})$$

Defining the magnetizing current as:

$$i_m = i_s + i_r' \quad (\text{II.21})$$

The final form of the equations is obtained:

$$V_s = (R_s + j\omega_g l_s) i_s + j\omega_g L_m' i_m'$$

$$V_r = 0 = \left(\frac{R_r'}{g} + j\omega_g l_r'\right) i_r' + j\omega_g (L_m' i_m')$$

The figure below illustrates the Steady-State equivalent circuit of the induction machine:

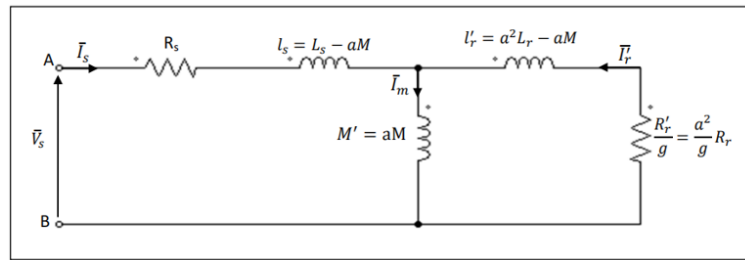


Figure II.4: Steady-State Equivalent Circuit [6]

The above circuit is analogous to that of a transformer with a short-circuited secondary winding, in which the rotor quantities are referred to the stator side.

In this representation:

- R_s represents the stator resistance,
- l_s is the stator leakage inductance,
- l_r' is the rotor leakage inductance referred to the stator,
- L_m' corresponds to the magnetizing inductance,
- $\frac{R_r'}{g}$ represents the equivalent rotor resistance, which depends on the slip.

II.5.1 Calculation of the Total Impedance Seen from the Stator

In steady-state operation, the induction motor can be represented by its per-phase equivalent circuit referred to the stator, as shown in the Figure above. In this model, all rotor quantities are expressed on the stator side using an appropriate transformation ratio [6].

The parameters of the equivalent circuit are defined as follows:

- R_s Stator resistance,

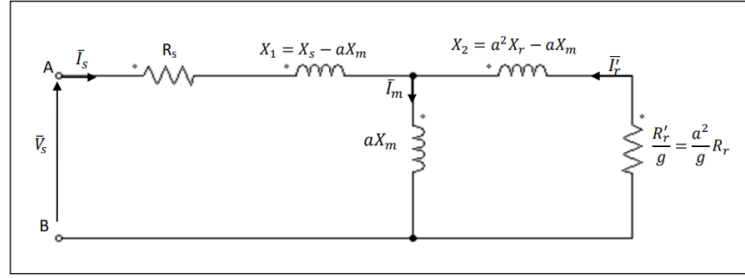


Figure II.5: Total Impedance Seen from the Stator [6]

- $R'_r = a^2 R_r$ rotor resistance referred to the stator,
- $X_r = \omega_g L_r$ rotor reactance,
- $X_s = \omega_g L_s$ stator reactance,
- $X_m = \omega_g L_m$ magnetizing reactance,
- a transformation constant adopted for referring the rotor to the stator frame.

The leakage reactances are given by:

$$X_1 = X_s - aX_m \quad (\text{II.24})$$

$$X_2 = a^2 X_r - aX_m \quad (\text{II.25})$$

Using this equivalent circuit, the total impedance per phase seen from the stator terminals can be determined. The stator impedance is:

$$Z_1 = R_s + jX_1 \quad (\text{II.26})$$

The rotor branch impedance is:

$$Z_2 = \frac{R'_r}{g} + jX_2 \quad (\text{II.27})$$

By combining the stator impedance with the parallel association of the magnetizing reactance jX_m and the rotor branch, the total input impedance is obtained as:

$$Z_e = \frac{V_s}{I_s} = R_s + jX_1 + \frac{jX_m \left(\frac{R'_r}{g} \right) + jX_2}{\frac{R'_r}{g} + jX_2 + jX_m} \quad (\text{II.28})$$

In steady-state operation, the per-phase equivalent circuit of the induction motor allows the definition of the input impedance seen from the stator terminals. This impedance can be expressed in the following form:

$$Z_e = R_e + jX_e \quad (\text{II.29})$$

where R_e and X_e represent the real and imaginary components of the input impedance, respectively.

Using the equivalent circuit parameters, these components can be written as:

$$R_e = R_s + \frac{jX_m + jX_2 \frac{R_r}{g}}{\left(\frac{R_r}{g}\right)^2 + \omega_g^2 L_r^2} \quad (\text{II.30})$$

$$X_e = \omega_g L_s + \frac{a^2 \omega_g^3 L_m^2 L_r}{\left(\frac{R_r}{g}\right)^2 + \omega_g^2 L_r^2} \quad (\text{II.31})$$

Where $X_1 = \omega_g (L_s - aL_m)$ and $X_2 = \omega_g (L_r' - aL_m)$

These expressions show that the input impedance depends on both electrical parameters (R_s, L_s, R_r, L_r, L_m) and the slip g , which reflects the operating condition of the machine.

It is important to note that the impedance Z_e is independent of the transformation factor a , while its components R_e and X_e are expressed in terms of parameters that must be identified experimentally.

II.5.2 Equivalent Circuit with Stator Leakage Concentration

Different equivalent circuit forms can be obtained by an appropriate choice of the transformation factor a . One commonly used configuration is the model in which the leakage inductances are entirely concentrated in the stator [6].

In this case, the transformation factor is chosen such that:

$$l_r' = 0 \Rightarrow a = \frac{L_m}{L_r} \quad (\text{II.32})$$

Under this assumption, the stator leakage inductance and the equivalent magnetizing inductance become:

$$l_s = L_s - \frac{L_m^2}{L_r} = \sigma L_s \quad (\text{II.33})$$

$$L_m' = \frac{L_m^2}{L_r} = (1 - \sigma) L_s \quad (\text{II.34})$$

The rotor resistance referred to the stator is then expressed as:

$$R_r' = \left(\frac{L_m^2}{L_r}\right) R_r = \frac{(1 - \sigma) L_s}{\tau_r} \quad (\text{II.35})$$

where σ is the leakage coefficient and τ_r is the rotor time constant given by $\sigma = 1 - \frac{L_m^2}{L_s L_r}$ and $\tau_r = \frac{L_r}{R_r}$ respectfully.

The figure below illustrates the equivalent circuit with stator leakage concentration:

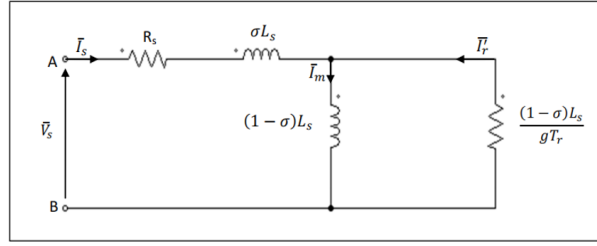


Figure II.6: Equivalent Circuit with Stator Leakage Concentration

II.5.3 Equivalent Circuit with Rotor Leakage Concentration

Induction Machine (IM) models are frequently formulated in terms of rotor flux, as this variable is essential for the implementation of Field-Oriented Control (FOC). To adapt the equivalent circuit for this objective, a transformation ratio is chosen such that $a = \frac{L_s}{L_m}$. This specific transformation effectively nullifies the stator leakage inductance ($l_s = 0$), resulting in a simplified circuit structure with the following derived parameters [6]:

Total rotor leakage inductance (l_s) is defined by the relationship:

$$l'_r = \left(\frac{L_s}{L_m} \right)^2 L_r - L_s = \frac{\sigma L_s}{1 - \sigma} \quad (\text{II.36})$$

Referred magnetizing inductance L_m :

$$L'_m = \left(\frac{L_s}{L_m} \right) L_m = L_s \quad (\text{II.37})$$

Referred rotor resistance R_r :

$$R'_r = \left(\frac{L_s}{L_m} \right)^2 R_r = \frac{L_s}{(1 - \sigma) \tau_r} \quad (\text{II.38})$$

The figure below illustrates the equivalent circuit with rotor leakage concentration

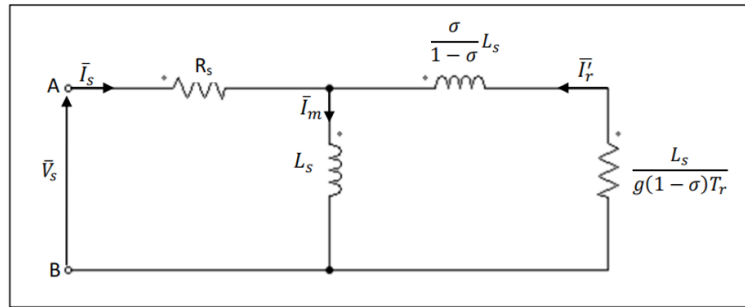


Figure II.7: Equivalent Circuit with Rotor Leakage Concentration [6]

II.5.4 Equivalent Circuit with Leakage Equally Distributed Between Stator and Rotor

In this configuration, the transformation constant a is selected based on the Alger hypothesis, which assumes that leakage is distributed equally between the stator and the

rotor ($l_s = l'_r$) Consequently, the relationship is established as $l_s = l'_r = a^2 l_r$, leading to a transformation ratio defined by [6]:

$$a = \sqrt{\frac{L_s}{L_r}} \quad (\text{II.39})$$

Under this assumption, the derived electrical parameters for the equivalent circuit are calculated as follows:

Referred magnetizing inductance L_m :

$$L'_m = \sqrt{\frac{L_s}{L_r}} L_m = \sqrt{1-\sigma} L_s \quad (\text{II.40})$$

Stator leakage inductance l_s :

$$l_s = L_s (1 - \sqrt{1-\sigma}) \quad (\text{II.41})$$

Referred rotor leakage inductance l'_r :

$$l'_r = l_s = L_s (1 - \sqrt{1-\sigma}) \quad (\text{II.42})$$

Referred rotor resistance R_r :

$$R'_r = \left(\sqrt{\frac{L_s}{L_r}} \right)^2 R_r = \frac{L_s}{L_r} R_r = \frac{L_s}{\tau_r} \quad (\text{II.43})$$

This specific model allows for the independent calculation of stator inductance L_s and rotor inductance L_r , as well as the determination of the mutual magnetizing inductance L_m . The figure below illustrates the equivalent circuit with leakage equally distributed between stator and rotor:

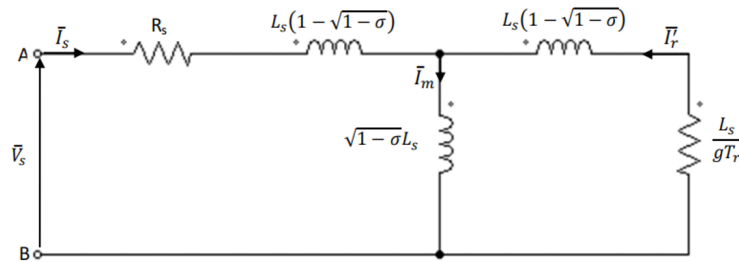


Figure II.8: Equivalent Circuit with Leakage Equally Distributed Between Stator and Rotor [6]

When supplied with a fixed voltage, all three previously discussed models, regardless of the non-zero value of a , exhibit the same total impedance as seen from the stator. As a result, they maintain identical global characteristics, such as current draw and power absorption [6].

Independent of the constant a , the resistive (R_e) and reactive (X_e) components of the total impedance Z_e are defined by the following expressions:

$$R_e = R_s + \omega_g L_s \frac{g\omega_g \tau_r}{1 + (g\omega_g \tau_r)^2} (1 - \sigma) \quad (\text{II.44})$$

$$X_e = \omega_g L_s \left[1 - \frac{(g\omega_g \tau_r)^2}{1 + (g\omega_g \tau_r)^2} (1 - \sigma) \right] \quad (\text{II.45})$$

In these equations, the term $g\omega_g = \omega_{sl}$ represents the **rotor pulsation** (or slip pulsation). These expressions demonstrate that the machine's behavior is governed by four primary electrical parameters (R_s , L_s , σ , and τ_r) and the rotor pulsation ω_{sl} , which varies according to the applied load torque [6].

II.5.5 Equivalent Circuit Including Iron Losses

To account for core losses within the machine, a shunt resistance R_F is typically integrated in parallel with the magnetizing inductance L_m . This modification is applied to the steady-state generalized model of the induction machine, as illustrated in the Figure below:

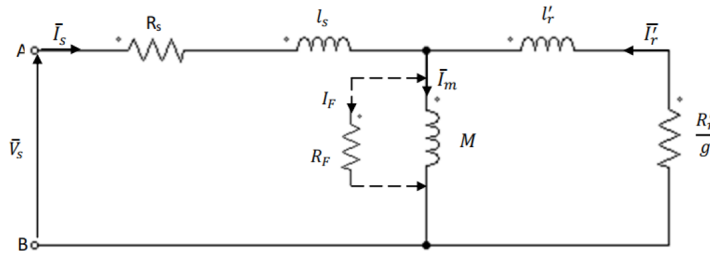


Figure II.9: Equivalent Circuit Including Iron Losses [6]

The resistance R_F is placed in parallel with the magnetizing inductance L_m to represent the iron losses.

The current distribution within this branch is governed by the following relationship:

$$i_m + i_F = i_s + i_r' \quad (\text{II.46})$$

Where:

$$R_F i_F = L_m i_m \quad (\text{II.47})$$

It is important to note that while this resistance is added to model energy dissipation in the iron, the fundamental voltage and flux equations for both the stator and rotor remain unchanged [6].

The use of equivalent circuit modeling allows the induction machine to be represented through various configurations derived from Park's model. However, the practical application of these models requires precise knowledge of their electrical parameters [6].

Often, the data provided by manufacturers is insufficient to fully characterize these parameters, and direct measurement can be technically challenging. Consequently, parametric identification through various experimental and analytical approaches becomes necessary. In the following sections, we will first detail conventional identification methods used to determine the electrical parameters, followed by the procedures for identifying mechanical parameters [6].

II.6 Classical Parameter Identification of the Induction Motor

This method consists in measuring, on a per-phase basis, the active power P and reactive power Q during steady-state tests performed at zero useful output power, namely the no-load test ($g \approx 0$) and the blocked-rotor test ($g = 1$). The measured powers are used to determine R_e and X_e of the input impedance, which depend on the parameters to be identified, as expressed below:

$$R_e = \frac{P}{P^2 + Q^2} V_s^2 = \frac{P}{I_s^2} \quad (\text{II.48})$$

$$X_e = \frac{Q}{P^2 + Q^2} V_s^2 = \frac{Q}{I_s^2} \quad (\text{II.49})$$

In motor operation, the squirrel-cage induction machine is the most commonly used in industry, but it poses a challenge due to limited access to its rotor for experimental measurements.

It is not possible to measure individually the actual parameters R_r , L_r , and M that appear in simulation models.

The classical tests considered enable the determination of all electrical parameters of the equivalent circuit with totalized leakage in the stator, namely: L_s , σ , and τ_r , from which L_m , L'_r , and R'_r can be deduced.

The stator resistance R_s can be easily obtained through a direct current (DC) test [6].

II.6.1 Measurement of Stator Resistance

The stator phase resistance can be determined either directly using measurement bridges (zero method), or indirectly using the volt-ammeter method. The latter is the most commonly used in practice [6].

In this method, a direct current (DC) voltage is applied to one phase of the stator, and the corresponding current and voltage across its terminals are measured. The resistance is then obtained using Ohm's law:

$$R_s = \frac{V}{I} \quad (\text{II.50})$$

Due to the relatively low value of the stator resistance, the downstream (ammeter-after-voltmeter) connection is preferred to improve measurement accuracy. In addition, the injected current must not exceed 10% of the rated current in order to avoid excessive heating and magnetic effects.

The measurement is typically performed under thermal steady-state conditions (i.e., when the machine is warm), since the stator resistance depends on temperature [6].

II.6.2 No-Load Test

In this test, the induction motor is supplied at its rated voltage and frequency while operating without any mechanical load. Under these conditions, the rotor rotates close to the synchronous speed, and the slip is very small ($g \approx 0$). As a result, the rotor current becomes negligible, and the air-gap flux is mainly established by the stator. The electromagnetic torque developed is only sufficient to overcome mechanical losses such as friction and ventilation [6].

Under no-load conditions, the following approximations can be made:

$$V_s = V_{s0}, \quad i_r \approx 0, \quad i_s = i_{s0} \approx i_m \quad (\text{II.51})$$

In the equivalent circuit, this corresponds to an open rotor branch, since:

$$\frac{R'_r}{g} \rightarrow \infty \quad \text{as} \quad g \rightarrow 0 \quad (\text{II.52})$$

Therefore, the no-load equivalent circuit is reduced to the stator impedance and the magnetizing branch only.

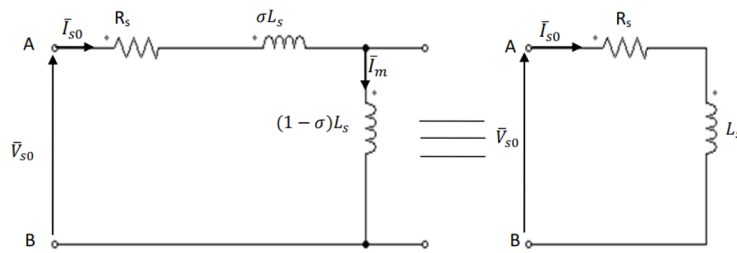


Figure II.10: No-Load Test

Parameter determination:

$$V = \frac{U_0}{\sqrt{3}}, P_{ph} = \frac{P_0}{3} \quad (\text{II.53})$$

The no-load power factor is:

$$\cos \varphi = \frac{P_0}{\sqrt{3} U_0 i_0} \quad (\text{II.54})$$

The current is decomposed into:

$$I_\omega = I_0 \cos \varphi_0 \quad (\text{II.55})$$

$$I_m = I_\omega \sin \varphi_0 \quad (\text{II.56})$$

The parameters of the magnetizing branch are then obtained as:

$$R_c = \frac{V}{I_\omega} \quad (\text{II.57})$$

$$X_m = \frac{V}{I_m} \quad (\text{II.58})$$

Impedance :

$$Z_{e0} = R_{e0} + jX_{e0} \quad (\text{II.59})$$

Where :

$$R_{e0} = \frac{P_0}{P_0^2 + Q_0^2} V_{s0}^2 = R_s \quad (\text{II.60})$$

$$X_{e0} = \frac{Q_0}{P_0^2 + Q_0^2} V_{s0}^2 = \omega_s L_s \quad (\text{II.61})$$

II.6.3 Slip Test

The slip test is performed in order to analyze the behavior of the induction motor as a function of the slip and to provide additional information about the rotor parameters. The model is expressed in the synchronous rotating reference frame (dq), rotating at the electrical angular speed ω_g , which corresponds to the synchronous speed of the stator magnetic field. The rotor rotates at the electrical angular speed ω_r [6].

The slip angular speed is defined as:

$$\omega_{sl} = \omega_g - \omega_r \quad (\text{II.62})$$

From this, the slip is expressed as:

$$s = \frac{\omega_g - \omega_r}{\omega_g} \quad (\text{II.63})$$

During the test, the motor is operated under different load conditions, causing variations in the rotor speed. For each operating point, the rotor speed, stator current, and input power are measured. As the slip increases, the rotor current increases, leading to higher rotor copper losses. This behavior reflects the dependence of the machine's electrical and mechanical performance on the slip. The slip test thus provides useful information for validating the identified parameters, particularly those related to the rotor, and for analyzing the operating characteristics of the induction motor [6].

II.6.4 Blocked-Rotor Test (Short-Circuit Test)

The blocked-rotor test, also known as the short-circuit test, is performed in order to determine the rotor and leakage parameters of the induction motor [6]. In this test, the rotor is mechanically locked so that its speed is zero: $\omega_r = 0$

Hence, $g = 1$. Under these conditions, the rotor circuit is fully active, and the motor behaves similarly to a short-circuited transformer

A reduced voltage is applied to the stator in order to circulate the rated current while

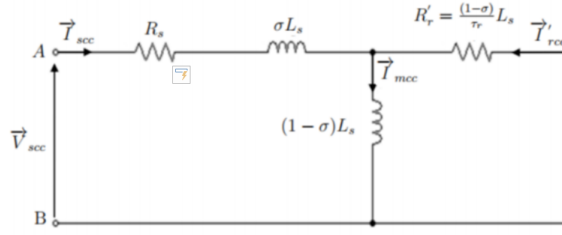


Figure II.11: Blocked-Rotor Test

avoiding excessive heating. During the test, the following quantities are measured:

- Applied voltage U_{cc} ,
- Current I_{cc} ,
- Input power P_{cc} .

Parameter determination :

$$R_{cc} = \frac{P_{cc}}{I_{cc}^2} V_{cc}^2 \quad (\text{II.64})$$

$$X_{cc} = \frac{Q_{cc}}{I_{cc}^2} V_{cc}^2 \quad (\text{II.65})$$

$$Z_{cc} = R_{cc} + jX_{cc} \quad (\text{II.66})$$

II.6.5 Deceleration Test (Routin's Method)

The deceleration (or slowing-down) test is used to identify the mechanical parameters of the induction motor. In this method, the machine is first brought to its rated speed, after which the power supply is disconnected. The rotor then decelerates naturally due to mechanical losses such as friction and ventilation [6]. The mechanical behavior of the system is governed by:

$$J \frac{d\omega_r}{dt} = -B\omega_r \quad (\text{II.67})$$

where:

- J is the moment of inertia,

- B is the viscous friction coefficient,
- ω_r is the rotor angular speed.

Solving this equation gives:

$$\omega_r(t) = \omega_{r0} e^{-\frac{B}{J}t} \quad (\text{II.68})$$

Taking the natural logarithm :

$$\ln(\omega_r) = \ln(\omega_{r0}) - \frac{B}{J}t \quad (\text{II.69})$$

II.7 Experimental Results

This section presents the experimental results obtained from the various tests performed on the induction motor. The measured data are used to determine the electrical and mechanical parameters of the machine using the classical identification method.

II.7.1 Nameplate Data of the Tested Motor

The experimental tests were carried out on a three-phase squirrel-cage induction motor manufactured by Siemens. The rated characteristics of the machine, obtained from the nameplate, are summarized in Table 100.

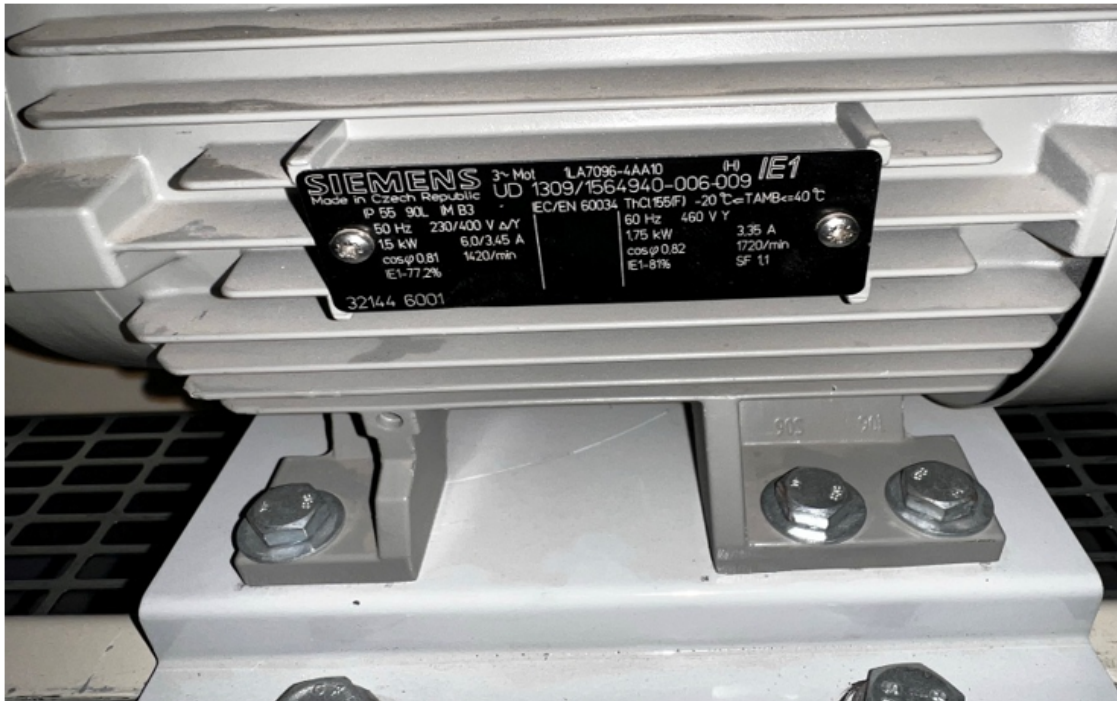


Figure II.12: Nameplate Data of the Tested Motor

Table II.1: Characteristics of the Machine

Parameter	Value
Manufacturer	Siemens
Motor type	Three-phase induction motor
Rated power	1.5 kW
Rated frequency	50 Hz
Rated voltage	230/400 V
Rated current	6.0/3.45 A
Rated speed	1420 rpm
Power factor	0.81
Efficiency class	1E1
Degree of protection	IP55
Insulation class	Class F
Mounting type	IM B3

The machine was operated under rated conditions during the experimental tests, with a star connection supplied at 400 V and 50 Hz.

Slip (s):

With the rated speed being 1420 rpm, synchronous speed=1500 rpm

$$s = \frac{\omega_g - \omega_r}{\omega_g} \quad (\text{II.70})$$

$$s = \frac{1500 - 1420}{1500} \approx 0.053 \quad (\text{II.71})$$

II.7.2 Measurement of Stator Resistance

The stator resistance is an important parameter for the identification of the induction motor model. It is required for the estimation of copper losses and for the determination of the electrical parameters of the equivalent circuit. In this work, the stator resistance was determined using two experimental methods:

- the voltammetric method,
- and direct measurement using a digital multimeter.

All measurements were carried out with the machine at standstill and under cold conditions.

II.7.2-a Measurement Using the Digital Multimeter

The resistance of one stator winding was first measured directly using a digital ohmmeter. Since the motor is connected in star configuration, the measured resistance between two terminals corresponds to two stator windings connected in series.

The measured resistance between two phases is:

$$R_{sa} = 29.20\Omega \quad (\text{II.72})$$

For one stator phase winding, the resistance is therefore given by:

$$R_s = \frac{R_{sa}}{2} = \frac{29.20\Omega}{2} \quad (\text{II.73})$$

$$R_s = 14.60\Omega \quad (\text{II.74})$$

To account for the temperature rise of the windings, a correction coefficient of 1.15 was applied. The corrected stator resistance becomes:

$$R_s = 1.15 * 14.60\Omega \quad (\text{II.75})$$

$$R_s = 16.79\Omega \quad (\text{II.76})$$

Hence, the corrected stator resistance obtained using the digital multimeter method is: $R_s = 16.79\Omega$

II.7.2-b Measurement Using the Voltammetric Method

The stator resistance was also determined experimentally using the voltammetric method. A DC voltage was applied to the stator winding and the corresponding current was measured. The resistance between two phases was calculated using Ohm's law:

$$R_s = \frac{U}{I} \quad (\text{II.77})$$

The table below show the different measurements taken and the value of resistance calculated:

$U =$	V	27.00	31.50	37.00	41.00	43.00	45.00
$I =$	A	2.50	2.84	3.15	3.45	3.60	3.70
$R_{sa} =$	Ω	9.51	10.00	10.72	11.39	11.62	12.16
$R_s =$	Ω	4.75	5.00	5.36	5.69	5.81	6.08

The average stator resistance obtained at cold temperature is:

$$R_s = \frac{(4.75 + 5.00 + 5.36 + 5.69 + 5.81 + 6.08)}{6} = 5.45\Omega \quad (\text{II.78})$$

After temperature correction:

$$R_s = 1.15 * 5.45\Omega = 6.27\Omega \quad (\text{II.79})$$

The results obtained using the two methods are relatively close, which confirms the validity of the measurements. Small differences may be caused by measurement uncertainties, contact resistances, and temperature variations during the experiments. The voltammetric method generally provides more reliable results because it is based on multiple measurements and reduces the influence of instrument precision errors.

II.7.3 No-Load Test

The no-load test was performed to study the behavior of the induction motor operating without mechanical load. During the experiment, the machine was supplied at rated frequency while the rotor rotated freely at a speed close to the synchronous speed. Under these conditions, the absorbed power mainly represents the iron losses, mechanical losses, and stator copper losses, while rotor copper losses remain negligible due to the very small slip. The measurements obtained during this test were used for the separation of collective losses and for the determination of the stator cyclic inductance. The experimental readings recorded during the test are presented in the following table.

$$R_s = 5.45\Omega \quad (\text{II.80})$$

Table II.2: Measured Parameters

$U_0(V)$	200.00	220.00	260.00	300.00	340.00	380.00	400.00	440.00
$I_0(A)$	0.85	0.94	1.10	1.36	1.60	1.88	2.26	3.12
$P_{a0}(W)$	130.70	149.60	198.80	269.20	350.00	462.00	552.00	810.80
$P_{b0}(W)$	-23.10	-36.40	-69.10	-115.30	-168.60	-246.30	-303.20	-461.80

Table II.3: Calculated Parameters

$V_0(V)$	115.47	127.02	150.11	173.21	196.30	219.39	230.94	254.03
$V_0^2(V^2)$	13333.33	16133.33	22533.33	30000.00	38533.33	48133.33	53333.33	64533.33
$P_0(W)$	107.60	113.20	129.70	153.90	181.40	215.70	248.80	349.00
$Q_0(VAR)$	266.39	322.16	464.02	665.97	898.24	1226.81	1481.25	2204.21
$\tan \varphi_0$	2.48	2.85	3.58	4.33	4.95	5.69	5.95	6.32
$\varphi_0(Rad)$	1.19	1.23	1.30	1.34	1.37	1.40	1.40	1.41
$\cos \varphi_0$	0.37	0.33	0.27	0.23	0.20	0.17	0.17	0.16
$P_{js0}(W)$	11.81	14.45	19.78	30.24	41.86	57.79	83.51	159.17
$P_c = P_{fer} + P_{mec}(W)$	95.79	98.75	109.92	123.66	139.54	157.91	165.29	189.83

$$V_0 = \frac{U_0}{\sqrt{3}} \quad (\text{II.81})$$

$$P_0 = P_{b0} + P_{a0} \quad (\text{II.82})$$

$$Q_0 = \sqrt{3} * (P_{a0} - P_{b0}) \quad (\text{II.83})$$

$$\tan \varphi_0 = \frac{Q_0}{P_0} \quad (\text{II.84})$$

$$\varphi_0 = \tanh^{-1}(\tan \varphi_0) \quad (\text{II.85})$$

$$P_{js} = 3 * R_s * I_0^2 \quad (\text{II.86})$$

$$P_c = P_0 - P_{js} \quad (\text{II.87})$$

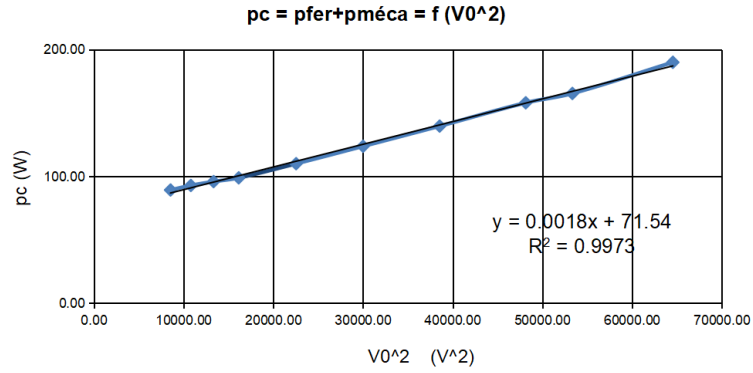


Figure II.13: No-Load Test Experimental Curve

From the curve above, $P_{mec} = 71.54W$

Under no load operation, $P_c = P_{mec} + P_{fer}$

$$P_{fer} = P_c - P_{mec} \quad (II.88)$$

$$P_{fer} = 165.29 - 71.54 \quad (II.89)$$

$$P_{fer} = 93.75W \quad (II.90)$$

$$L_s = \frac{3V^2}{(Q_0 \cdot \omega_g)} \quad (II.91)$$

$$L_s = \frac{3 * 53333.33}{1481.25 * 2 * \pi * 50} \quad (II.92)$$

$$L_s = 0.344H \quad (II.93)$$

$$X_s = L_s \cdot \omega_g \quad (II.94)$$

$$X_s = 0.344 * 2 * \pi * 50 \quad (II.95)$$

$$X_s = 108.07\Omega \quad (II.96)$$

$$R_{fer} = \frac{3V^2}{P_{fer}} \quad (II.97)$$

$$R_{fer} = \frac{3 * 53333.33}{93.75} = 1076.67\Omega \quad (II.98)$$

II.7.4 Blocked-rotor test

The blocked-rotor test is performed by mechanically locking the rotor and applying a reduced three-phase voltage to the stator until the rated current is reached. Under these conditions, the rotor speed is zero and the slip is equal to unity $g = 1$. Since the applied voltage is low, the magnetic flux remains weak and the magnetizing current, iron losses, and mechanical losses can be neglected. The equivalent circuit is therefore reduced to the series combination of the stator resistance, the rotor resistance referred to the stator, and the total

leakage reactance. This test makes it possible to determine the rotor resistance referred to the stator R'_r , the total leakage inductance L_σ , and the Blondel leakage coefficient σ . The experimental results and the corresponding calculations are presented in the Figure below.

Table II.4: Measured Parameters

U_{cc}	V	53.00	59.00	64.20	69.50	75.00	81.90	88.20	90.00	96.00
I_{cc}	A	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.45	3.60
P_{acc}	W	109.00	133.90	155.00	184.00	209.10	249.90	288.30	302.00	337.50
P_{bcc}	W	32.20	41.10	48.60	56.80	68.60	84.20	100.20	110.00	122.90

Table II.5: Calculated Parameters

V_{cc}	V	30.60	34.06	37.07	40.13	43.30	47.28	50.92	51.96	55.43
P_{cc}	W	141.20	175.00	203.60	240.80	277.70	334.10	388.50	412.00	460.40
Q_{cc}	VAR	133.02	160.73	184.29	220.32	243.35	287.00	325.80	332.55	371.70
$x = \frac{P_{bcc}}{P_{acc}}$	/	3.39	3.26	3.19	3.24	3.05	2.97	2.88	2.75	2.75
$\cos \varphi_{cc}$	/	0.73	0.74	0.74	0.74	0.75	0.76	0.77	0.78	0.78

II.7.4-a Determination of the Rotor Copper-Loss Resistance Referred to the Stator

The total resistance per phase during the blocked-rotor test is given by:

$$R_{cc} = \frac{P_{cc}}{3I_{cc}^2} \quad (\text{II.99})$$

Substituting the measured values:

$$R_{cc} = \frac{412}{3 \times 3.45^2} = 11.54\Omega \quad (\text{II.100})$$

The rotor resistance referred to the stator is then obtained from:

$$R'_r = R_{cc} - R_s \quad (\text{II.101})$$

$$R_s = 5.45\Omega \quad (\text{II.102})$$

$$R'_r = 6.09\Omega \quad (\text{II.103})$$

II.7.4-b Determination of the Total Leakage Reactance

The Magnetizing Reactance :

$$X_m = \frac{3 \times V_0^2}{Q_0} = \frac{3 \times 230.94^2}{1481.25} \approx 108.01\Omega \quad (\text{II.104})$$

The Mutual Inductance :

$$L_m = \frac{X_m}{\omega} = \frac{108.01}{100\pi} = 0.3438H \quad (\text{II.105})$$

Total Leakage Reactance :

$$X_{cc} = \frac{Q_{cc}}{3 \times I_{cc}^2} = \frac{332.55}{3 \times 3.45^2} \approx 9.31\Omega \quad (\text{II.106})$$

Total Leakage Inductance :

$$L_{cc} = \frac{X_{cc}}{\omega} = \frac{9.31}{100\pi} = 0.0296H \quad (\text{II.107})$$

Blondel Dispersion Coefficient :

$$\sigma = 1 - \frac{L_m^2}{L_s L_r} = 1 - \frac{0.3438^2}{0.3586^2} = 0.081N \quad (\text{II.108})$$

II.7.4-c Determination of Blondel's Leakage Coefficient

The Blondel leakage coefficient is obtained from:

$$\sigma = \frac{L_\sigma}{L_s} \quad (\text{II.109})$$

$$\sigma = \frac{3.08 \times 10^{-2}}{0.3438} = 8.9 \times 10^{-2} \quad (\text{II.110})$$

II.7.5 Slip Test

The slip test was carried out in order to determine the rotor time constant τ_r , which is a key parameter in the dynamic model of the induction motor. The machine was connected in star and supplied at rated voltage and frequency. Once steady-state operation was established, the stator supply was suddenly disconnected and the transient response of the stator voltage U_{sa} was recorded. The Figure below shows the damped sinusoidal waveform obtained after the interruption of the supply. The amplitude of the signal decreases exponentially with time, and the corresponding envelope is characterized by the rotor time constant τ_r . Two complementary methods were used to determine this parameter: a direct graphical reading and an analytical calculation based on two points selected on the exponential envelope.

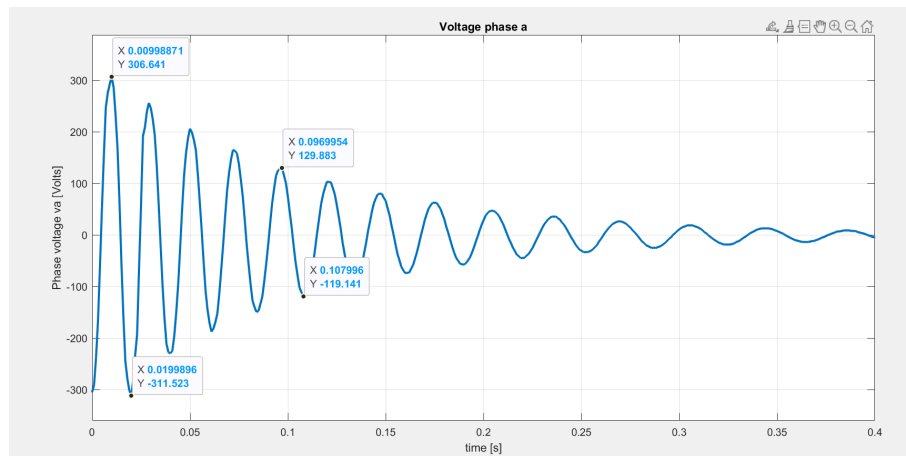


Figure II.14: Slip Test Graphical Reading

II.7.5-a Method 1: Direct Reading of the Rotor Time Constant τ_r

In this method, the rotor time constant is obtained by identifying two points on the positive or negative envelope such that the second amplitude is approximately equal to 36.8% of the first amplitude. The rotor time constant is then given by:

$$\tau_r = t_2 - t_1 \quad (\text{II.111})$$

For the positive envelope, the selected points were:

$$t_1 = 0.00998871s, V_1 = 306.641V \quad (\text{II.112})$$

$$t_2 = 0.0969954s, V_2 = 129.883V \quad (\text{II.113})$$

$$\frac{V_2}{V_1} = 42.4\% \quad (\text{II.114})$$

The corresponding rotor time constant is:

$$\tau_{r\text{positive}} = 0.0969954 - 0.00998871 = 0.087s \quad (\text{II.115})$$

For the negative envelope, the selected points were:

$$t_1 = 0.0199896s, V_1 = -311.523V \quad (\text{II.116})$$

$$t_2 = 0.107996s, V_2 = -119.141V \quad (\text{II.117})$$

The corresponding rotor time constant is:

$$\tau_{r\text{negative}} = 0.107996 - 0.0199896 = 0.088s \quad (\text{II.118})$$

Average value:

$$\tau_r = \frac{\tau_{r\text{negative}} + \tau_{r\text{positive}}}{2} \quad (\text{II.119})$$

$$\tau_r = \frac{0.088 + 0.087}{2} \quad (\text{II.120})$$

$$\tau_r = 0.0875s \quad (\text{II.121})$$

II.7.5-b Method 2: Calculation of the Rotor Time Constant τ_r

Using the analytical expression of the exponential decay:

$$v(t) = V_0 e^{-\frac{t}{\tau_r}} \quad (\text{II.122})$$

The rotor time constant is calculated from two selected points according to:

$$\tau_r = -\frac{t_1 - t_2}{\ln\left(\frac{V_1}{V_2}\right)} \quad (\text{II.123})$$

Applying this relation to the selected points yields:

Positive envelope:

$$\tau_r = -\frac{0.00998871 - 0.0969954}{\ln\left(\frac{306.641}{129.883}\right)} \quad (\text{II.124})$$

$$\tau_r = 0.101s \quad (\text{II.125})$$

Negative envelope:

$$\tau_r = - \frac{0.0199896 - 0,107996}{\ln \left(\frac{-311.523}{-119.141} \right)} \quad (\text{II.126})$$

$$\tau_r = 0.092s \quad (\text{II.127})$$

II.7.6 Deceleration Test (Routin Method)

The deceleration test was performed to identify the mechanical parameters of the induction motor, namely the total moment of inertia $\mathbf{J}$ and the viscous friction coefficient $\mathbf{B}$. The motor was first operated at no load under rated conditions. The stator supply was then disconnected, and the decrease in rotor speed was recorded as a function of time.

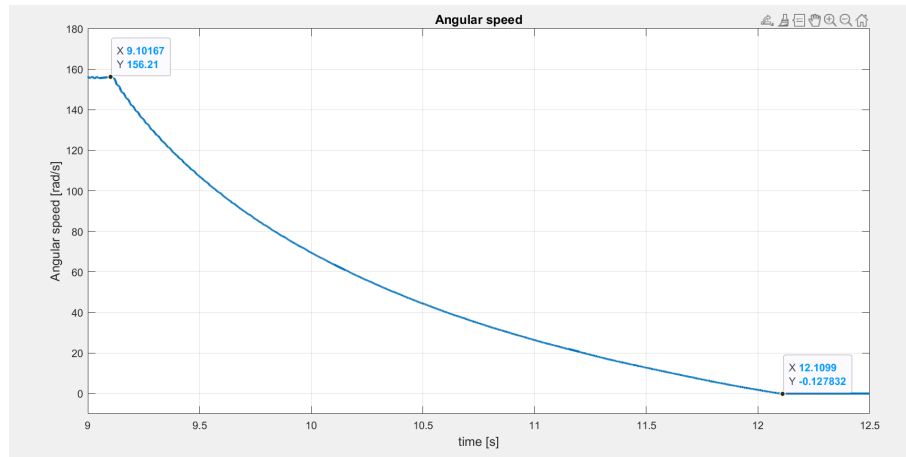


Figure II.15: Experimental Variation of Rotor Angular

The Figure above shows the experimental variation of the rotor angular speed during the slowing-down process. At the instant of supply interruption, the rotor angular speed was approximately:

$$\omega_{r0} = 156 \text{ rad/s} \quad (\text{II.128})$$

The mechanical losses previously obtained from the no-load test were:

$$P_{mec} = 71.54 \text{ W} \quad (\text{II.129})$$

The corresponding resistant torque is:

$$C_{r0} = \frac{P_{mec}}{\omega_{r0}} \quad (\text{II.130})$$

$$C_{r0} = \frac{71.54}{156} = 0.4586 \text{ N.m} \quad (\text{II.131})$$

The mechanical time constant was determined graphically by drawing the tangent to the deceleration curve at the initial operating point. The intersection of this tangent with the time axis yielded:

$$\tau_m = 1.10s \quad (\text{II.132})$$

II.7.6-a Determination of the Moment of Inertia

The total moment of inertia is calculated using:

$$J = \tau_m \frac{C_{r0}}{\omega_{r0}} \quad (\text{II.133})$$

Substituting the measured values gives:

$$J = 1.10 * \frac{0.4586}{156} \quad (\text{II.134})$$

$$J = 3.23 \times 10^{-3} \text{Kg.m}^2 \quad (\text{II.135})$$

II.7.6-b Determination of the Viscous Friction Coefficient

The viscous friction coefficient is determined from:

$$B = \frac{J}{\tau_m}$$

Substituting the previously calculated values:

$$B = \frac{3.23 \times 10^{-3}}{1.10} \quad (\text{II.136})$$

$$B = 2.94 \times 10^{-3} \text{N.m.s/rad} \quad (\text{II.137})$$

Validation of Identified Parameters

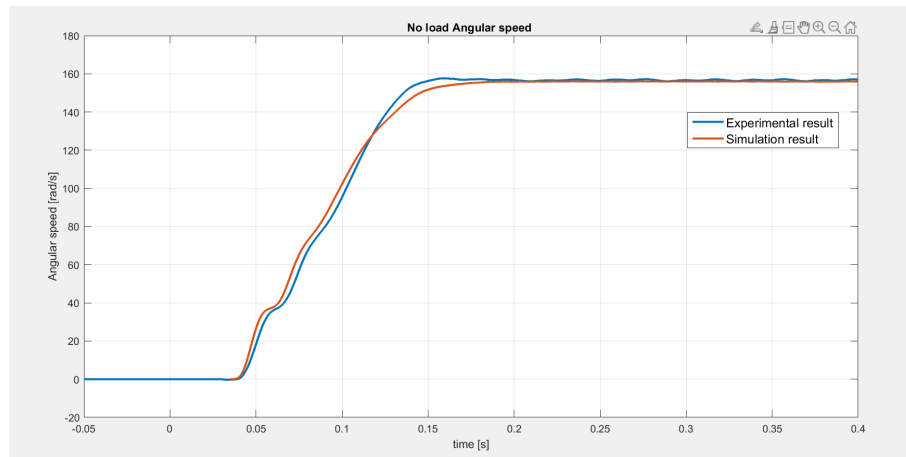


Figure II.16: Comparing the Angular Speed of Experimental and Simulation Result

The experimental validation was performed by comparing the angular speed obtained from the dSPACE platform with the corresponding Simulink simulation results. The comparison shows a very good agreement between the two responses during both the transient and steady-state operating regimes. The small discrepancies observed during the acceleration phase can be attributed to modeling simplifications, measurement noise, and parameter uncertainties. Nevertheless, the close overlap of the two curves confirms the validity of the identified parameters and demonstrates the accuracy of the developed induction machine model.

II.8 Conclusion

In this chapter, the identification of the induction machine parameters was carried out through a series of experimental tests. The no-load and blocked-rotor tests, together with the analysis of the machine responses, made it possible to determine the main electrical and mechanical parameters required for the development of an accurate machine model. The experimental procedures allowed the estimation of parameters such as the stator resistance, rotor resistance, leakage inductance, magnetizing inductance, and rotor time constant. These parameters constitute the basis for the mathematical representation of the machine and play a crucial role in the analysis of its dynamic behavior. The obtained results provide a satisfactory characterization of the induction machine and establish a reliable parameter set for simulation and control purposes. However, due to measurement uncertainties and modeling assumptions, the identified parameters may still be affected by estimation errors. Consequently, it becomes necessary to employ optimization techniques capable of refining the identified parameter values and improving the agreement between the mathematical model and the actual machine behavior. This aspect will be addressed in the following chapter through the application of the Particle Swarm Optimization (PSO) algorithm.

Chapter III

Identification of the Induction Motor by the Particle Swarm Metaheuristic Method

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III.1 Introduction

The induction motor is widely used in industry due to its reliability and low maintenance. High-performance control of these motors is typically achieved using vector control, which depends on accurate mathematical models and precise knowledge of internal parameters [10].

However, key motor parameters (like resistances and inductances) vary during operation due to thermal effects, magnetic saturation, and other factors. This variation causes detuning, leading to poorer performance such as slow response, oscillations, and reduced efficiency [10].

To address this, parameter identification methods are used. Traditional techniques (like MRAC or standard tests) have limitations, including sensitivity to errors or restrictive conditions [10].

Particle Swarm Optimization (PSO) provides a more advanced solution by treating parameter estimation as an optimization problem. Inspired by swarm behavior, PSO efficiently searches for the best parameter set, making it well-suited for handling the complex, nonlinear nature of induction motor models and improving overall drive performance [10].

III.2 Sophisticated Methods for Identifying Induction Motor Parameters

Identifying induction motor parameters (like stator/rotor resistance, leakage inductances, magnetizing inductance, inertia, etc.) can go far beyond simple tests (no-load, locked-rotor). Sophisticated methods usually mean model-based, optimization-based, or AI-driven approaches that estimate parameters from measured signals (current, voltages, speed).

III.2.1 Numerical and Statistical Approaches

III.2.1-a Least Square (LS) Method

Principle: LS is a statistical approach to data processing that aims to find the best-fitting parameter set for a machine model. It achieves this by minimizing the sum of the squares of the difference between the measured data and the model's predictions [10].

Application: It is primarily used as a numerical tool to process experimental data obtained from the motor to calculate specific parameters based on an adopted machine model, and offline identification off resistance and inductance [10].

Limits: Sources suggest that for certain identification tasks, this statistical approach may not be simple or accurate enough compared to direct experimental methods due to being sensitive to measurement noise [10].

III.2.1-b Recursive Least Squares (RLS) Method

RLS is an iterative version of the Least Square method . Instead of processing all data in a single batch, RLS processes data points sequentially as they become available, updating the parameter estimates at each step [10].

Application: In modern drive systems, RLS is often utilized for self-commissioning and identifying the magnetising curve at standstill. It is particularly useful when the system needs to "learn" parameters while the motor is not in motion [10].

Limits: The accuracy of the RLS method significantly deteriorates below a certain magnetising current value . Because of this sensitivity to current levels [10].

III.2.1-c Extended Kalman Filter (EKF)

The Extended Kalman Filter (EKF) is a sophisticated tool from modern control theory used for identifying asynchronous motor parameters.

Principle: It functions as an optimal observer for non-linear processes, providing state and parameter estimation [10].

Application: It is primarily utilized for on-line identification of rotor resistance (R_r), which helps increase the overall accuracy and robustness of vector-controlled drives [10].

Limits: The method is computationally intensive and the compensation can be slow, which often limits its successful application to steady-state operation. Additionally, its accuracy is sensitive to the precision of other parameters in the machine model [10].

III.2.2 Methods Based on Artificial Intelligence Algorithms

III.2.2-a Fuzzy Logic (FL)

Principle: Fuzzy Logic theory is used to handle the inherent non-linearity and complexity of induction machine structures by providing a framework that can process imprecise information. Use of linguistic rules to approximate uncertain behaviors [10].

Applications: It is used to vary the gains of standard PI controllers to improve dynamic performance. It is applied for the optimal efficiency control of machine flux [10].

Limits: It has low noise susceptibility and provide fast response times without overshoot [10].

III.2.2-b Artificial Neural Networks (ANN)

Principles: ANNs are extremely suitable for modeling and controlling highly non-linear processes like electromechanical energy conversion. They attempt to biologically emulate the adaptive evolutionary nature of living beings, such as reasoning, decision-making, and

learning. In the context of parameter identification, ANNs function by learning the relationship between a system's electrical signals and parameters to minimize an error function [10] [2].

Applications: ANNs are explicitly used for identifying motor parameters, including the on-line identification of rotor resistance (R_r). They are applied to feedback signal processing and fault diagnosis in electric drives [10].

Limits: The calculation required for these algorithms is described as "huge". For non-linear parameter modeling, the network must learn from extensive sets of training data [10] [2].

III.2.2-c Genetic Algorithms (GA)

Principle: This optimization technique is based on the principles of natural selection and genetics. It represents potential solutions as a population of individuals (chromosomes) composed of parameters (genes). The algorithm evolves these solutions over successive generations using operators such as selection (favoring fitter individuals), crossover (combining parent genes to create offspring), and mutation (randomly altering genes to maintain diversity). Each individual is evaluated using a fitness function that measures how well it minimizes the error in the machine model [2].

Applications: It is primarily used for off-line parameter identification, particularly for determining electromagnetic parameters (resistances and inductances) from startup transient data such as free acceleration tests. It is also effective for identifying parameters in nonlinear machine models [2].

Limits: The method is computationally expensive and requires significant CPU time compared to conventional techniques. Its performance is sensitive to the selection of optimization parameters, such as population size and mutation rate; for instance, a population that is too small can lead to premature convergence on sub-optimal solutions [2].

III.2.2-d Particle Swarm Optimization (PSO)

Principle: PSO is inspired by the social behavior of swarms, such as flocks of birds or schools of fish. It maintains a swarm of "particles" that move through a multi-dimensional search space representing potential solutions. Each particle updates its position and velocity based on its own personal best experience and the global best experience of the entire swarm [9].

Applications: It is utilized to estimate equivalent circuit parameters from manufacturer nameplate and performance data (such as torque-slip and current-slip characteristics) for steady-state analysis [9].

Limits: A common drawback is premature convergence to local minima rather than the global optimum [9].

III.2.2-e Bee Colony Optimization (BEE)

Principle: This algorithm mimics the foraging behavior of honey bees. The colony consists of three groups: employed bees (who exploit known food sources), onlooker bees (who select sources based on quality shared in the hive), and scout bees (who randomly search for new sources). Food source locations represent potential solutions, and the quality of the food corresponds to the solution's fitness [8].

Applications: It is particularly effective for estimating parameters in double-cage induction motors by minimizing the differences between model outputs (torque, current, power factor) and manufacturer-provided data [8].

Limits: Like other meta-heuristic methods, it can suffer from slow convergence or require a large number of tuning parameters in certain cases [8].

III.2.2-f Ant Colony Optimization (ANT)

Principle: ACO uses a swarm of "ants" to search for motor parameters that minimize the error between measured and modeled behavior. Better solutions attract more search via "pheromones," leading to convergence [1].

Advantages: Finds global (near-optimal) solutions without a need for derivatives and works well with nonlinear models and noisy data [1].

Limitations: Computationally slow and needs careful parameter tuning. It is also less efficient for continuous problems with risk of premature convergence [1].

III.2.2-g Comparative Table of Different Methods

Table III.1: Comparative Table

Method	Principle	Application	Limits
Least Squares (LS)	Minimizes error between data and model	Offline parameter identification (R, L)	Sensitive to noise, less accurate
Recursive LS (RLS)	Updates parameters step by step	Self-commissioning, standstill learning	Poor accuracy at low current
Extended Kalman Filter (EKF)	Observer for non-linear systems	Online rotor resistance estimation	High computation, slow response
Fuzzy Logic (FL)	Uses rules for uncertain systems	Control improvement, efficiency optimization	Implementation details unclear
Neural Networks (ANN)	Learns nonlinear relationships	Parameter ID, fault diagnosis	Needs large data, heavy computation
Genetic Algorithm (GA)	Evolution-based optimization	Offline parameter estimation	High CPU time, parameter sensitive
Particle Swarm (PSO)	Swarm-based search method	Steady-state parameter estimation	May converge to local minima
Bee Colony (BEE)	Mimics bee foraging behavior	Double-cage motor parameters	Slow convergence, tuning needed
Ant Colony (ANT)	Uses pheromone-based search	Nonlinear parameter estimation	Slow, risk of premature convergence

III.3 Particle Swarm Optimization (PSO)

III.3.1 Definition of an Optimization Problem

An optimization problem is a mathematical formulation used to determine the best possible solution for a specific set of variables by minimizing or maximizing an objective function while satisfying a given set of conditions.

In the context of asynchronous motor parameter identification, an optimization problem is defined as a mathematical formulation used to determine the best-fitting set of machine parameters such as resistances and inductances by minimizing the error between a computed model's performance and reference data.

A standard optimization problem for motor parameter determination consists of the following core elements:

- **The Objective Function:** This is the primary mathematical expression designed to be minimized,
- **Decision Variables:** These represent the unknown motor parameters being sought, typically including stator and rotor resistances (R_1, R_2) and various inductances (L_1, L_2, L_m),
- **Constraints:** These are specific conditions or limits that the potential solution must satisfy to be physically and operationally valid.

III.3.2 Heuristic and Meta heuristic Methods

Some optimization problems are considered difficult requiring the use of sophisticated algorithms approximate methods called: Heuristics and Metaheuristics.

Heuristic and Meta-heuristic methods are advanced optimization strategies used to find "global solutions" for complex, non-linear problems(physical, biological, or socio-psychological phenomena) where traditional mathematical methods might fail.

III.3.2-a Heuristic Methods

A heuristic is a practical problem-solving approach designed to find a sufficiently good solution in a reasonable timeframe, rather than an absolute mathematical proof of optimality.

Principle: These methods use specific knowledge or intuitive strategies related to the problem.

III.3.2-b Meta-heuristic Methods

A meta-heuristic is a higher-level framework, they are typically inspired by natural or biological systems, such as evolution or foraging.

Common meta-heuristics include:

- Genetic Algorithms (GA): Based on natural selection,
- Ant Colony Optimization (ACO): Based on the foraging behavior of real ants,
- Particle Swarm Optimization (PSO): Mimics the movement of bird flocks or fish schools.

III.3.3 Principle of Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a stochastic optimization technique based on swarm, which was proposed by James Kennedy and Russell C. Eberhart in 1995. It is a population-based meta-heuristic optimization algorithm inspired by the social foraging behavior of swarms, such as bird flocks or schools of fish. It is designed to navigate a multi-dimensional search space to find a "global solution" for complex, non-linear problems like motor parameter identification [9].

The principle of PSO is defined by the following key components:

- **Swarm of Particles:** The algorithm maintains a "swarm" of software agents called particles. Each particle represents a potential solution to the problem,
- **The Search Space:** These particles move through a dimensional space, where unknown decision variables are being sought,
- **Particle Attributes:** Each individual particle is characterized by two main vectors:
Position (X): Its current coordinates in the search space,
Velocity (V): The direction and rate of change of its position.
- **Fitness Evaluation:** Every particle has a fitness value calculated via an objective function. This value indicates how "fit" or accurate that specific solution is compared to the desired performance data.

Unlike simple random searches, PSO utilizes a memory-based mechanism to guide the swarm toward the best solution. It tracks two critical pieces of information:

- **Personal Best (pbest):** Each particle remembers the best position it has personally visited so far,
- **Global Best (gbest):** The swarm as a whole tracks the best position ever found by any individual in the entire population.

The trajectory of each particle is updated iteratively. At each step, a particle adjusts its velocity and position based on three factors:

- **A physical component (Inertia):** Its previous velocity, which keeps it moving in its current direction,
- **Cognitive Component:** Its attraction to its own personal best experience,
- **Social Component:** Its attraction to the global best experience of the entire swarm.

By balancing these components, the particles "fly" through the search space, gradually clustering around the region with the highest fitness (the global optimum).

III.3.3-a General Formulation

In a dimensional space, where n is the number of unknown decision variables, each particle i in the swarm is characterized by the following vectors [9]: " . " . "

- Position(x_i): Represented as $x_i = [x_{i1}, x_{i2}, \dots, x_{in}]$, which signifies the particle's current location in the search space,
- Velocity(v_i): Represented as $v_i = [v_{i1}, v_{i2}, \dots, v_{in}]$, which signifies the direction and speed of the particle's movement,
- Fitness($f(x)$): A value calculated from an objective function that indicates the quality of the solution at the current position,
- Acceleration Factors (C_1, C_2): C_1 is the cognitive learning factor and C_2 is the social learning factor,
- r_1, r_2 : Random numbers uniformly distributed between 0 and 1.

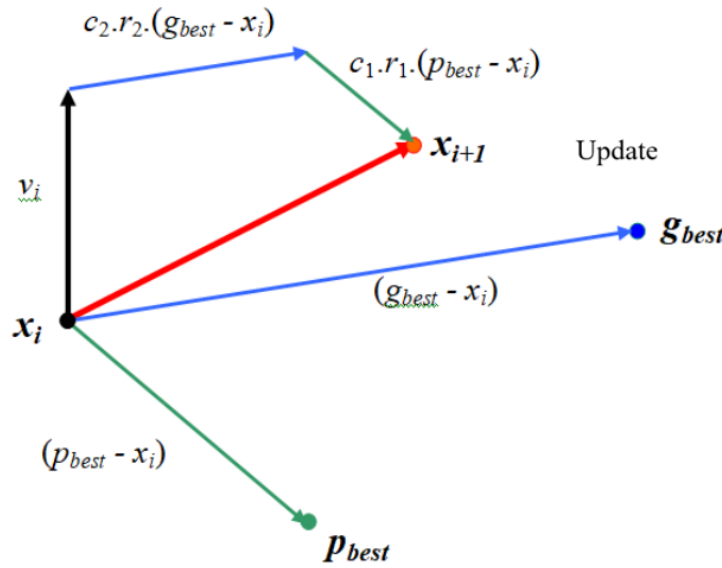


Figure III.1: General Formulation

Velocity Update Formula:

$$v_{ij}(k+1) = v_{ij}(k) + C_1 \cdot r_1 \cdot [pbest_{ij}(k) - x_{ij}(k)] + C_2 \cdot r_2 \cdot [gbest_j(k) - x_{ij}(k)] \quad (III.1)$$

In this formula the first part ($v_{ij}(k)$) represents inertia, the second part ($C_1 \cdot r_1 \cdot [pbest_{ij}(k) - x_{ij}(k)]$) represents the cognitive component, and the third part ($C_2 \cdot r_2 \cdot [gbest_j(k) - x_{ij}(k)]$) represents the social component.

Position Update Formula:

$$x_{ij}(k+1) = x_{ij}(k) + v_{ij}(k+1) \quad (\text{III.2})$$

The new position is simply the sum of the current position and the newly calculated velocity.

III.3.3-b Standard PSO Algorithm

The algorithm begins by initializing a population (swarm) of n particles. Each individual is randomly assigned an initial position vector (x_i) and velocity vector (v_i) within the multi-dimensional search space [9].

Each particle's current position uses an objective function ($f(x)$) to determine its "fitness," this shows how close it is to the optimal solution for the specific problem [9].

The algorithm compares the current fitness of each particle to the best fitness it has personally achieved in previous iterations. If the current position is better, it updates the individual's optimal position (pbest) [9].

The swarm identifies the best position achieved by any particle in the entire population. This global optimal position (gbest) is updated if a superior solution is found in the current generation [9].

New velocities are calculated for each dimension based on the particle's previous velocity, its distance from its own pbest, and its distance from the swarm's gbest [9].

$$v_{ij}(k+1) = v_{ij}(k) + C_1 \cdot r_1 \cdot [pbest_{ij}(k) - x_{ij}(k)] + C_2 \cdot r_2 \cdot [gbest_j(k) - x_{ij}(k)] \quad (\text{III.3})$$

The particle's new position is computed by adding the updated velocity vector to its current position vector.

$$x_{ij}(k+1) = x_{ij}(k) + v_{ij}(k+1) \quad (\text{III.4})$$

The algorithm checks if the ending criteria have been met such as reaching a predefined optimal state, achieving balance, or exceeding the calculation limits [9].

- If the condition is not satisfied: The process loops back to be evaluated using the objective function ($f(x)$) for a new generation,
- If the condition is satisfied: The search ends, and the final gbest is provided as the solution.

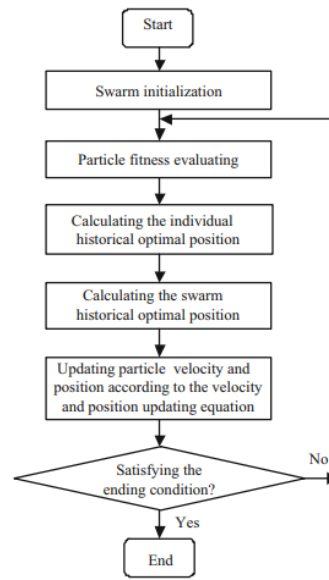


Figure III.2: PSO Algorithm

III.3.4 Topology Models

In Particle Swarm Optimization (PSO), the topology structure defines how particles interact and share information with one another, directly influencing the swarm's diversity and performance. There are two primary ways to determine these neighborhoods: social (index-based) and geographic (distance-based) [9].

III.3.4-a Social Neighborhoods (Index-Based)

In this model, the relationship between particles has nothing to do with their physical or topological distance in the search space. Instead, a particle is connected to other particles based on their numerical identifier (e.g., Particle 1 is a neighbor of Particle 2 and Particle 10). These connections are typically static, meaning the structure remains fixed throughout the optimization process [9].

Common social topologies include the star, ring, and von Neumann structures. For instance, in a ring topology, each particle only communicates with its immediate numerical neighbors, which slows down the spread of information and can help prevent premature convergence on complex problems [9].

III.3.4-b Geographic Neighborhoods (Distance-Based)

Geographic (or spatial) neighborhoods are determined according to the topological distance between particles in the search space. A particle's neighbors are those that are physically closest to it at any given iteration. This mimics "spatial clustering," where individuals only learn from those in their immediate vicinity [9].

Unlike index-based models, geographic neighborhoods are often dynamic. As particles move through the search space to find optimal solutions, their "nearest neighbors" change, requiring the topology to be recalculated periodically [9].

One method mentioned involves selecting the nearest m particles to be the neighbors for a specific individual in each generation. Another advanced approach uses K-means clustering to define spatial clusters where particles update themselves based on the common experience of their current cluster rather than the entire swarm [9].

III.3.5 Parameter Configuration

The configuration of parameters in Particle Swarm Optimization (PSO) is critical because it directly influences the algorithm's convergence speed, its ability to find a global optimum, and its overall robustness. The primary parameters requiring configuration include the number of particles, the confidence coefficients (acceleration factors), and the inertia coefficient [9].

III.3.5-a Number of Particles

The number of particles, or population size, represents the number of potential solutions being evaluated simultaneously in the search space. Selection of the population size is often related to the complexity of the problem, though the algorithm is not always highly sensitive to this value. A common range for most optimization tasks is 10 to 50 particles. In specific studies for induction motor parameter determination, a population size of 20 particles has been proven effective for identifying equivalent circuit parameters and while larger populations can explore more of the search space at once, they also increase the computational burden of each iteration [9].

III.3.5-b Confidence and Inertia Coefficients

Confidence Coefficients (Acceleration Factors C_1 and C_2) These coefficients, also known as acceleration factors, determine the influence of a particle's own experience and the experience of the swarm on its next movement [9].

- Cognitive Learning Factor (C_1): This represents "individual thinking". It pulls the particle toward its personal best (pbest) position, signifying the particle's confidence in its own previous success.
- Social Learning Factor (C_2): This represents "information sharing" or cooperation. It pulls the particle toward the global best (gbest) position found by any member of the swarm.

Historically, both are often set to 2.0 to allow the search to cover the regions between the particle and its targets. In motor identification experiments, values of 1.5 for both factors are frequently utilized [9]. **Inertia Coefficient (Inertia Weight ω)** The inertia coefficient or weight regulates the trade-off between global exploration and local exploitation. It controls the impact of a particle's previous velocity on its current velocity. It signifies the particle's "confidence" in its current moving state; it conducts inertial movement based on its own existing velocity [9].

- If the speed is too low ω the algorithm is too slow,
- If the speed is too high ω the algorithm is too unstable.

Maximum Velocity Constraint The maximum velocity constraint (V_{max}) is a control parameter in Particle Swarm Optimization (PSO) used to limit the speed of particles as they move through the search space. Its primary purpose is to prevent particle velocities from rapidly increasing to extremely high values, which can negatively affect the algorithm's performance.

It regulates the global search ability of the swarm, ensuring that particles do not "fly" past optimal regions too quickly. It helps manage the trade-off between exploring new areas of the search space and exploiting known good solutions [9].

III.3.6 Variants of the PSO Algorithm

III.3.6-a PSO Algorithm with Inertia Coefficient

This variant, often referred to as the Inertia Weights Approach (IWA) was introduced by Shi and Eberhart in 1998. It introduces an inertia weight (ω) into the velocity update formula to regulate the impact of a particle's previous velocity on its current movement. The velocity update equation becomes:

$$v_{ij}(k+1) = \omega \cdot v_{ij}(k) + C_1 \cdot r_1 \cdot [pbest_{ij}(k) - x_{ij}(k)] + C_2 \cdot r_2 \cdot [gbest_j(k) - x_{ij}(k)] \quad (\text{III.5})$$

The inertia weight balances global exploration and local exploitation [9].

- Large ω : Facilitates global exploration, allowing particles to search wide areas of the solution space,
- Small ω : Tends to facilitate local exploitation, helping the swarm to fine-tune and settle on a specific solution.

In many applications, such as motor parameter determination, ω is a time-varying function that decreases linearly from a maximum (ω_{max}) to a minimum (ω_{min}) over the course of the iterations. Shi and Eberhart originally suggested a range of [0.9, 1.2] for the weight [9].

$$\omega = \omega_{max} - (\omega_{max} - \omega_{min}) \cdot \left(\frac{iter}{iter_{max}} \right) \quad (\text{III.6})$$

III.3.6-b PSO Algorithm with Constriction Factor

Introduced by Clerc and Kennedy in 2002, this variant utilizes a constriction factor (χ) to control the swarm's behavior. The constriction factor was developed through a formal analysis of the algorithm's stability to ensure convergence and improve the overall convergence rate. The velocity update equation is modified as follows:

$$v_{ij}(k+1) = \chi \cdot [v_{ij}(k) + \phi_1 \cdot r_1 \cdot [pbest_{ij}(k) - x_{ij}(k)] + \phi_2 \cdot r_2 \cdot [gbest_j(k) - x_{ij}(k)]] \quad (\text{III.7})$$

$$\chi = \frac{2}{\left| \phi - 2 + \sqrt{\phi^2 - 4 \cdot \phi} \right|} \quad (\text{III.8})$$

$$\phi = \phi_1 + \phi_2, \phi > 4 \quad (\text{III.9})$$

One unique advantage of the constriction factor is that it can realize the purpose of limiting particle velocity without the absolute necessity of a fixed maximum velocity constraint (V_{max}), although experiments show that using both together can still provide better results [9].

III.3.7 Advantages and Disadvantages of the PSO Algorithm

Advantages of PSO

- Simple and easy to implement through programming,
- It is often more computationally efficient and demonstrates a fast convergence rate, allowing it to reach optimal values quickly,
- It can be adapted to various application environments with minimal modification.

Disadvantages of PSO

- It can converge prematurely,
- PSO does not sufficiently utilize all information gathered during the calculation,
- The performance of traditional PSO can deteriorate quickly as the dimensionality of the search space increases exponentially,
- PSO has difficulty dealing with discrete or binary variables.

III.3.8 Example of Optimization Using the PSO Algorithm

This section aims to apply the PSO method to solve an optimization problem, by varying the algorithm parameters (number of particles n and acceleration parameters (C_1 , C_2)) to observe its behavior and analyze the impact of the different parameters on the search. We consider the optimization problem: $\min \{f(x_1, x_2, x_3)\}$ which consists of minimizing the function defined by the expression:

$$f(x_1, x_2, x_3) = 10 \cdot (1 - x_1)^2 + 20 \cdot (2 - x_2)^2 + 30 \cdot (3 - x_3)^2 \quad (\text{III.10})$$

with the following constraints:

$$x_1 + x_2 + x_3 \leq 5 \text{ and } x_1^2 + 2 \cdot x_2 \leq x_3 \quad (\text{III.11})$$

As an indication, this function has a global optimum defined at the point:
 $x_1 = 0.4377$; $x_2 = 1.4569$; $x_3 = 3.1054$
 $f(x_1, x_2, x_3) = 9.3941$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      DEFINITION OF THE FITNESS FUNCTION
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function f=ofun(x)
%FUNCTIONAL OBJECTIVE (FITNESS) TO MINIMIZE
of=10*(x(1)-1)^2+20*(x(2)-2)^2+30*(x(3)-3)^2;
```

This is the function the PSO wants to minimize.
Minimum occurs at:

$$x_1 = 1; \quad x_2 = 2; \quad x_3 = 3$$

where: $f(x) = 0$

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% DEFINITION OF CONSTRAINTS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% All constraints must be converted to the form (c0 <= 0)
% If there are no constraints, then comment out (Start with %)
% the 3 lines 'c0' below

c0=[];
c0(1)=x(1)+x(2)+x(3)-5;          % Constraint 1
c0(2)=x(1)^2+2*x(2)-x(3);       % Constraint 2

% Defining penalties for each constraint

for i=1:length(c0)
    if c0(i)>0
        c(i)=1;
    else
        c(i)=0;
    end
end
%
penalty=10000;                    % Penalty for each violation of constraint
f=of+penalty*sum(c);              % Fitness function
end

```

If constraint is violated, the penalty activated.
The final fitness is then given as $f = objective + penalty$. So infeasible solutions become very large.

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% SEARCH FOR THE GLOBAL MINIMUM OF THE FUNCTION
% f(x1,x2,x3)=10*(x1-1)^2+20*(x2-2)^2+30*(x3-3)^2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

tic
clc; clear all; close all

rng default
LB=[0 0 0];                      % Lower Bounds of the variables
UB=[10 10 10];                   % Upper Bounds of the variables

```

This gives the lower and upper bounds of variables.

$$0 \leq x_1, x_2, x_3 \leq 10 \quad (\text{III.12})$$

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               PSO PARAMETER VALUES
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

m=3;                               % Number of variables
n=100;                             % Population size
wmax=0.9;                          % Max value of the inertia coefficient
wmin=0.4;                          % Min value of the inertia coefficient
c1=2;                              % Personal acceleration factor
c2=2;                              % Global acceleration factor

```

This gives the PSO parameters

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               START MAIN PSO PROGRAM
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

maxite=1000;                       % Set maximum number of iterations
maxrun=10;                         % Set maximum number of trials

for run=1:maxrun
    run
    % Start PSO initialisation -----
    for i=1:n
        for j=1:m
            x0(i,j)=round(LB(j)+rand()*(UB(j)-LB(j)));
        end
    end
    x=x0;                          % Initial Population
    v=0.1*x0;                      % Initial Vitesse
    for i=1:n
        f0(i,1)=ofun(x0(i,:));
    end
    [fmin0,index0]=min(f0);
    pbest=x0;                      % Initial valeur of pbest
    gbest=x0(index0,:);            % Initial valeur of gbest

    % End PSO initialisation -----
end

```

This code runs the PSO multiple times, randomly initializes particles within bounds, assigns initial velocities, evaluates fitness, and stores initial personal and global best.

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               START PSO ALGORITHM
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

ite=1;
tolerance=1;
while ite<=maxite && tolerance>10^-12

    w=wmax-(wmax-wmin)*ite/maxite;    % inertia coefficient update

    % Velocity update
    for i=1:n
        for j=1:m
            v(i,j)=w*v(i,j)+c1*rand()*(pbest(i,j)-x(i,j))+...
                +c2*rand()*(gbest(1,j)-x(i,j));
        end
    end

    % position update
    for i=1:n
        for j=1:m
            x(i,j)=x(i,j)+v(i,j);
        end
    end

    ite=ite+1;
    tolerance=tolerance/10;
end

```

This code initializes iteration counter and stopping criterion(until max iteration or convergence is achieved), linearly decreases inertia weight(improves convergence), and implements velocity and position update formula.

```
% Managing boundary (limit) violations of variables
for i=1:n
    for j=1:m
        if x(i,j)<LB(j)
            x(i,j)=LB(j);
        elseif x(i,j)>UB(j)
            x(i,j)=UB(j);
        end
    end
end

% Fitness function assessment
for i=1:n
    f(i,1)=ofun(x(i,:));
end
```

This code keeps particles inside search limits and computes fitness at new positions.

```
% Updated pbest values and fitness function
for i=1:n
    if f(i,1)<f0(i,1)
        pbest(i,:)=x(i,:);
        f0(i,1)=f(i,1);
    end
end
[fmin,index]=min(f0);           % Search for the best particle
ffmin(ite,run)=fmin;           % Store the best fitness
ffite(run)=ite;                % Store the number of iterations

% Updated gBest values and best fitness function
if fmin<fmin0
    gbest=pbest(index,:);
    fmin0=fmin;
end
```

If current position better than previous best this code updates its personal best. This code also finds the best particle among all and stores it and if a better solution is found it updates it as the global best.

```
% Tolerance calculation
if ite>100;
    tolerance=abs(ffmin(ite-100,run)-fmin0);
end

% Displaying the results of the iterations
if ite==1
    fprintf('Iteration Best particle Objective fun\n');
end
fprintf('%8g %8g %8.4f\n',ite,index,fmin0);
ite=ite+1;
end

%%%%%%%%%%%%%%
% END OF PSO ALGORITHM
%%%%%%%%%%%%%%
```

This code checks improvement over last 100 iterations and displays iteration number, best particle index and best fitness value.

```
gbest;
fvalue=10*(gbest(1)-1)^2+20*(gbest(2)-2)^2+30*(gbest(3)-3)^2;
fff(run)=fvalue;
rgbest(run,:)=gbest;
fprintf('-----\n');
end

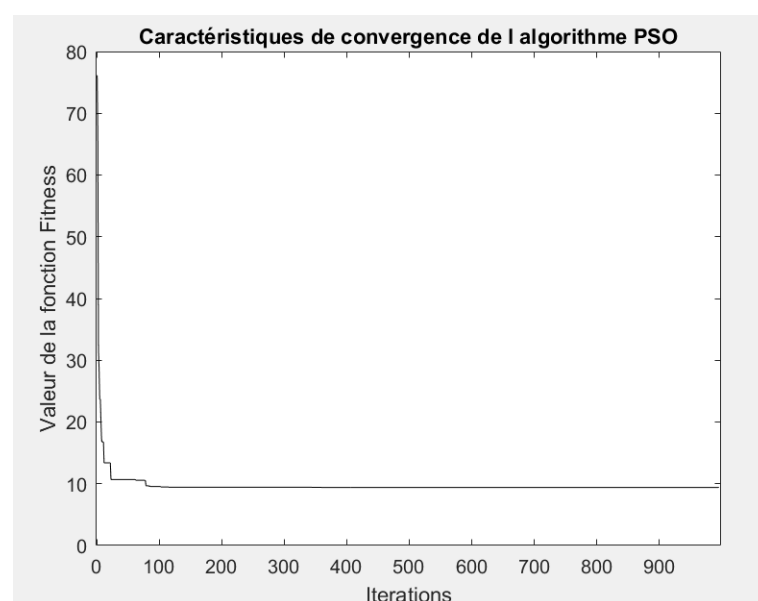
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               END OF MAIN PSO PROGRAM
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

This code computes objective function without penalties then stores the final solution and the best variables.

```
% Displaying results
disp(newline);
fprintf('*****\n');
fprintf('RESULTAT FINAL-----\n');
[bestfun,bestrun]=min(fff)
best_variables=rgbest(bestrn,:)
fprintf('*****\n');
toc

% Convergence characteristics
plot(ffmin(1:ffite(bestrn),bestrun),'-k');
xlabel('Iterations');
ylabel('Valeur de la fonction Fitness');
title('Caractéristiques de convergence de l algorithme PSO')
```

The figure below displays the optimal variables



The convergence curve demonstrates that the PSO algorithm rapidly decreases the fitness function during the first iterations, indicating efficient exploration of the search space. After approximately 100 iterations, the fitness stabilizes around its minimum value, showing convergence toward an optimal feasible solution.

```

RESULTAT FINAL-----
|
bestfun =

    9.3941

bestrun =

    7

best_variables =

    0.4377    1.4569    3.1054

*****
Elapsed time is 7.372866 seconds.

```

III.4 Identification of Asynchronous Motor Parameters by Particle Swarms

III.4.1 Principle of Identification using the PSO Algorithm

The principle of parameter identification using Particle Swarm Optimization (PSO) is based on formulating the determination of motor parameters as an optimization problem designed to minimize the error between a computed mathematical model and reference data.

III.4.2 Parameters to be Identified

The identification process typically seeks to determine the electrical equivalent circuit parameters of the asynchronous motor to ensure the machine model accurately represents electrical and electromagnetic interactions.

The primary parameters identified include:

- **Stator Resistance (R_1 or R_s):** The electrical resistance of the stator windings,
- **Rotor Resistance (R_2 or R_r):** The resistance of the rotor windings,
- **Stator Leakage Reactance/Inductance (L_1 or L_s):** The reactance associated with the stator's leakage flux,
- **Rotor Leakage Reactance/Inductance (L_2 or L_r):** The reactance associated with the rotor's leakage flux,

- **Magnetizing Reactance/Inductance (L_m):** The reactance representing the main flux path in the motor.

For double-cage induction motors, the identification task expands to include specific parameters for both cages, such as R_1d , L_1d , R_2d and L_2d .

In some sophisticated identification schemes, additional mechanical parameters are also sought to better predict system behavior, such as:

- **Total Inertia (J):** The polar inertia of the motor and its load,
- **Viscous Friction/Damping (β_v):** The mechanical losses due to friction,
- **Rotor Time Constant (Tr):** Often derived from the identified L_r and R_r , this parameter is critical for vector control accuracy.

III.5 Simulation Results

A Particle Swarm Optimization (PSO) algorithm was implemented to estimate induction motor parameters (L_s , L_r , L_m).

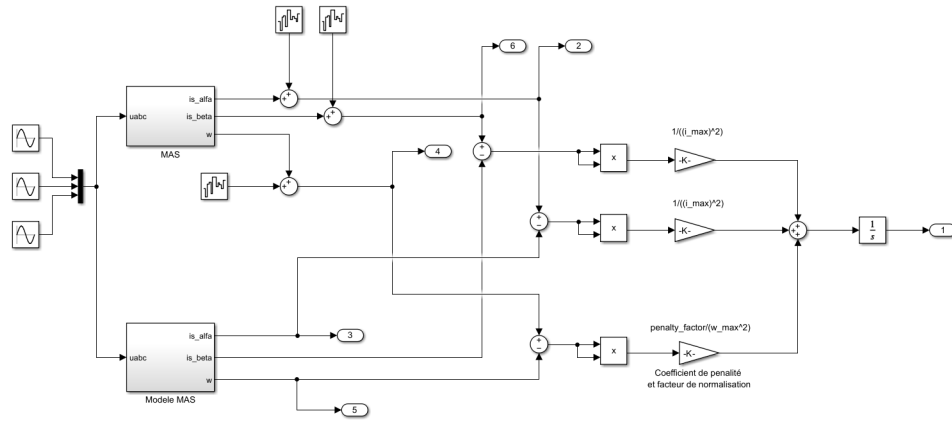


Figure III.3: PSO Induction Motor Simulation

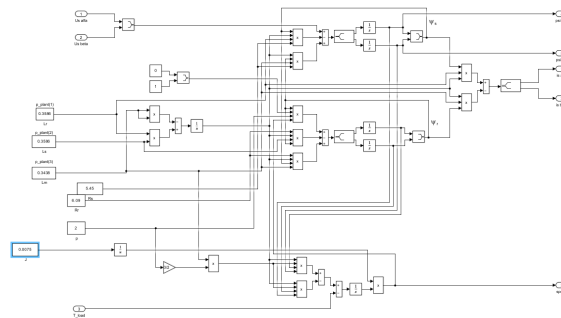


Figure III.4: Induction Motor (Real Model)

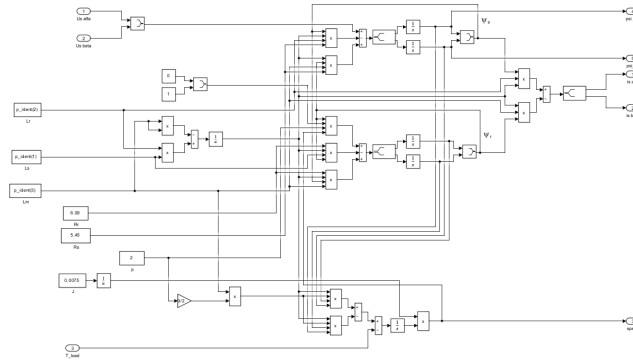


Figure III.5: Induction Motor to be Estimated using PSO

The simulation was carried out with a swarm size of **40 particles**, with **180 iterations**. The objective of the PSO algorithm is to minimize the error between the outputs of the real induction motor model and those of the estimated model.

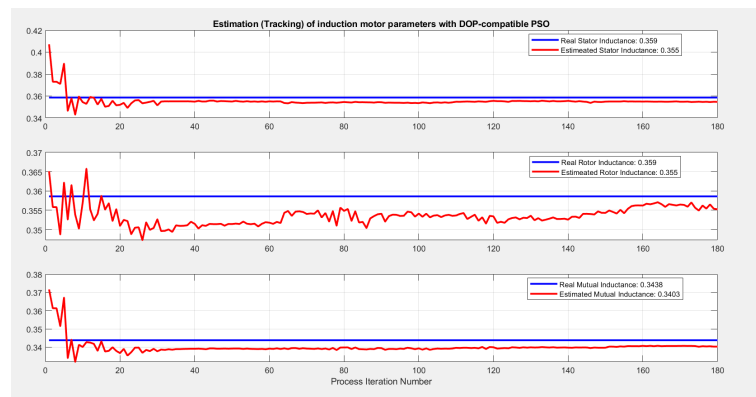


Figure III.6: Estimated Parameters

The figure above shows the evolution of the estimated parameters during the PSO optimization process. Initially, the estimates exhibit oscillations due to the random initialization of the particles and the exploration phase of the algorithm. As the iterations progress, the swarm converges toward the optimal solution, reducing these oscillations. After approximately 3050 iterations, the estimated parameters stabilize and remain close to the actual values, demonstrating the effectiveness of PSO in achieving accurate parameter estimation.

The final estimated values obtained after 180 iterations are:

Table III.2: Comparative Table

Parameter	Real Value	Estimated Value	Absolute Error
Stator Inductance	0.3586 H	0.3550 H	0.0036 H
Rotor Inductance	0.3586 H	0.3550 H	0.0036 H
Mutual Inductance	0.3438 H	0.3403 H	0.0035 H

The relative errors are approximately:

$$\varepsilon_{Ls} = \frac{|0.3586 - 0.3550|}{0.3586} * 100 \approx 1.0\% \quad (\text{III.13})$$

$$\varepsilon_{Lr} = \frac{|0.3586 - 0.3550|}{0.3586} * 100 \approx 1.0\% \quad (\text{III.14})$$

$$\varepsilon_{Lm} = \frac{|0.3438 - 0.3403|}{0.3438} * 100 \approx 1.0\% \quad (\text{III.15})$$

Therefore, all estimated parameters were identified with an error close to 1%, indicating a high estimation accuracy.

III.6 Conclusion

This chapter focused on identifying induction motor parameters using optimization techniques, with emphasis on Particle Swarm Optimization (PSO). After reviewing classical and intelligent methods, PSO was introduced as a simple and effective metaheuristic for nonlinear parameter estimation.

The PSO principle, including particle updates and key parameters influencing convergence, was presented. The method was then applied by formulating an objective function that minimizes the error between the real and estimated models.

Simulation results showed that PSO achieves accurate parameter estimation with good convergence speed and robustness compared to classical approaches.

In conclusion, the PSO-based method proved to be an efficient tool for induction motor parameter identification, improving modeling accuracy for advanced control applications.

Chapter IV

Observer Synthesis for the Induction Motor

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IV.1 Introduction

Effective control of an induction machine requires accurate knowledge of its internal state variables. However, some important quantities, such as rotor fluxes and mechanical speed, are not directly measurable or require expensive and fragile sensors. To overcome this limitation, two approaches can be used:

An estimator, which is a replica of the system without feedback correction;

An observer, which is a replica of the system including a feedback term that continuously corrects the estimation error.

In industrial applications, observers are widely used to reconstruct non-measurable variables from easily accessible quantities, namely stator voltages and stator currents. They are particularly important in sensorless control systems, where they improve reliability, reduce cost, and simplify maintenance. This chapter presents the theory and implementation of several observer structures applied to the induction machine, including full-order observers, reduced-order observers, and high-gain observers. Their performance is evaluated through simulation and experimental results [5] [4].

IV.2 Observer Theory

IV.2.1 Principle of an Observer

The structure of an observer consists first of an estimator operating in open loop, also called a predictor, whose dynamics are identical to those of the real system. By introducing a correction term based on the difference between the measured outputs and the estimated outputs, a closed-loop structure is obtained. This feedback mechanism improves the convergence of the estimated states toward their true values [5] [4]. In general, the observer uses:

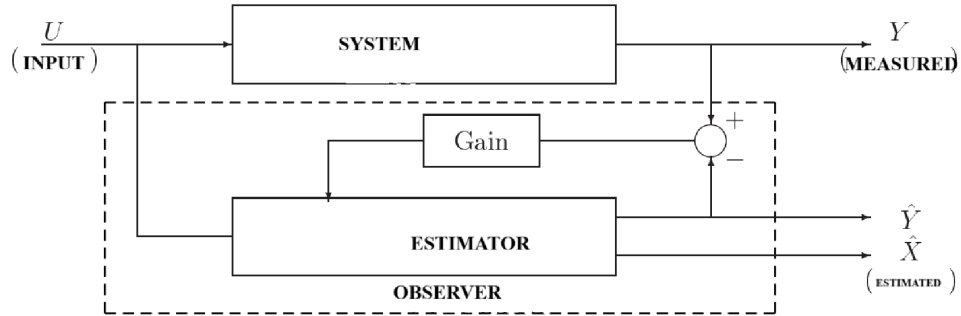
- an input vector U , common to both the real system and the observer;
- a state vector X , containing the variables to be estimated;
- an output vector Y , whose components are directly measurable;
- an estimated output vector $\hat{Y}$, generated by the observer;
- an estimated state vector $\hat{X}$.

The observation error is defined as:

$$e_y = Y - \hat{Y} \quad (\text{IV.1})$$

This error is multiplied by a gain matrix L and injected into the observer dynamics to correct the estimated states. By appropriately selecting the gain matrix, the convergence speed and robustness of the observer can be adjusted so that the estimation error tends toward zero. Before designing an observer, it is necessary to verify the observability of the system and to choose a suitable reference frame. It is also important to evaluate

the robustness of the observer with respect to parameter variations and measurement uncertainties [5] [4].



IV.2.2 Observer Structure

The architecture of an observer is generally composed of a copy of the plant dynamics supplemented by a corrective feedback term. Based on the provided documentation, the most common mathematical structure is expressed as:

$$\begin{cases} \dot{\hat{x}} = f(\hat{x}, u) - G(g)(\hat{y} - y) \\ \dot{\hat{g}} = \varphi(\hat{x}, u, y, g) \end{cases} \quad (\text{IV.2})$$

Where:

- $f(\hat{x}, u)$ represents the predictor or the open-loop estimator.
- $G(g)(\hat{y} - y)$ is the correction term, where G is the observer gain.
- $(\hat{y} - y)$ is the innovation, which drives the estimation error toward zero.

IV.2.3 Observer Implementation

The implementation of an observer for the induction machine involves several essential steps. First, it is necessary to verify the observability of the system, which ensures that the internal state variables can be uniquely reconstructed from the measured outputs. This preliminary analysis is fundamental before any observer design. The second step consists in selecting an appropriate mathematical model and reference frame. In the case of the induction machine, the stationary ($\alpha\beta$) reference frame or the synchronous rotating (dq) reference frame is generally adopted, depending on the intended application and the desired simplifications. The synthesis of the observer gain matrix (L) is then carried out in order to impose the desired convergence dynamics of the estimation error. Several design methods may be used, the most common being pole placement, in which the observer poles are chosen significantly faster than the natural poles of the machine, and the Lyapunov method, which guarantees the asymptotic stability of the estimation error [5] [4].

Finally, particular attention must be paid to the robustness of the observer with respect to parameter variations. Since certain machine parameters, such as the rotor resistance (R_r), may vary with operating conditions and temperature, the observer must maintain satisfactory convergence and accuracy despite these uncertainties [5] [4].

IV.3 Different Types of Observers

The choice of an observer depends on several factors, including the nature of the system (linear or nonlinear), the presence of disturbances and measurement noise, and the available computational resources. In the case of induction machines, a wide variety of observer structures have been developed to estimate non-measurable state variables such as rotor fluxes and mechanical speed. These observers can be classified according to different criteria, which are presented in the following sections [5] [4].

IV.3.1 Classification of Observers

Observers are generally classified according to the way they model system dynamics and handle uncertainties [5] [4]. The main classification criteria are:

- According to the treatment of uncertainties: deterministic observers and stochastic observers;
- According to the nature of the system model: linear observers and nonlinear observers;
- According to the order of the model: full-order observers and reduced-order observers.

Each category offers specific advantages depending on the application requirements in terms of estimation accuracy, robustness, and computational complexity. In the following sections, the principal observer families used for induction machine applications are presented [5] [4].

IV.3.2 Full-Order and Reduced-Order Observers

Observers may also be classified according to the dimension of the estimated state vector relative to that of the system model.

A **full-order observer** estimates all state variables of the system. In the case of the induction machine, the state vector generally includes the two stator current components and the two rotor flux components, resulting in a fourth-order model, to which the mechanical speed may be added depending on the adopted formulation. This type of observer offers a complete reconstruction of the machine states and generally provides high estimation accuracy. However, it requires greater computational effort and may involve the measurement or estimation of additional variables such as rotor speed [5] [4].

A **reduced-order observer** estimates only the non-measurable variables, while directly

measured quantities, such as stator currents, are used as known inputs. In induction machine applications, reduced-order observers are commonly used to estimate rotor fluxes and, in some cases, rotor speed. By decreasing the dimension of the observer, the computational burden is significantly reduced, making this approach particularly suitable for real-time industrial applications [5] [4].

The design of reduced-order observers is often based on the separation of fast and slow dynamics, for example using singular perturbation techniques. This approach preserves good estimation performance while simplifying implementation [5] [4].

IV.3.3 Deterministic and Stochastic Observers

This classification is based on the manner in which the observer takes into account uncertainties and measurement noise. In practice, the signals available from the induction machine are often affected by disturbances, modeling errors, and sensor noise, which can significantly influence the quality of state estimation [5] [4].

Deterministic observers are designed under the assumption that the mathematical model is exact and that measurements are free from random disturbances. Their objective is to ensure the asymptotic convergence of the estimation error through an appropriate choice of the correction gain matrix. Typical examples include the David G. Luenberger observer, full-order and reduced-order observers, and high-gain observers. These structures are relatively simple to implement and are widely used in induction machine control [5] [4].

Stochastic observers, on the other hand, explicitly account for the presence of process and measurement noise. They model these disturbances using probabilistic descriptions and determine the estimated states according to an optimal statistical criterion, usually by minimizing the estimation error covariance. The most representative technique of this family is the Rudolf E. Kálmán filter, which provides highly accurate estimates even in noisy environments [5] [4].

In summary, deterministic observers are preferred when the model is sufficiently accurate and computational simplicity is required, whereas stochastic observers are particularly suitable when measurements are corrupted by significant noise and a statistically optimal estimation is desired [5] [4].

IV.3.4 Linear and Nonlinear Observers

The induction machine is inherently a nonlinear system due to the coupling between electrical and mechanical variables and the dependence of certain parameters on the operating conditions. Consequently, observers can be classified into linear and nonlinear structures according to the nature of the mathematical model used for state estimation [5] [4].

Linear observers are designed on the basis of a linearized state-space model. Their correction gains are determined from constant system matrices, which simplifies both analysis and implementation. These observers are effective when the machine operates around a given operating point, but their performance may deteriorate when the system is subjected to large variations in speed, load, or parameter values [5] [4].

Nonlinear observers are specifically developed to account for the intrinsic nonlinearities of the induction machine. They generally provide improved robustness and accuracy over a wide operating range. Two principal approaches are commonly employed. The first consists in linearizing the system locally around each operating point and updating the observer gains in real time. The second approach constructs the observer directly from the nonlinear model, leading to structures such as high-gain observers, adaptive observers, and sliding mode observers [5] [4].

Because of their ability to cope with model nonlinearities and parameter variations, nonlinear observers are particularly well suited to the estimation of rotor fluxes and mechanical speed in induction machine drives.

IV.4 Deterministic observers

IV.4.1 Full-Order Deterministic Observer

IV.4.1-a System State Equation

For the synthesis of the full-order observer, we utilize a 4th-order state-space representation. This is based on the assumption that the mechanical speed Ω is a slow-varying variable compared to the electrical dynamics, allowing the system to be treated as pseudo-linear [5] [4]. The observer, which runs a parallel model of the machine with a correction term, is defined by the following state equations:

$$\begin{bmatrix} \dot{\hat{i}}_s \\ \dot{\hat{\psi}}_r \end{bmatrix} = \begin{bmatrix} -\gamma I & \frac{K}{T_r} I - pK\Omega J \\ \frac{M}{T_r} I & -\frac{1}{T_r} I + p\Omega J \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \hat{\psi}_r \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma L_s} I \\ 0_{2,2} \end{bmatrix} v_s - \begin{bmatrix} L_i \\ L_\psi \end{bmatrix} (\hat{i}_s, \hat{i}_s) \quad (\text{IV.3})$$

Where:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{ are the identity and orthogonal rotation matrices.}$$

L_i and L_ψ are the complex observation gains, expressed as

$$L_i = IL_{i\Re} + Jp\Omega L_{i\Im} \text{ and } L_\psi = IL_{\psi\Re} + Jp\Omega L_{\psi\Im}.$$

The observation gain matrix is structured as:

$$L = \begin{bmatrix} L_i \\ L_\psi \end{bmatrix} = \begin{bmatrix} L_{i\Re} & -p\Omega L_{i\Re} \\ p\Omega L_{i\Re} & L_{i\Re} \\ L_{\psi\Re} & -p\Omega L_{\psi\Re} \\ p\Omega L_{\psi\Re} & L_{\psi\Re} \end{bmatrix} \quad (\text{IV.4})$$

IV.4.1-b Choice of Observer Poles

The dynamics of the observation error $e = x - \hat{x}$ determine how quickly the estimated states converge to the real states. From the error dynamics equation $\dot{e} = (A - LC)e$, we can define the characteristic behavior of the observer. The system utilizes the property that the eigenvalues of the product of two matrices $XQ(\Omega)$ are the products of their respective

eigenvalues [5] [4].

$$\begin{bmatrix} \dot{e}_i \\ \dot{e}_\psi \end{bmatrix} = XQ(\Omega) \begin{bmatrix} e_i \\ e_\psi \end{bmatrix} \text{ where } X = \begin{bmatrix} -L_{i\beta}I & -KI \\ -L_{\psi\beta}I & I \end{bmatrix} \text{ and } Q(\Omega) = \begin{bmatrix} -\frac{1}{T_r}I + p\Omega J & 0 \\ 0 & -\frac{1}{T_r}I + p\Omega J \end{bmatrix}$$

The observer poles are determined through the eigenvalues of the observation error matrix. Considering the characteristic equation of matrix $\mathbf{X}$:

$$\det(\phi I - X) = 0 \quad (\text{IV.5})$$

The characteristic polynomial becomes:

$$\phi^2 - (L_{i\beta} + 1)\phi + (-KL_{\psi\beta} + L_{i\beta}) = 0 \quad (\text{IV.6})$$

The roots of this polynomial define the observer poles:

$$\phi_{1,2} = \frac{1 - L_{i\beta}}{2} \pm \frac{\sqrt{1 + 2L_{i\beta} + L_{i\beta}^2 + 4KL_{\psi\beta}}}{2} \quad (\text{IV.7})$$

The matrix $Q(\Omega)$ also contributes to the dynamic characteristics of the observer. Its eigenvalues are obtained from:

$$\det(\lambda I - Q(\Omega)) = 0 \quad (\text{IV.8})$$

Which yields :

$$\lambda_{Q1,2} = -\frac{1}{T_r}I \pm jp\Omega \quad (\text{IV.9})$$

The obtained eigenvalues consist of a real and an imaginary component. The real part, $-\frac{1}{T_r}$, governs the attenuation of the estimation error, while the imaginary part $p\Omega$ reflects the rotational behavior of the machine. To ensure asymptotic stability of the observer, all poles of the observation error dynamics must possess negative real parts [5] [4]. Therefore, the following stability condition must be satisfied:

$$L_{i\Im} + KL_{\psi\Im} < 0 \quad (\text{IV.10})$$

This condition guarantees that the observation error decreases with time and that the estimated states converge toward their actual values. Proper tuning of the observer gains $L_{i\Im}$ and $L_{\psi\Im}$ allows achieving a compromise between fast dynamic response and system stability [5] [4].

IV.4.1-c Determination of Correction Gains

The gains $L_{i\Im}$ and $L_{\psi\Im}$ are determined by Pole Placement. By imposing a desired pole $x_{\Re} + jx_{\Im}$ and comparing it with the theoretical eigenvalues derived in the previous section, we can solve for the specific gain values [5] [4]. For a general case involving complex poles, the gains are calculated as:

$$L_{i\Im} = \frac{T_r^2 x_{\Re}^2 p^2 \Omega^2 + T_r x_{\Re} p^2 \Omega^2 + x_{\Im} p \Omega - x_{\Im}^2}{p \Omega (p \Omega x_{\Re} T_r + x_{\Im})} \quad (\text{IV.11})$$

$$L_{\psi\Im} = -\frac{(x_{\Re} T_r + 1)(p \Omega - x_{\Im})}{p \Omega K} \quad (\text{IV.12})$$

Simplified Real Pole Case:

If the design objectives allow for the choice of purely real poles $x_{\Im} = 0$, the gain expressions simplify significantly to:

$$L_{i\Im} = T_r x_{\Re} + 1 \quad (\text{IV.13})$$

$$L_{\psi\Im} = -\frac{T_r x_{\Re} + 1}{K} \quad (\text{IV.14})$$

This simplification is often preferred in industrial implementations to reduce computational complexity while maintaining robust convergence.

IV.4.1-d Simulation

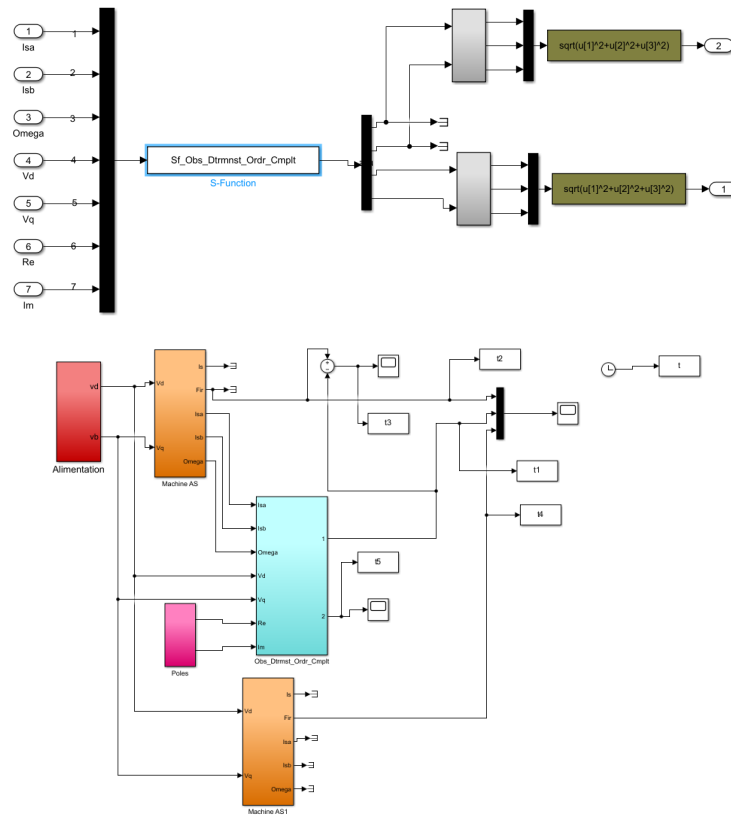


Figure IV.1: Simulation of the Full-order Observer

The figure above is the structure of a full-order deterministic observer. To evaluate the tracking performance of the designed deterministic observers, a high-fidelity simulation model was constructed in MATLAB/Simulink. The physical induction motor is modeled using its state equations expressed in the stationary $\alpha - \beta$ reference frame, obtained via the Clarke Transformation. This approach simplifies the observer equations by avoiding the need for continuous rotor position tracking (θ_s) during coordinate transformations. The nominal electrical and mechanical parameters of the physical machine used throughout all

simulation profiles were obtained in chapter 2:

$$J = 0.0075 \text{ Kg.m}^2 \quad (\text{IV.15})$$

$$L_r = 0.3586 \text{ H} \quad (\text{IV.16})$$

$$L_s = 0.3586 \text{ H} \quad (\text{IV.17})$$

$$L_m = 0.3438 \text{ H} \quad (\text{IV.18})$$

$$R_s = 5.45 \Omega \quad (\text{IV.19})$$

$$R_r = 6.09 \Omega \quad (\text{IV.20})$$

$$f = 0 \text{ Kg.m/s}^2 \quad (\text{IV.21})$$

While the physical induction machine exhibits natural mechanical losses due to friction, the coefficient of viscous friction (f) inside the observer's internal mathematical model was deliberately set to zero ($f = 0$). This simplification deviates from the empirical value calculated during the parameter identification stage in Chapter 2. By assuming an unmodeled mechanical loss scenario within the S-Function loops, the simulation acts as a rigorous test for parametric robustness. Eliminating the friction coefficient forces the observer to rely entirely on its feedback correction gains to overcome the intentional modeling mismatch. Consequently, successful state reconstruction under these conditions validates the observer's immunity to parameter variations and unmodeled mechanical dynamics.

The performance, stability, and convergence rate of the observers depend directly on the placement of their eigenvalues (poles). To comprehensively evaluate the tracking boundaries and structural sensitivity of the designs, the observer poles (p_1) were varied systematically across different configurations:

Table IV.1: Simulation Results

Pole p_1	Response time of the rotor flux observation error	Maximum flux error
-10+j.250	0.615 s	0.029 Wb
-100+j.250	0.3 s	0.016 Wb
-250+j.250	0.32 s	0.053 Wb
-400+j.250	0.23 s	0.051 Wb
-250+j.10	/	0.111 Wb
-250+j.100	0.45 s	0.007 Wb
-250+j.400	0.17 s	0.055 Wb

The simulation results for the full-order deterministic observer reveal a distinct trade-off between convergence speed and initial transient overshoot. Moving the real part of the poles further left (from -10 to -400) successfully accelerates the flux error decay, reducing the response time from 0.615s down to 0.23s , but nearly doubles the maximum transient error due to increased high-gain sensitivity. Conversely, altering the imaginary component directly dictates tracking stability; a weak imaginary part $-250 + j10$ causes a complete failure to converge, while an overly aggressive imaginary frequency tracking loop causes higher peak overshoots. Among all tested configurations, $-250 + j400$ is selected as the

optimal pole configuration. It delivers the fastest overall convergence speed of $0.17s$ while maintaining a well-contained maximum flux error of only $0.055Wb$, providing the most robust dynamic performance for high-speed tracking applications.

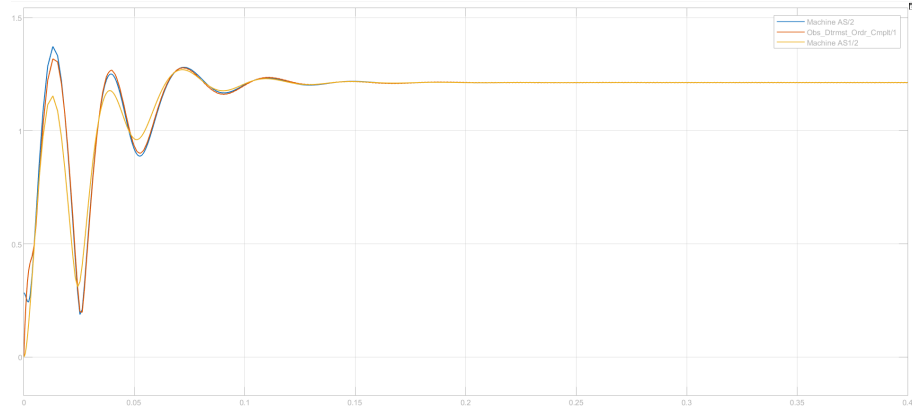


Figure IV.2: Rotor Flux Tracking

The graph of the rotor flux tracking for $p_1 = -250 + j400$.

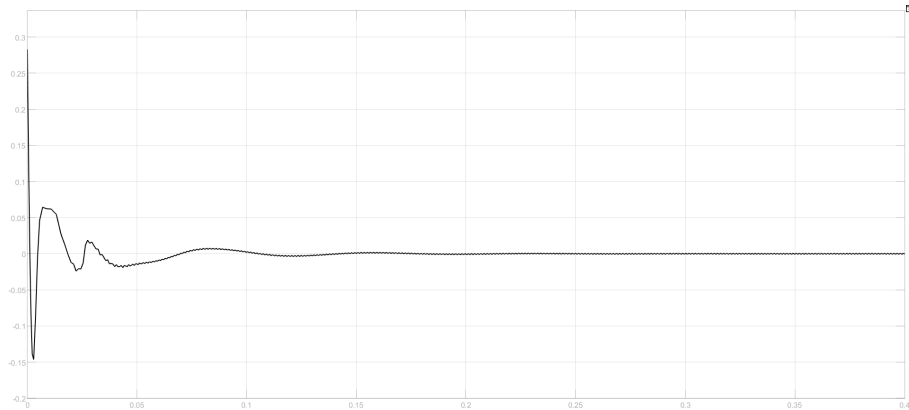


Figure IV.3: Rotor Flux Estimation Error $\Delta(\psi_r)$

IV.4.2 Reduced-Order Deterministic Observer

While a full-order observer reconstructs all state variables, a reduced-order observer focuses exclusively on the non-measurable states. In the case of the induction machine, since the stator currents $(i_{s\alpha}, i_{s\beta})$ are already available through physical sensors, it is more efficient to design an estimator solely for the rotor flux components $(\psi_{r\alpha}, \psi_{r\beta})$ [5] [4].

IV.4.2-a System State Equation

The synthesis of the reduced-order observer begins by isolating the rotor dynamics from the general model of the machine. By considering the stator currents as known inputs to

the rotor subsystem, the state equation for the rotor flux is expressed as follows:

$$\begin{bmatrix} \dot{\psi}_{r\alpha} \\ \dot{\psi}_{r\beta} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_r} & -p\Omega \\ p\Omega & -\frac{1}{T_r} \end{bmatrix} \begin{bmatrix} \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix} + \begin{bmatrix} \frac{L_m}{T_r} & 0 \\ 0 & \frac{L_m}{T_r} \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} \quad (\text{IV.22})$$

To effectively observe the flux, we must define a measurable output that is directly linked to these variables. By manipulating the stator voltage equations and the derivative of the stator currents, we can define a "calculated output" vector y_s . This vector is derived by combining the stator voltages and currents:

$$\begin{bmatrix} y_{s\alpha} \\ y_{s\beta} \end{bmatrix} = \begin{bmatrix} v_{s\alpha} - \sigma L_s \frac{di_{s\alpha}}{dt} \\ v_{s\beta} - \sigma L_s \frac{di_{s\beta}}{dt} \end{bmatrix} \quad (\text{IV.23})$$

Substituting the machine's parameters, this measurement equation can be rewritten in terms of the rotor flux:

$$\begin{bmatrix} y_{s\alpha} \\ y_{s\beta} \end{bmatrix} = \frac{L_m}{L_r} \begin{bmatrix} -\frac{1}{T_r} & -p\Omega \\ p\Omega & -\frac{1}{T_r} \end{bmatrix} \begin{bmatrix} \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix} + \begin{bmatrix} R'_r & 0 \\ 0 & R'_r \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} \quad (\text{IV.24})$$

Where R'_r represents the modified resistance, defined as:

$$R'_r = R_s - R_r \left(\frac{L_m}{L_r} \right)^2 \quad (\text{IV.25})$$

This resistance reflects the stator resistance augmented by the rotor resistance referred to the stator side. This compact representation allows us to treat the rotor flux as the state vector, while the currents and voltages serve as the inputs to a 2nd-order estimation problem.

IV.4.2-b Antisymmetric Gain Matrix

For the synthesis of the reduced-order observer, an antisymmetric structure is chosen for the gain matrix L . This choice is motivated by the inherent antisymmetric nature of the induction machine's dynamics, allowing for a simplified analytical approach to stability and pole placement [5] [4].

Structure of the Gain Matrix The gain matrix L is formulated using a real component $L_{\Re}$ and an imaginary component $L_{\Im}$:

$$L = \begin{bmatrix} L_{\Re} & -L_{\Im} \\ L_{\Im} & L_{\Re} \end{bmatrix} = L_{\Re} J + L_{\Im} J \quad (\text{IV.26})$$

Error Dynamics and Stability The evolution of the estimation error $e_{\psi_r} = \psi_r - \widehat{\psi_r}$ is governed by the state matrix M . Given the rotor dynamics, the error differential equation is:

$$\dot{e}_{\psi_r} = M e_{\psi_r} \quad (\text{IV.27})$$

Where M is defined as :

$$M = \left(\left(1 - \frac{L_m L_{\Re}}{L_r} \right) I + \frac{L_m p \Omega L_{\Im}}{L_r} J \right) \left(-\frac{1}{T_r} I + p \Omega J \right) \quad (\text{IV.28})$$

$$\dot{e}_{\psi_r} = \left(\left(1 - \frac{L_m L_{\Re}}{L_r} \right) I + \frac{L_m p \Omega L_{\Im}}{L_r} J \right) \left(-\frac{1}{T_r} I + p \Omega J \right) e_{\psi_r} \quad (\text{IV.29})$$

Choice of Observer Poles Assuming that the rotor speed Ω remains constant, the observer behaves as a linear system. The observer poles are therefore obtained from the characteristic equation:

The eigenvalues of matrix M are given by:

$$m_1 = - \left(\frac{g_1}{T_r} + g_2 p \Omega \right) + j \left(g_1 p \Omega - \frac{g_2}{T_r} \right) \quad (\text{IV.30})$$

$$m_2 = - \left(\frac{g_1}{T_r} + g_2 p \Omega \right) - j \left(g_1 p \Omega - \frac{g_2}{T_r} \right) \quad (\text{IV.31})$$

Where :

$$g_1 = 1 - \frac{L_m}{L_r} L_{\Re} \text{ and } g_2 = -p \Omega \frac{L_m}{L_r} L_{\Im}$$

The observer poles are selected such that their real parts remain negative, ensuring asymptotic convergence of the estimation error.

Determination of Correction Gains To ensure the error converges with the desired dynamics, we equate the components of M to the real and imaginary parts of the desired poles ($x_{\Re}$ and $x_{\Im}$):

$$L_{\Re} = \frac{L_r}{L_m} \left[1 - \frac{x_{\Re} (1/T_r) + x_{\Im} p \Omega}{(1/T_r)^2 + (p \Omega)^2} \right] \quad (\text{IV.32})$$

$$L_{\Im} = \frac{L_r}{L_m} \left[\frac{x_{\Re} p \Omega - x_{\Im} (1/T_r)}{(1/T_r)^2 + (p \Omega)^2} \right] \quad (\text{IV.33})$$

IV.4.2-c Simulation

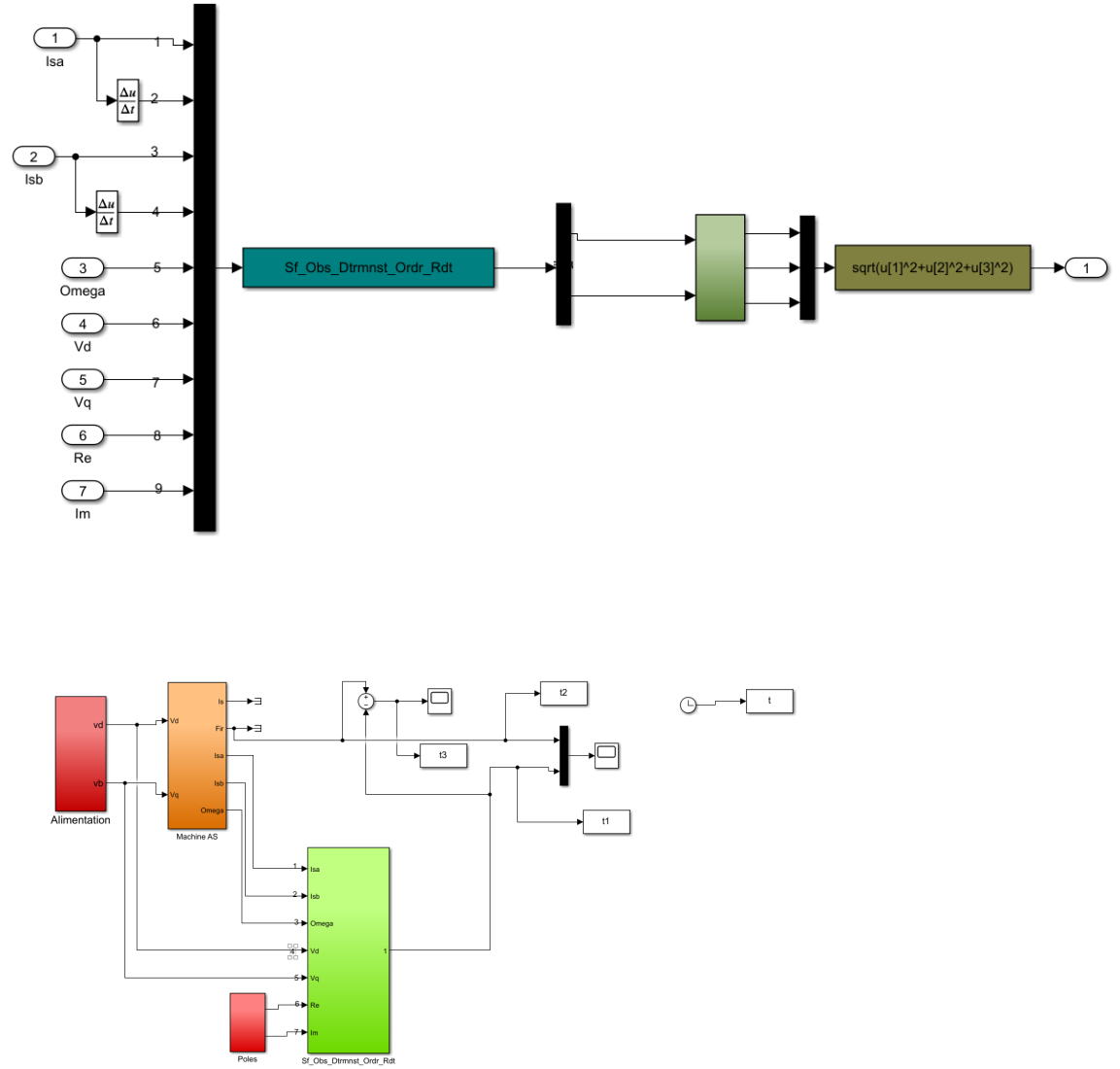


Figure IV.4: Simulation of the Reduced-order Observer with an Antisymmetric Gain Matrix

The figure above is the structure of a reduced-order deterministic observer with skew-symmetric gain matrix. We evaluate the tracking performance using the same method as the previous simulation. The nominal parameters remain the same too. The performance, stability, and convergence rate of the observers depend directly on the placement of their eigenvalues (poles). To comprehensively evaluate the tracking boundaries and structural sensitivity of the designs, the observer poles (p_1) were varied systematically across different configurations:

Table IV.2: Simulation Results

Pole p_1	Response time of the rotor flux observation error	Maximum flux error
$-10+j.250$	0.725 s	1.627 Wb
$-100+j.250$	0.15 s	0.974 Wb
$-250+j.250$	0.13 s	0.532 Wb
$-400+j.250$	0.17 s	0.298 Wb
$-250+j.10$	0.14 S	0.195 Wb
$-250+j.100$	0.17 s	0.320 Wb
$-250+j.400$	0.15 s	0.641 Wb

The reduced-order observer exhibits a large initial transient overshoots due to its reliance on measured inputs. However, it displays a good structural robustness, maintaining stability and achieving fast convergence speeds across all tested pole configuration.

The best balance is $-250 + j250$.

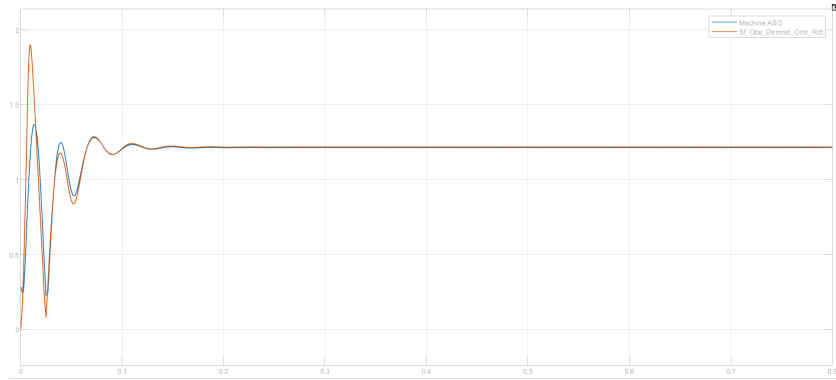
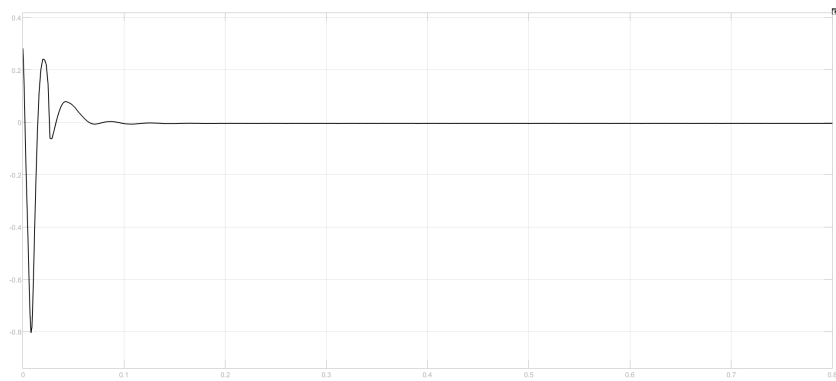


Figure IV.5: Rotor Flux Tracking

The graph of the rotor flux tracking for $p_1 = -250 + j250$.

Figure IV.6: Rotor Flux Estimation Error $\Delta(\psi_r)$

IV.4.2-d Diagonal Gain Matrix

In this configuration, the gain matrix L is chosen to be diagonal. This structure is often utilized to simplify the implementation of the correction terms in the (α, β) stationary frame [5] [4].

Choice of Observer Poles The error dynamics for the reduced-order observer remain governed by the matrix M :

$$\dot{e}_{\psi_r} = M e_{\psi_r} \quad (\text{IV.34})$$

For the diagonal gain matrix $L = \text{diag}(L_{\Re}, L_{\Im})$, the characteristic equation is determined by $\det(\phi I - M) = 0$. Based on the analytical expansion, the eigenvalues of the system are found to be:

$$m_{1,2} = -\left(\frac{g_1}{T_r} + g_2 p \Omega\right) \pm j \left(g_1 p \Omega - \frac{g_2}{T_r}\right) \quad (\text{IV.35})$$

By imposing the desired poles $\lambda_{1,2} = -x_{\Re} \pm j x_{\Im}$, we identify the real and imaginary parts:

$$x_{\Re} = \frac{g_1}{T_r} + g_2 p \Omega \quad (\text{IV.36})$$

$$x_{\Im} = g_1 p \Omega - \frac{g_2}{T_r} \quad (\text{IV.37})$$

Determination of Correction Gains By solving the linear system formed by the equations above for g_1 and g_2 , and then substituting back to find $L_{\Re}$ and $L_{\Im}$, we obtain the following correction gains:

$$L_{\Re} = \frac{L_r}{L_m} \left[1 - \frac{x_{\Re} (1/T_r) + x_{\Im} p \Omega}{(1/T_r)^2 + (p \Omega)^2} \right] \quad (\text{IV.38})$$

$$L_{\Im} = \frac{L_r}{L_m} \left[\frac{x_{\Re} p \Omega - x_{\Im} (1/T_r)}{(1/T_r)^2 + (p \Omega)^2} \right] \quad (\text{IV.39})$$

These expressions allow for the exact placement of the observer poles while accounting for the coupling effects induced by the mechanical speed Ω .

IV.4.2-e Simulation Real Part

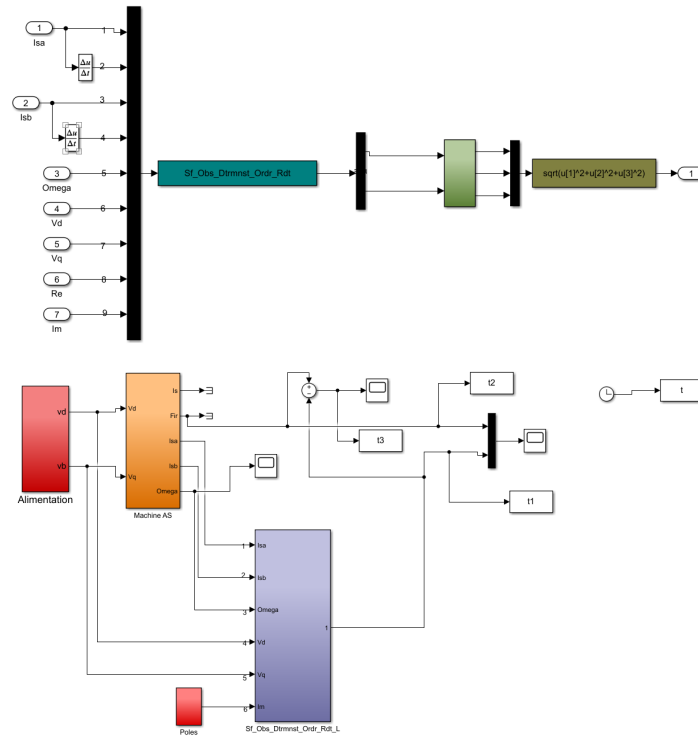


Figure IV.7: Simulation of the Reduced-order Observer with a Diagonal Gain Matrix fixing the Real part

The figure above represents the observer used for the simulation of the reduced-order deterministic observer with a diagonal gain matrix fixing the real part.

This simulation was performed to test a Diagonal Gain Matrix structure, which simplifies real-time processor calculations by mathematically uncoupling the observer's state equations. The goal was to fix the real part of the observer's eigenvalues to a single target number p_1 to see how restricting the matrix layout affects tracking speed and startup overshoot.

Table IV.3: Simulation Results

Pole p_1	Response time of the rotor flux observation error	Maximum flux error
-400	0.006 s	0.382 Wb
-250	0.17 s	0.561 Wb
-100	0.13 s	0.587 Wb
-10	0.6 s	0.001 Wb

The simulation results for the reduced-order observer using a diagonal gain matrix to fix the real part reveal a highly non-linear relationship between eigenvalue placement and

convergence dynamics. Pushing the pole aggressively to the left ($p_1 = -400$) yields an ultra-fast response time of $0.006s$ with a well-contained maximum flux error of $0.382Wb$. Interestingly, shifting the pole closer to the origin slows the response but exposes a structural property: at $p_1 = -10$, the observer takes $0.6s$ to settle but achieves a near-perfect maximum flux tracking error of $0.001Wb$, showing that slow eigenvalue constraints heavily suppress initial transient overshoots in this specific topology.

Among the tested choices, $p_1 = -400$ is selected as the optimal pole configuration for high-performance applications due to its near-instantaneous error decay rate $0.006s$ while maintaining a highly acceptable transient error bound, making it ideal for tracking fast machine transitions.

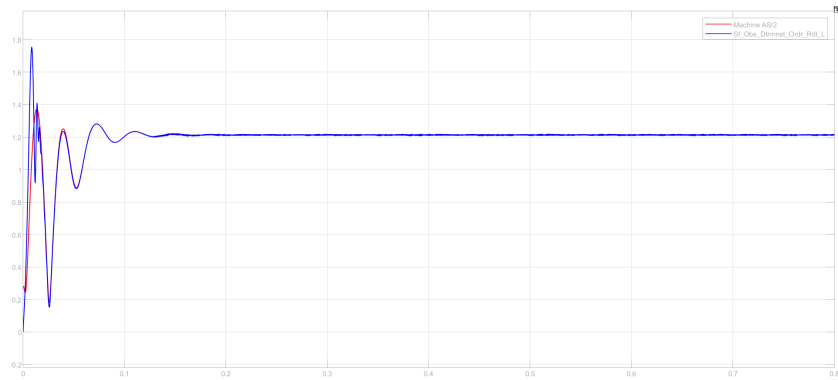


Figure IV.8: Rotor Flux Tracking

The graph of the rotor flux tracking for $p_1 = -400$.

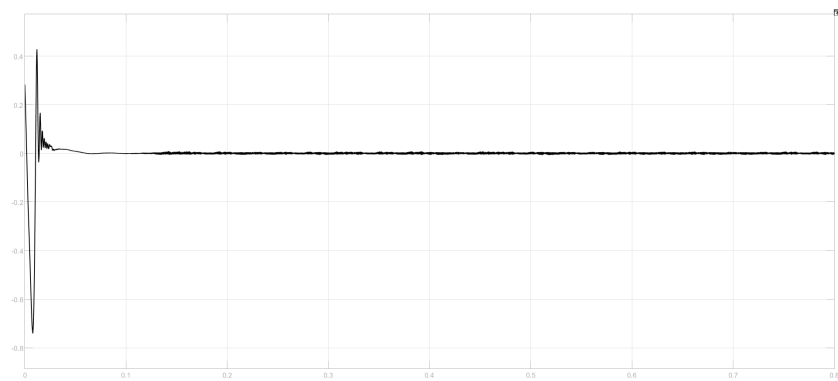


Figure IV.9: Rotor Flux Estimation Error $\Delta(\psi_r)$

IV.4.2-f Simulation Imaginary Part

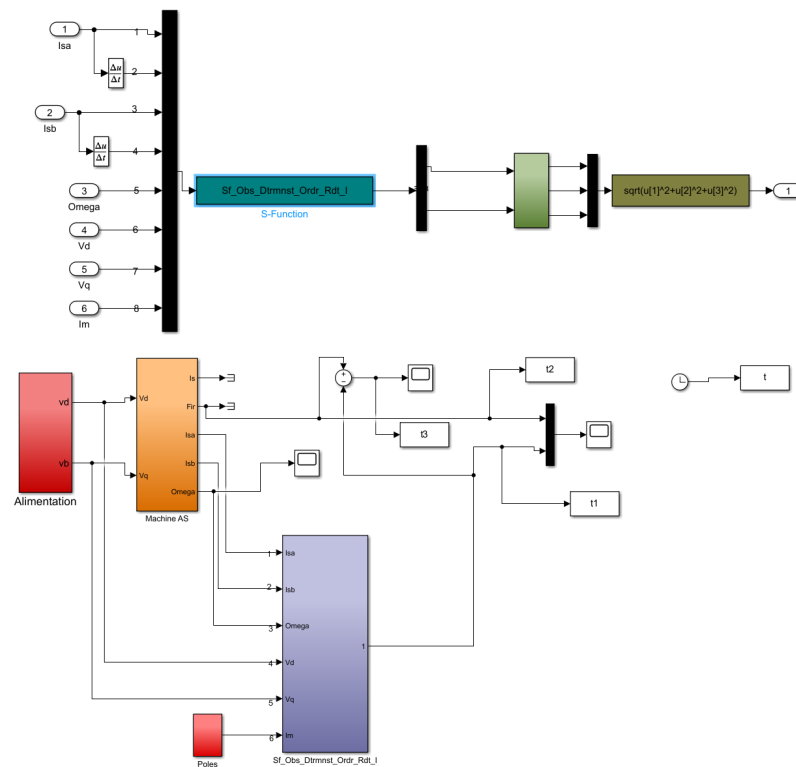


Figure IV.10: Simulation of the Reduced-order Observer with a Diagonal Gain Matrix fixing the Imaginary part

The figure above represents the observer used for the simulation of the reduced-order deterministic observer with a diagonal gain matrix fixing the imaginary part.

This section evaluates a structural variation of the simplified Diagonal Gain Matrix layout. While the previous section constrained the real component of the eigenvalues to enforce specific exponential decay rates, this configuration fixes the imaginary part to a designated frequency target.

Table IV.4: Simulation Results

Pole p_1	Response time of the rotor flux observation error	Maximum flux error
-400	0.4 s	0.295 Wb
-250	0.6 s	0.686 Wb
-100	1.4 s	0.659 Wb
-10	11 s	0.147 Wb

Among all configurations tested in this section, $p_1 = -400$ is selected as the optimal pole configuration. It yields the most efficient speed-to-error trade-off by cutting down the

settling delay to a fraction of a second ($0.4s$) while ensuring that the high feedback gains do not compromise the structural stability or trigger an uncontrolled transient overshoot at startup.

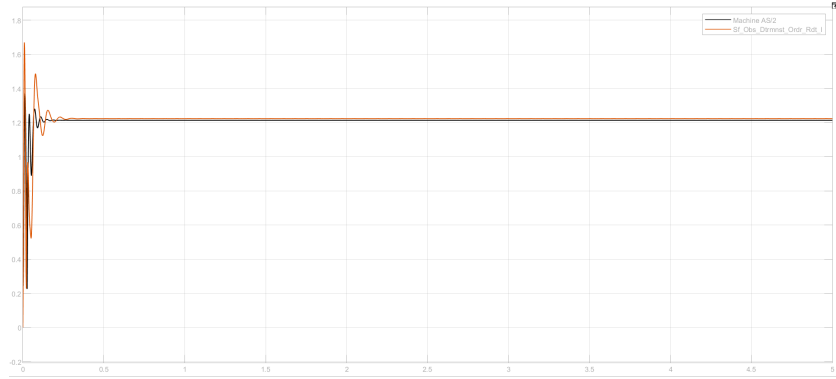


Figure IV.11: Rotor Flux Tracking

The graph of the rotor flux tracking for $p_1 = -400$.

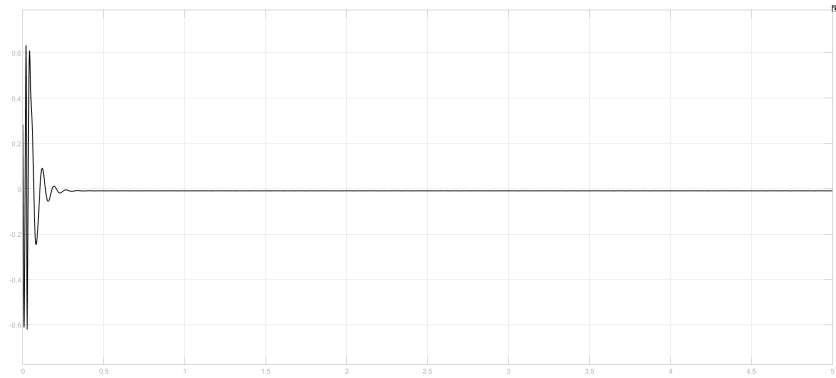


Figure IV.12: Rotor Flux Estimation Error $\Delta(\psi_r)$

IV.5 High-gain observer

IV.5.1 High-Gain Observer for a Special Class of Non-Linear Systems

The induction machine is naturally a non-linear system due to the coupling between stator currents, rotor fluxes, and mechanical speed. In order to estimate the non-measurable states with fast convergence, a high-gain observer can be employed. This observer structure is particularly suitable for systems that can be expressed in a triangular observable form and whose non-linearities satisfy the Lipschitz condition. Starting from the state model of the induction machine in the stationary $\alpha - \beta$ reference frame, the state vector is defined as [5] [4]:

$$z = \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix} = \begin{bmatrix} i_s \\ \psi_r \end{bmatrix} \quad (\text{IV.40})$$

The system can then be written under the compact form:

$$\dot{z} = F(\Omega) z + G(v_s, \Omega, z) \quad (\text{IV.41})$$

With the measured output :

$$y = Cz \quad (\text{IV.42})$$

Where :

$$C = \begin{bmatrix} 1 & 0_{2,2} \end{bmatrix} \quad (\text{IV.43})$$

The matrix $F(\Omega)$ is given by:

$$F(\Omega) = \begin{bmatrix} 0 & F(\Omega) \\ 0 & 0 \end{bmatrix} \quad (\text{IV.44})$$

With :

$$F(\Omega) = \begin{bmatrix} \frac{K}{T_r} & Kp\Omega \\ -Kp\Omega & \frac{K}{T_r} \end{bmatrix} \quad (\text{IV.45})$$

The non-linear term $G(v_s, \Omega, z)$ is expressed as:

$$G(v_s, \Omega, z) = \begin{bmatrix} g_1(v_s, \Omega, i_s) \\ g_2(v_s, \Omega, i_s, \psi_r) \end{bmatrix} \quad (\text{IV.46})$$

Where :

$$g_1(v_s, \Omega, i_s) = \begin{bmatrix} -\gamma i_{s\alpha} + \frac{1}{\sigma L_s} v_{s\alpha} \\ -\gamma i_{s\beta} + \frac{1}{\sigma L_s} v_{s\beta} \end{bmatrix} \quad (\text{IV.47})$$

And :

$$g_2(v_s, \Omega, i_s, \psi_r) = \begin{bmatrix} \frac{M}{T_r} i_{s\alpha} - \frac{1}{T_r} \psi_{r\alpha} - p\Omega \psi_{r\beta} \\ \frac{M}{T_r} i_{s\beta} + p\Omega \psi_{r\alpha} - \frac{1}{T_r} \psi_{r\beta} \end{bmatrix} \quad (\text{IV.48})$$

A diagonal matrix $\Lambda(\Omega)$ is introduced as:

$$\Lambda(\Omega) = \begin{bmatrix} I & 0 \\ 0 & F(\Omega) \end{bmatrix} \quad (\text{IV.49})$$

Which gives :

$$\Lambda(\Omega) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{K}{T_r} & Kp\Omega \\ 0 & 0 & -Kp\Omega & \frac{K}{T_r} \end{bmatrix} \quad (\text{IV.50})$$

The proposed high-gain observer is therefore written as:

$$\dot{\hat{z}} = F(\Omega) \hat{z} + G(v_s, \Omega, \hat{z}) - \Lambda^{-1}(\Omega, \hat{z}) S_\theta^{-1} C^T (C\hat{z} - y) \quad (\text{IV.51})$$

Where S_θ is obtained from the Lyapunov equation:

$$\theta S_\theta + A^T S_\theta + S_\theta A - C^T C = 0 \quad (\text{IV.52})$$

With :

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \quad (\text{IV.53})$$

The matrix S_θ can be expressed as:

$$S_\theta = \begin{bmatrix} \theta^{-1} I & -\theta^{-2} I \\ -\theta^{-2} I & 2\theta^{-3} I \end{bmatrix} \quad (\text{IV.54})$$

The complete panoramic form of the observer becomes:

$$\begin{bmatrix} \dot{\widehat{i_{s\alpha}}} \\ \dot{\widehat{i_{s\beta}}} \\ \dot{\widehat{\psi_{r\alpha}}} \\ \dot{\widehat{\psi_{r\beta}}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{K}{T_r} & Kp\Omega \\ 0 & 0 & -Kp\Omega & \frac{K}{T_r} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \widehat{i_{s\alpha}} \\ \widehat{i_{s\beta}} \\ \widehat{\psi_{r\alpha}} \\ \widehat{\psi_{r\beta}} \end{bmatrix} + G(v_s, \Omega, \widehat{z}) - L_\theta \begin{bmatrix} \widehat{i_{s\alpha}} - i_{s\alpha} \\ \widehat{i_{s\beta}} - i_{s\beta} \end{bmatrix} \quad (\text{IV.55})$$

Where L_θ represents the correction gain matrix depending on the parameter θ .

The estimation error is defined by:

$$e = \widehat{z} - z \quad (\text{IV.56})$$

Thus :

$$e = \begin{bmatrix} e_i \\ e_\psi \end{bmatrix} = \begin{bmatrix} \widehat{i_s} - i_s \\ \widehat{\psi_r} - \psi_r \end{bmatrix} \quad (\text{IV.57})$$

The corresponding error dynamics are written as:

$$\dot{e} = \{F(\Omega) - \Lambda^{-1}(\Omega) S_\theta^{-1} C^T C\} e + G(v_s, \Omega, \widehat{z}) - G(v_s, \Omega, z) \quad (\text{IV.58})$$

To study the convergence of the observer, the following Lyapunov function is considered:

$$V(\bar{e}) = \bar{e}^T S_1 \bar{e} \quad (\text{IV.59})$$

$$\dot{V} \leq -\theta V + 2p\delta\alpha_g\lambda b_{S_1} V + 2\lambda' b_{S_1} V \quad (\text{IV.60})$$

Where :

$$\delta = \sup \{ \Lambda^{-1}(\Omega) \} \quad (\text{IV.61})$$

$$\delta = \sup \left\{ 1, \left| \frac{\frac{K}{T_r}}{\left(\frac{K}{T_r}\right)^2 + (Kp\Omega)^2} \right| + \left| \frac{-Kp\Omega}{\left(\frac{K}{T_r}\right)^2 + (Kp\Omega)^2} \right| \right\} \quad (\text{IV.62})$$

$$\lambda = \sup \{ \|\Lambda(\Omega)\| \} \quad (\text{IV.63})$$

$$\lambda = \sup \left\{ 1, \left| \frac{K}{T_r} \right| + |Kp\Omega| \right\} \quad (\text{IV.64})$$

$$\lambda' = \sup \left\{ \left\| \dot{\Lambda}(\Omega) \Lambda^{-1}(\Omega) \right\| \right\} \quad (\text{IV.65})$$

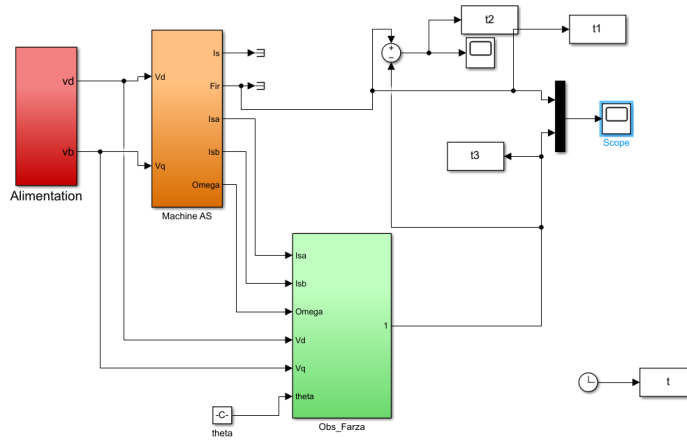
$$\lambda' = \left| -\frac{pT_r}{1 + p^2\Omega^2 T_r^2} \right| + \left| \frac{\Omega}{1 + p^2\Omega^2 T_r^2} \right| \quad (\text{IV.66})$$

Consequently, the convergence condition of the observer is ensured if:

$$\theta > 2p\delta\alpha_g\lambda b_{S_1} + 2\lambda' b_{S_1} \quad (\text{IV.67})$$

This condition guarantees the asymptotic stability of the estimation error and ensures rapid convergence of the estimated states toward the real states of the induction machine.

IV.5.2 Simulation



To validate state reconstruction under non-linear system constraints, a high-gain Farza observer structure was implemented within the MATLAB/Simulink environment via a specialized S-function block labeled ObsFarza. Unlike the linear deterministic models, this non-linear framework dynamically updates its estimation trajectory by tracking continuous physical couplings across six input channels: the stationary stator voltages, the measured stator currents ($I_{s\alpha}, I_{s\beta}$), the mechanical rotor speed (Ω), and a structural tracking argument (θ). The output estimation vector is evaluated using a dual-channel validation topology consisting of a real-time tracking overlay scope on the right, alongside a dedicated algebraic subtraction circuit at the top center designed to isolate and log the absolute non-linear estimation error vector ($\Delta\Phi$) directly to the workspace memory workspace block (t_2).

Table IV.5: Simulation Results

Pole θ	Response time of the rotor flux observation error	Maximum flux error
0	0.105 s	0.093 Wb
1000	0.01 s	0.001 Wb
1050	0.01 s	0 Wb
1100	0.01 s	0 Wb
1500	0.008 s	0 Wb

The simulation metrics for the non-linear Farza observer highlight the exceptional tracking power of high-gain non-linear estimation loops. The parameter θ acts as a design scaling

factor; when $\theta = 0$, the observer behaves similarly to a standard baseline estimator, yielding a moderate response time of 0.105 s and a maximum flux error of 0.093 Wb. However, increasing θ to 1000 and above triggers an aggressive mathematical correction path that systematically drives tracking errors down to zero. For values between 1050 and 1500, the observer completely suppresses initial startup overshoots, yielding a flawless 0 Wb maximum flux error and forcing the state reconstruction loop to converge almost instantly.

Among the tested values, $\theta = 1500$ is selected as the optimal tuning parameter. It achieves the fastest overall convergence speed in the entire lab, settling the state observation error in an ultra-fast 0.008 s with 0 Wb of transient overshoot, demonstrating that the non-linear observer is highly effective for high-performance, instantaneous state estimation.

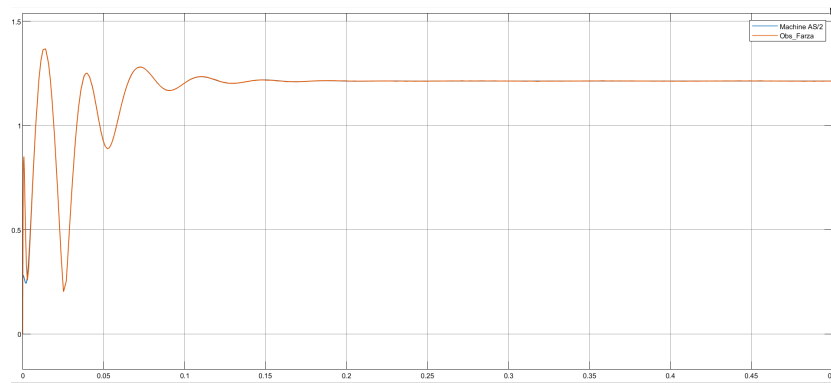


Figure IV.13: Rotor Flux Tracking

The graph of the rotor flux tracking for $\theta = 1500$.

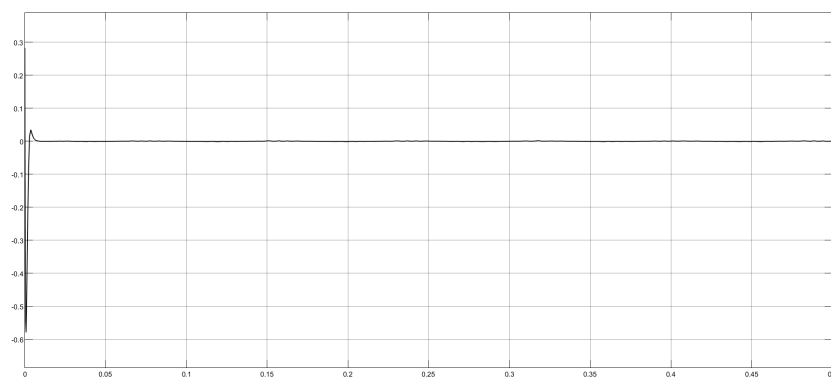


Figure IV.14: Rotor Flux Estimation Error $\Delta(\psi_r)$

IV.5.3 High-Gain Observer for General Nonlinear Systems

Unlike the particular class of nonlinear systems presented previously, a general nonlinear system requires a more flexible observer design approach. For uniformly observable systems, a coordinate transformation can be introduced in order to express the model in a suitable form for high-gain observer synthesis. For the induction machine model considered in this

work, the following transformation is introduced:

$$z = \Phi(x) = \begin{bmatrix} h_1 \\ h_2 \\ L_{f+ug}(h_1) \\ L_{f+ug}(h_2) \end{bmatrix} \quad (\text{IV.68})$$

Which leads to the following state variables:

$$z_1 = i_{s\alpha} \quad (\text{IV.69})$$

$$z_2 = i_{s\beta} \quad (\text{IV.70})$$

$$z_3 = -\gamma i_{s\alpha} + \frac{K}{T_r} \psi_{r\alpha} + pK\Omega \psi_{r\beta} \quad (\text{IV.71})$$

$$z_4 = -\gamma i_{s\beta} - pK\Omega \psi_{r\alpha} + \frac{K}{T_r} \psi_{r\beta} \quad (\text{IV.72})$$

Applying the coordinate transformation allows the induction machine model to be rewritten in a new state representation:

$$\dot{z}_1 = z_3 + \frac{1}{\sigma L_s} v_{s\alpha} \quad (\text{IV.73})$$

$$\dot{z}_2 = z_4 + \frac{1}{\sigma L_s} v_{s\beta} \quad (\text{IV.74})$$

$$\dot{z}_3 = \left(\frac{KM}{T_r T_r} - \frac{\gamma}{T_r} \right) z_1 + p\Omega \left(\frac{KM}{T_r} - \gamma \right) z_2 - \left(\gamma + \frac{1}{T_r} \right) z_3 - p\Omega z_4 \quad (\text{IV.75})$$

$$\dot{z}_4 = -p\Omega \left(\frac{KM}{T_r} - \gamma \right) z_1 + \left(\frac{KM}{T_r T_r} - \frac{\gamma}{T_r} \right) z_2 + p\Omega z_3 - \left(\gamma + \frac{1}{T_r} \right) z_4 \quad (\text{IV.76})$$

$$(\text{IV.77})$$

The above equations can be written in compact form as:

$$\dot{z} = Az + \varphi(v_s, \Omega, z) \quad (\text{IV.78})$$

With the output equation :

$$y = Cz \quad (\text{IV.79})$$

where A and C are the state and output matrices, respectively, while $\varphi(v_s, \Omega, z)$ represents the nonlinear part of the system:

$$\varphi(v_s, \Omega, z) = \begin{bmatrix} \varphi_i(v_s, \Omega, z_i) \\ \varphi_\psi(v_s, \Omega, z_i, z_\psi) \end{bmatrix} \quad (\text{IV.80})$$

With:

$$\varphi_i(v_s, \Omega, z_i) = \begin{bmatrix} \frac{1}{\sigma L_s} v_{s\alpha} \\ \frac{1}{\sigma L_s} v_{s\beta} \end{bmatrix} \quad (\text{IV.81})$$

And:

$$\varphi_\psi(v_s, \Omega, z_i, z_\psi) = \begin{bmatrix} \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} \quad (\text{IV.82})$$

The following observer is then proposed:

$$\dot{\hat{z}} = A\hat{z} + \varphi(\hat{z} - z) - S_\theta^{-1}C^T(C\hat{z} - y) \quad (\text{IV.83})$$

This observer is an exponential observer for the considered system, where $S_\theta = S_\theta^T$ is given by:

$$S_\theta = \begin{bmatrix} \theta^{-1}I & -\theta^{-2}I \\ -\theta^{-2}I & 2\theta^{-3}I \end{bmatrix} \quad (\text{IV.84})$$

With $\theta > 0$, obtained as the unique solution of the following algebraic Lyapunov equation:

$$\theta S_\theta + A^T S_\theta + S_\theta A = C^T C \quad (\text{IV.85})$$

The nonlinear function φ is globally Lipschitz with respect to z , uniformly with respect to v_s :

$$\|\varphi(\hat{z}, v_s) - \varphi(z, v_s)\| \leq k \|\hat{z} - z\| \quad (\text{IV.86})$$

Where the Lipschitz constant is defined by:

$$k = \|A\|_\infty \quad (\text{IV.87})$$

With:

$$k = \max \left\{ \left| \frac{KM}{T_r T_r} - \frac{\gamma}{T_r} \right| + \left| p\Omega \left(\frac{KM}{T_r} - \gamma \right) \right| + \left| \gamma + \frac{1}{T_r} \right| + |p\Omega| \right\} \quad (\text{IV.88})$$

Stability Analysis

Let the estimation error be defined as:

$$e = \hat{z} - z \quad (\text{IV.89})$$

Its dynamics are given by:

$$\dot{e} = (A - S_\theta^{-1}C^T C) e + \varphi(\hat{z}, v_s) - \varphi(z, v_s) \quad (\text{IV.90})$$

Consider the following Lyapunov candidate function:

$$V(e) = e^T S_\theta e \quad (\text{IV.91})$$

Its derivative is:

$$\dot{V}(e) = e^T \dot{S}_\theta e + e^T S_\theta \dot{e} \quad (\text{IV.92})$$

Substituting the error dynamics leads to:

$$\dot{V}(e) = -e^T (\theta S_\theta + C^T C) e + 2e^T S_\theta [\varphi(\hat{z}, v_s) - \varphi(z, v_s)] \quad (\text{IV.93})$$

Using the positivity of $C^T C$ together with the Lipschitz property, the following inequality is obtained:

$$\dot{V}(e) \leq -\theta e^T S_\theta e + 2k e^T S_\theta e \quad (\text{IV.94})$$

Hence:

$$\dot{V}(e) \leq -(\theta - 2k) V(e) \quad (\text{IV.95})$$

Therefore, exponential stability of the observer is guaranteed if:

$$\theta > 2k \quad (\text{IV.96})$$

This condition ensures asymptotic convergence of the estimation error toward zero.

Observer in the Original Coordinates

The estimated state vector in the original coordinates is obtained through the inverse transformation:

$$\hat{x} = \Phi^{-1}(z) \quad (\text{IV.97})$$

Since:

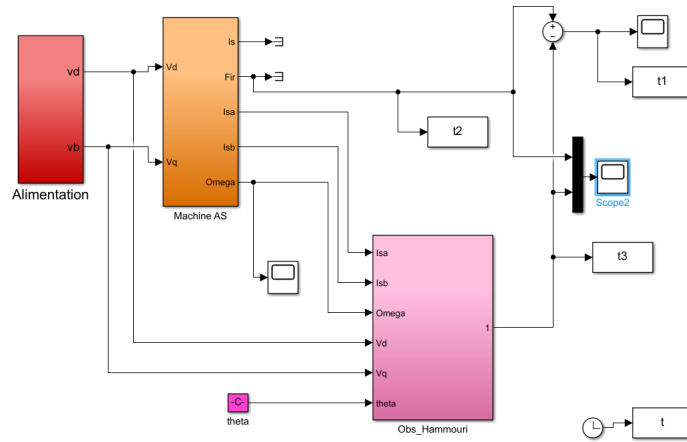
$$\frac{dz}{dt} = \frac{\partial \Phi(x)}{\partial x} \frac{dx}{dt} \quad (\text{IV.98})$$

Another formulation of observer synthesis expressed in the original coordinates can be written as:

$$\dot{\hat{x}} = f(x) + g(x, u) - \left(\frac{\partial \Phi(x)}{\partial x} \right)^{-1} S(\theta)^{-1} C^T (C\hat{x} - y) \quad (\text{IV.99})$$

Finally, the tuning of this observer depends only on the single parameter θ . Although increasing θ improves the convergence speed of the observer, excessively large values may amplify measurement noise and degrade practical performance.

IV.5.4 Simulation



To evaluate generalized non-linear estimation strategies, a Hammouri observer structure was implemented within the MATLAB/Simulink environment using a specialized S-function block labeled ObsHammouri. The block diagram maintains the same six-input channel topology as the previous high-gain implementation, feeding the stationary stator variables, the mechanical rotor speed (Ω), and a structural tuning argument θ into the estimation loop. The estimation accuracy is verified using a real-time tracking overlay scope (Scope2) on the right, paired with an algebraic subtraction loop at the top right designed to isolate the absolute non-linear observation error and save it directly to the workspace memory block t_1 .

Table IV.6: Simulation Results

Pole θ	Response time of the rotor flux observation error	Maximum flux error
10	0.11 s	0.095 Wb
20	0.11 s	0.093 Wb
27.5	0.09 s	0.065 Wb
40	0.075 s	0.023 Wb
50	0.075 s	0.001 Wb
67.5	0.075 s	0.027 Wb

The simulation results for the Hammouri observer demonstrate a very clear optimization curve as the tuning parameter θ increases. At lower gain values ($\theta = 10$ to 20), the tracker exhibits a sluggish response time of 0.11 s and a high transient error profile near 0.095 Wb. Increasing to 40 and beyond significantly sharpens the tracking dynamics, locking the response time to a fast baseline of 0.075 s. However, raising the parameter excessively to 67.5 causes a slight degradation, with the maximum tracking error bouncing back up to 0.027 Wb due to characteristic high-gain transient peaking.

Among all evaluated settings, $\theta = 50$ represents the optimal tuning parameter. It yields the absolute lowest maximum flux tracking error in the dataset 0.001 Wb while locking onto the actual states within the fastest recorded response time of 0.075 s, striking a flawless balance between rapid estimation and transient stability.

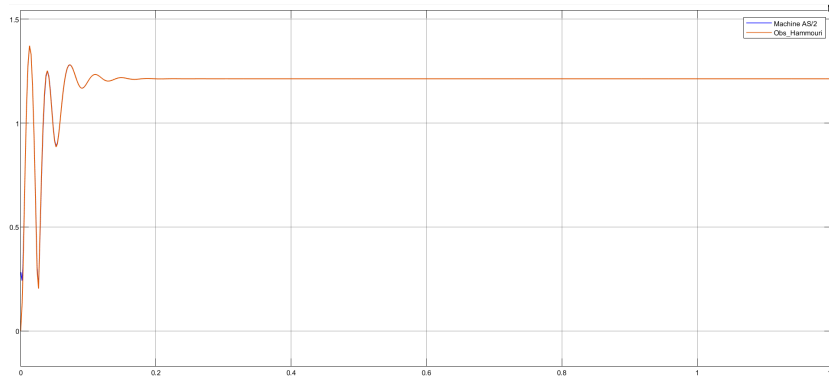
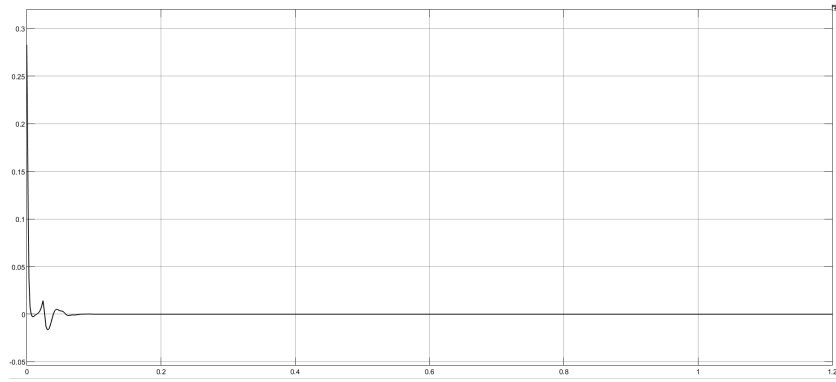


Figure IV.15: Rotor Flux Tracking

The graph of the rotor flux tracking for $\theta = 50$.

Figure IV.16: Rotor Flux Estimation Error $\Delta(\psi_r)$

IV.6 Conclusion

This chapter presented the design and analysis of observers for induction motors, focusing on the estimation of non-measurable state variables required for advanced control. Different observer structures were reviewed, with emphasis on deterministic full-order and reduced-order observers developed from the machine's state-space model.

Observer gains were selected to ensure stability and rapid error convergence. Simulation results confirmed the ability of the observers to accurately estimate internal variables, particularly rotor flux. The high-gain observer was also studied as a nonlinear estimation approach, offering faster convergence at the cost of greater noise sensitivity.

Overall, the chapter demonstrated that observer-based techniques provide reliable state estimation, making them essential for high-performance control and fault diagnosis applications.

General Conclusion

This thesis focused on the modeling, identification, and state observation of a three-phase induction machine, with particular emphasis on rotor flux estimation.

The first part of this work was devoted to the development of the mathematical model of the induction machine in the stationary reference frame. This model constitutes the foundation for the analysis, simulation, and implementation of advanced estimation techniques. The study highlighted the importance of accurate machine parameters in obtaining reliable simulation results and achieving satisfactory observer performance. Subsequently, an experimental identification procedure was carried out to determine the electrical and mechanical parameters of the machine. Several tests were performed, allowing the estimation of the stator and rotor parameters. In order to improve the quality of the identified parameters, the Particle Swarm Optimization (PSO) algorithm was employed. The obtained results demonstrated the effectiveness of this optimization technique in minimizing the identification error and improving the agreement between the model and the real machine behavior.

The identified parameters were then used to perform an experimental validation of the machine model. The comparison between simulation and experimental results showed a satisfactory correspondence, confirming the validity of the identified parameters and the accuracy of the developed model. Despite the inevitable discrepancies caused by modeling assumptions, measurement uncertainties, and unmodeled phenomena, the obtained results remained within acceptable limits.

The final part of this work focused on the design and implementation of state observers for rotor flux estimation. Different observer structures were studied, including deterministic full-order observers, reduced-order observers, and high-gain observers. Their theoretical foundations were presented, and their performance was evaluated through simulation. The obtained results demonstrated the ability of these observers to accurately reconstruct the rotor fluxes from measurable quantities such as stator currents, stator voltages, and rotor speed.

The convergence analysis and simulation results confirmed the effectiveness of the pro-

posed observers. In particular, the estimation errors converged toward zero after a short transient period, demonstrating the stability and robustness of the adopted approaches. Furthermore, the influence of observer pole placement on convergence speed and estimation accuracy was investigated, highlighting the trade-off between rapid convergence and sensitivity to disturbances.

Overall, the results obtained throughout this work demonstrate that the combination of accurate parameter identification and advanced observer design provides an effective solution for state estimation in induction machines. Such techniques are of significant interest for monitoring, control, and high-performance drive applications.

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Appendix A: Observability of the Induction Motor

OBSERVABILITY

Observability is the property that enables the reconstruction of the internal state of a dynamic system from its inputs and measured outputs. In the case of the induction machine, this concept plays a fundamental role in sensorless control, where the mechanical speed is not directly measured and must therefore be estimated from electrical quantities. The problem becomes particularly challenging at low speeds, since the electrical signals contain less exploitable information about the mechanical dynamics, which may lead to a degradation of the estimation performance [5] [4].

The observability matrix ϑ is constructed using the gradient of the Lie derivatives:

$$\vartheta(x) = \begin{bmatrix} \nabla L_f^0 h \\ \nabla L_f^1 h \\ \vdots \\ \nabla L_f^{n-1} h \end{bmatrix} \quad (100)$$

The system is locally observable at a point x_0 if the observability matrix has a full rank of n (where n is the dimension of the state space) at that point:

$$\text{rank}(\vartheta(x_0)) = n \quad (101)$$

In this section, two cases will be considered:

- Large a system with a speed sensor,
- Small a sensorless system.

With speed sensor

When the rotor speed ω_r is measured via an encoder, it is treated as a known time-varying parameter. The state vector reduces to the electromagnetic variables [5] [4].

State vector ($n = 4$) :

$$x = [i_{s\alpha}, i_{s\beta}, \psi_{r\alpha}, \psi_{r\beta}]^T \quad (102)$$

Measurement :

$$y = [i_{s\alpha}, i_{s\beta}]^T \quad (103)$$

The system matrices are :

$$A(\omega_r) = \begin{bmatrix} -\gamma & 0 & \frac{K}{T_r} & K\omega_r \\ 0 & -\gamma & -K\omega_r & \frac{K}{T_r} \\ \frac{L_m}{T_r} & 0 & -\frac{1}{T_r} & -\omega_r \\ 0 & \frac{L_m}{T_r} & \omega_r & -\frac{1}{T_r} \end{bmatrix} \quad (104)$$

Where $\gamma = \left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \right)$, $T_r = \frac{L_r}{R_r}$, $K = \frac{L_m}{\sigma L_s L_r}$.

The rank of $\vartheta = [C, CA, CA^2, CA^3]^T$ is always 4, provided the rotor flux is non-zero. This system is globally observable. Even at zero speed ($\omega_r = 0$), the back-EMF terms do not vanish in a way that creates a singularity, allowing for flux estimation in all regimes [5] [4].

The sensorless systems

In the sensorless system, the speed ω_r is an unknown to be estimated. It is promoted to a state variable, introducing a high degree of nonlinearity.

State vector ($n = 5$) :

$$x = [i_{s\alpha}, i_{s\beta}, \psi_{r\alpha}, \psi_{r\beta}, \omega_r]^T \quad (105)$$

Measurement :

$$y = [i_{s\alpha}, i_{s\beta}]^T \quad (106)$$

The nonlinear vector field $f(x)$ is:

$$\vartheta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -\gamma & 0 & \frac{K}{T_r} & K\omega_r & K\psi_{r\beta} \\ 0 & -\gamma & -K\omega_r & \frac{K}{T_r} & -K\psi_{r\alpha} \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \quad (107)$$

The observability analysis of the induction machine highlights the strong dependence of state reconstruction on operating conditions and available measurements. In the case where the rotor speed is measured, the system reduces to an electromagnetic subsystem whose observability is generally guaranteed as long as the rotor flux is sufficiently excited. This configuration ensures reliable estimation of the flux variables over a wide operating range. In contrast, the sensorless configuration introduces an additional state variable corresponding to the mechanical speed, significantly increasing the nonlinear coupling within the system. The resulting observability matrix depends explicitly on operating conditions such as rotor speed, stator frequency, and flux magnitude. In particular, at low stator frequencies or near zero speed, the coupling terms that carry mechanical information become

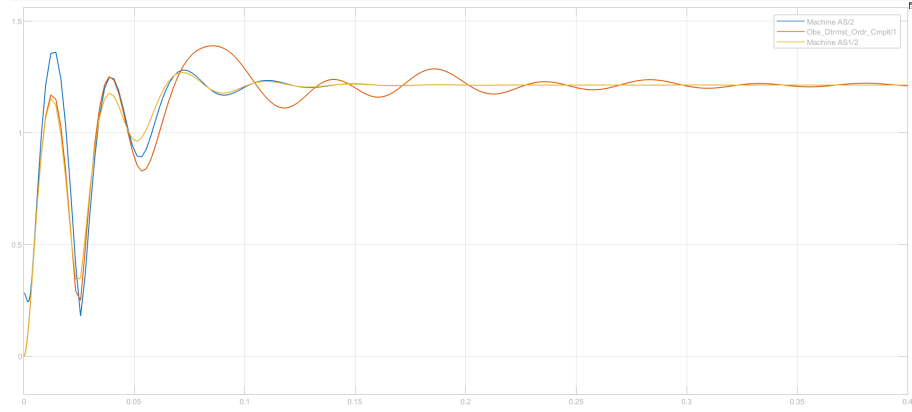
weak or vanish, leading to a degradation of observability.

This analysis shows that observability is not an intrinsic binary property, but rather a condition-dependent characteristic of the system. As a result, the performance of sensorless control strategies is directly linked to the excitation level of the machine and the richness of the measured electrical signals.

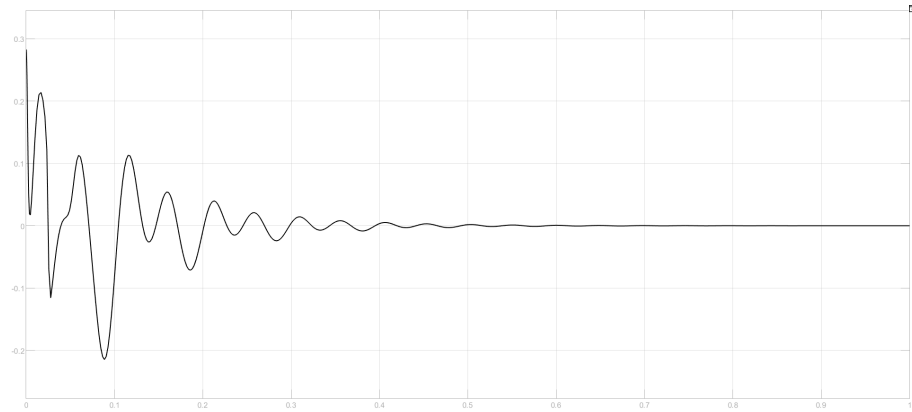
Therefore, the design of observers for induction machines must explicitly take into account these limitations, especially in low-speed regimes. This motivates the use of advanced estimation techniques such as adaptive observers, high-gain observers, or nonlinear observers (e.g., sliding mode or extended Kalman filtering), which aim to maintain satisfactory state reconstruction despite variations in observability conditions [5] [4].

.1 RESULT OF OBSERVER SIMULATION

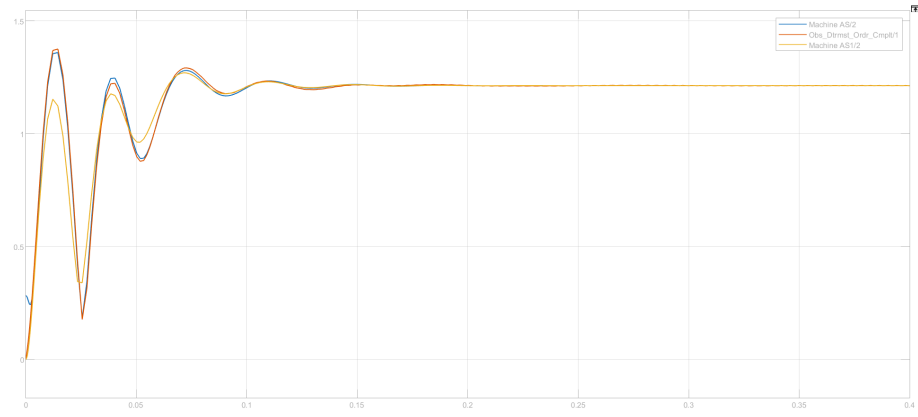
Full-order deterministic observer



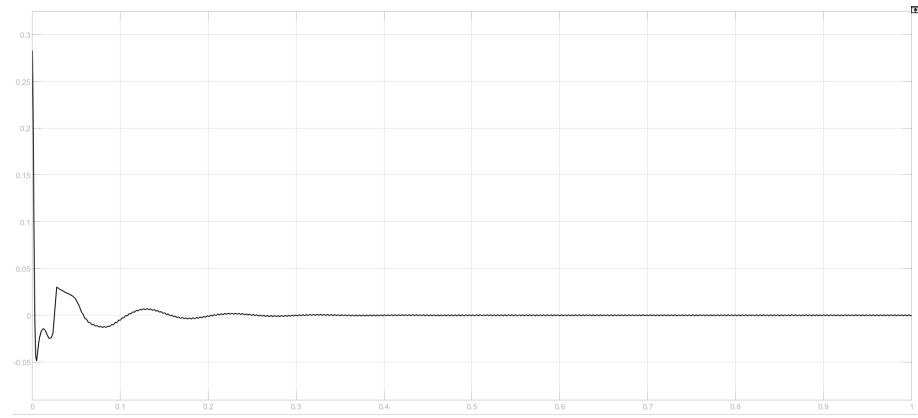
The graph of the rotor flux tracking for $p_1 = -10 + j250$



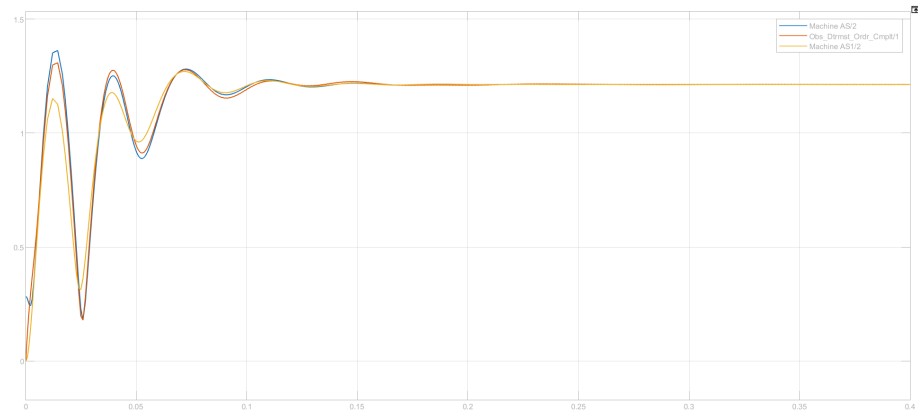
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



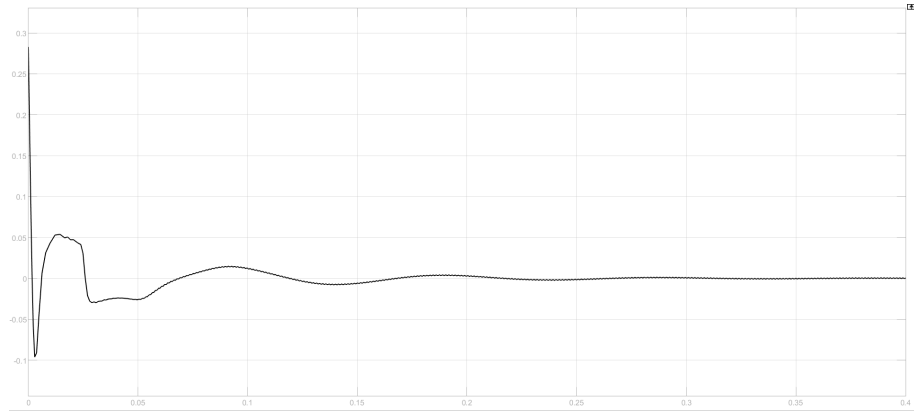
The graph of the rotor flux tracking for $p_1 = -100 + j250$



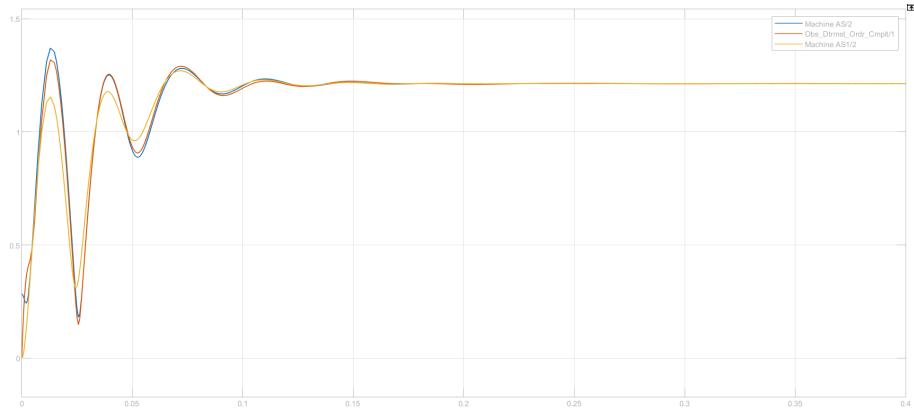
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



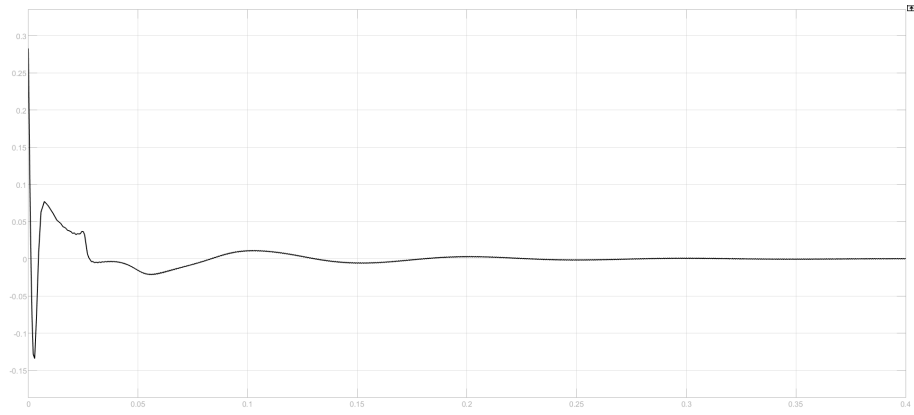
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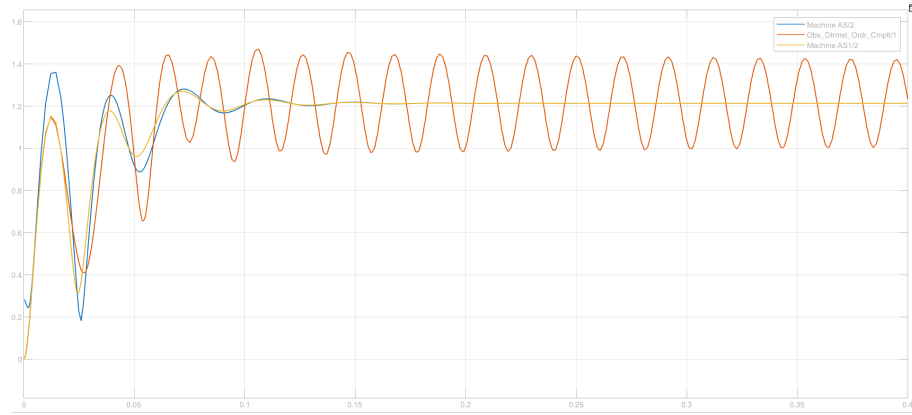
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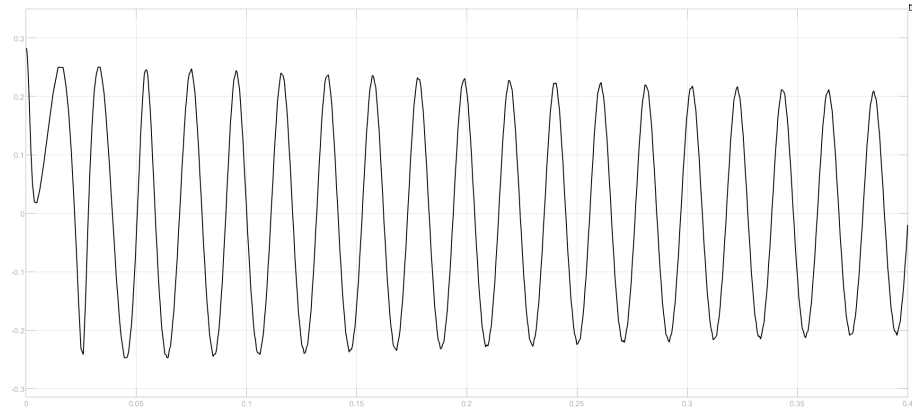
The graph of the rotor flux tracking for $p_1 = -400 + j250$



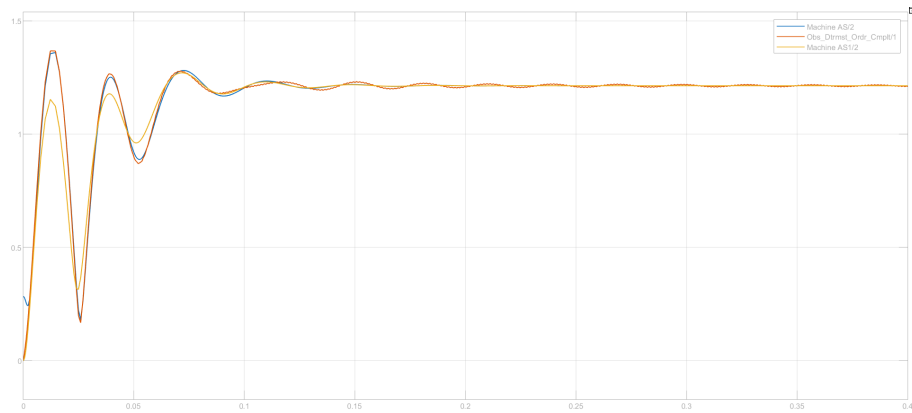
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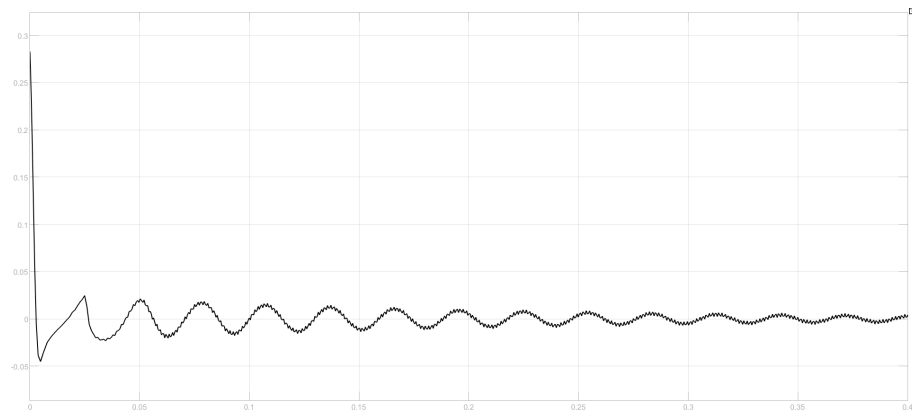
The graph of the rotor flux tracking for $p_1 = -250 + j10$



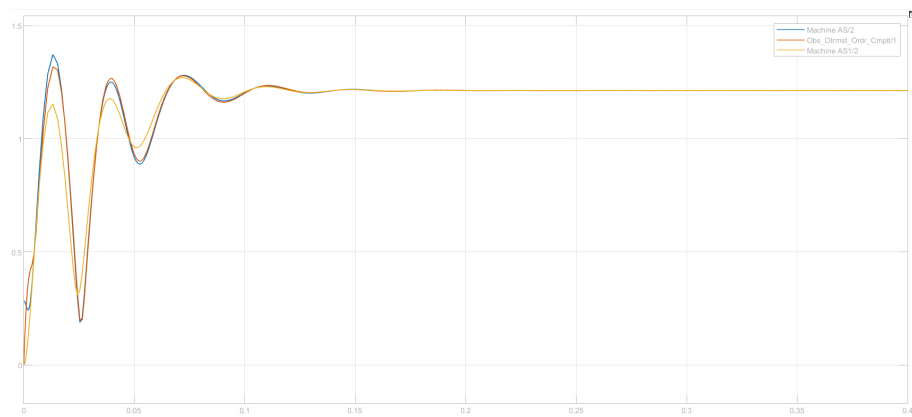
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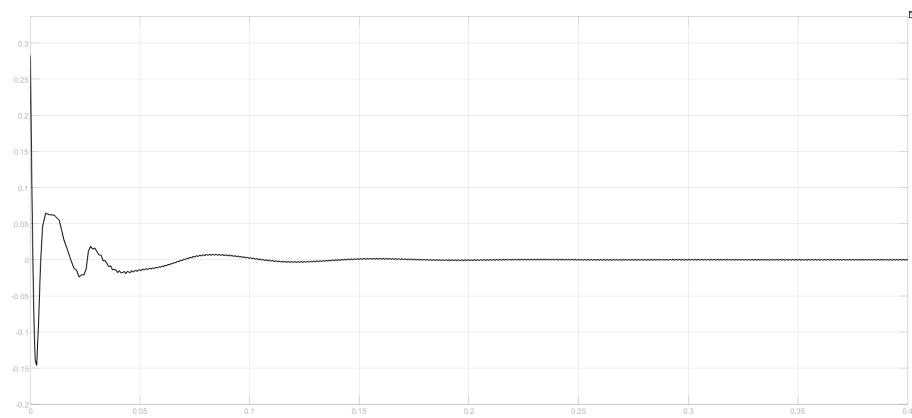
The graph of the rotor flux tracking for $p_1 = -250 + j100$



The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$

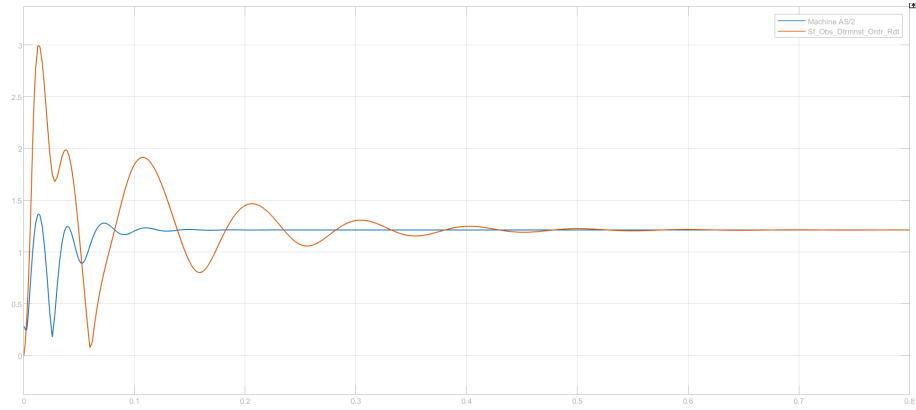


The graph of the rotor flux tracking for $p_1 = -400 + j100$

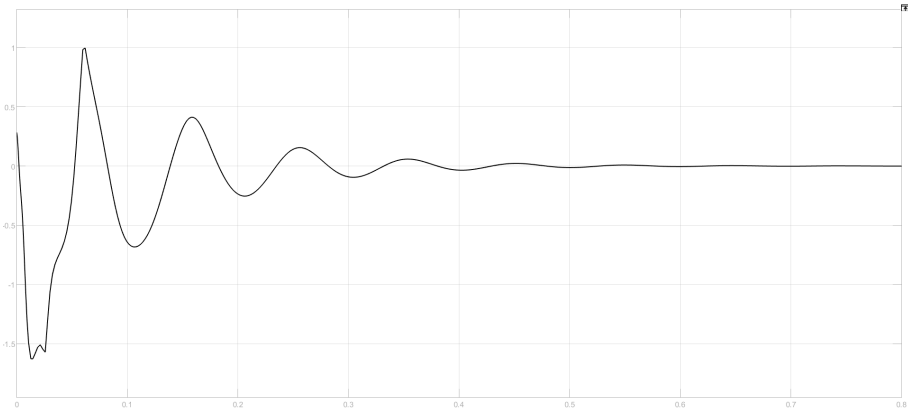


The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$

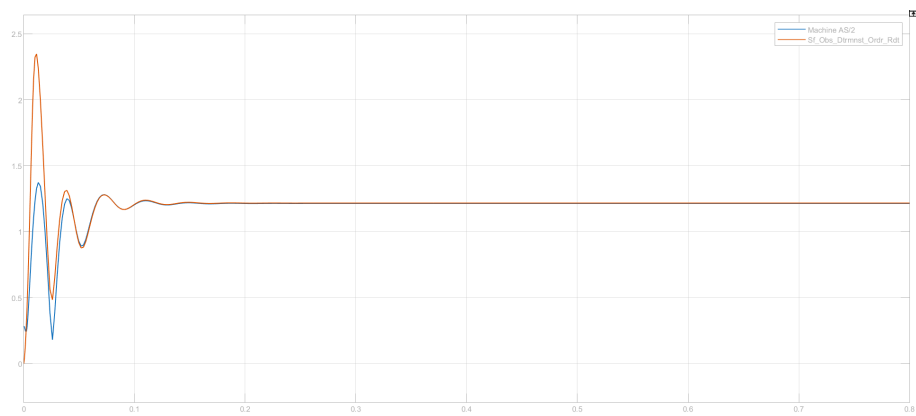
Reduced-order Deterministic Observer with Skew-symmetric Gain Matrix



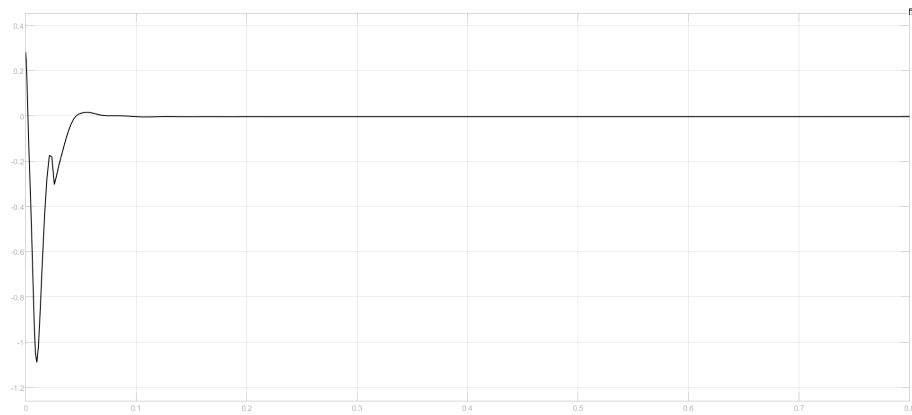
The graph of the rotor flux tracking for $p_1 = -10 + j250$



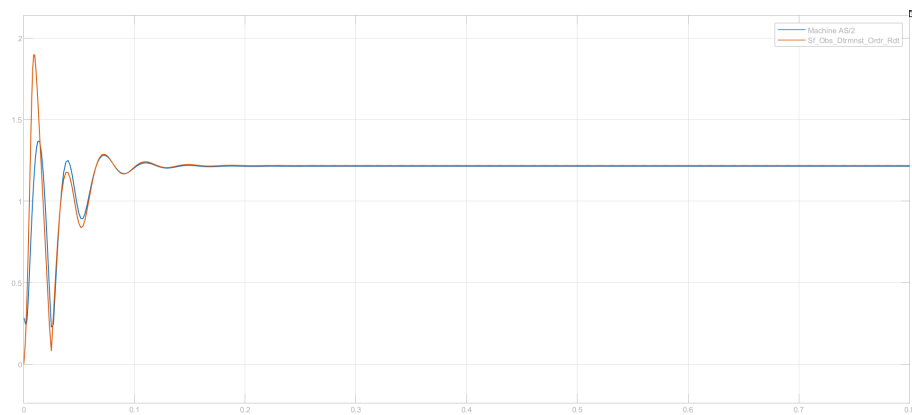
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



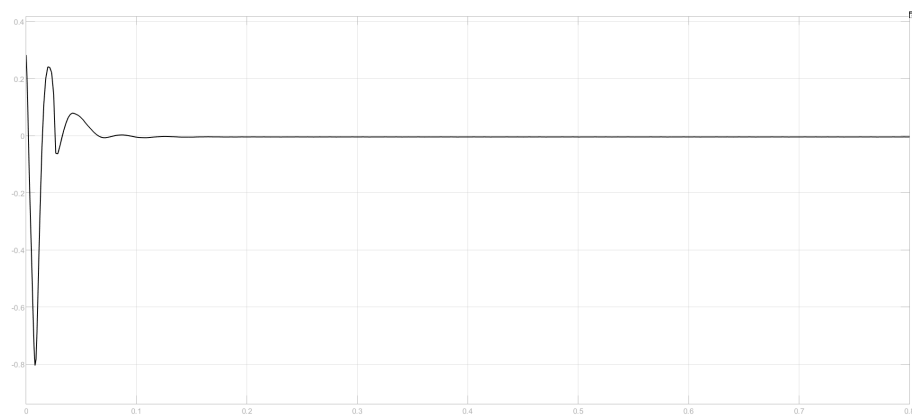
The graph of the rotor flux tracking for $p_1 = -100 + j250$



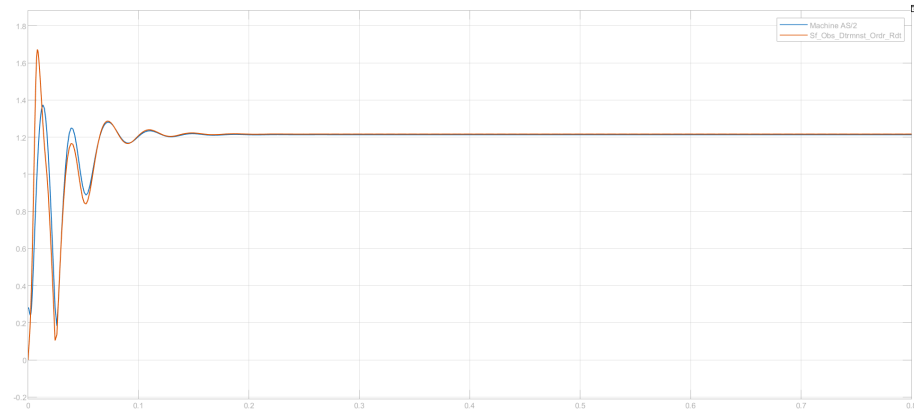
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



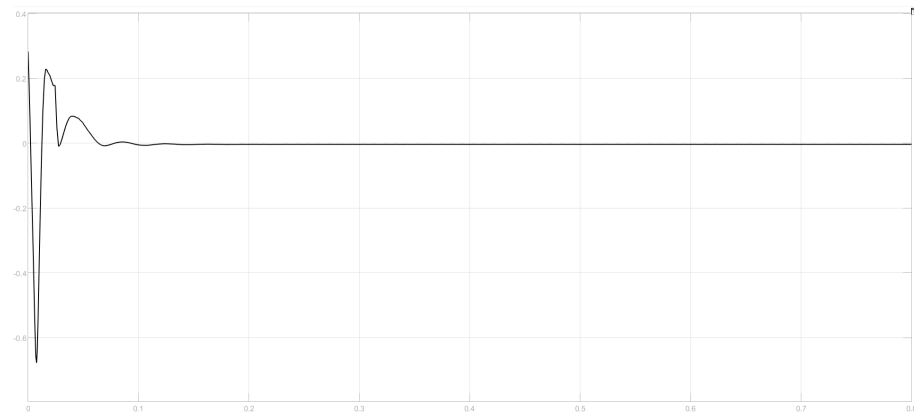
The graph of the rotor flux tracking for $p_1 = -250 + j250$



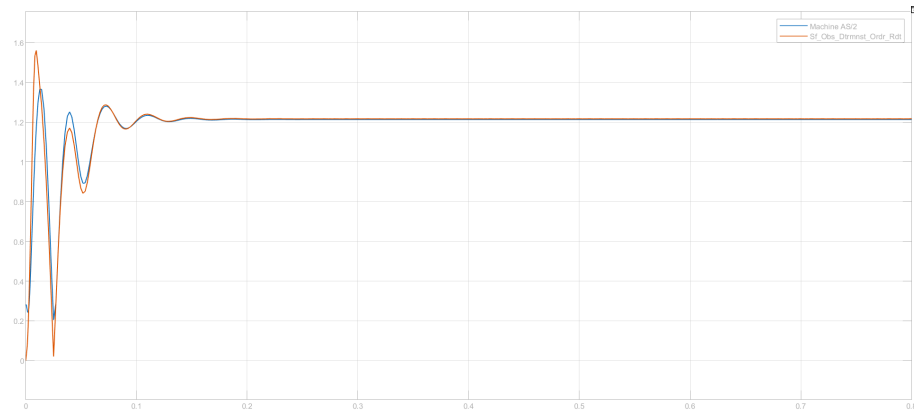
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



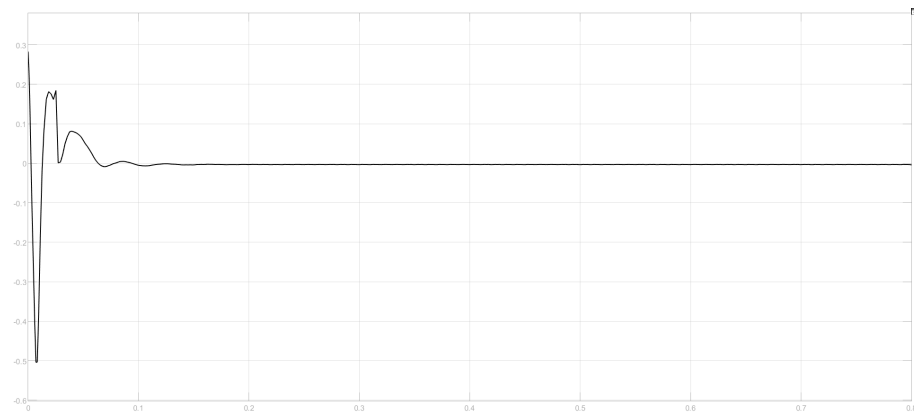
The graph of the rotor flux tracking for $p_1 = -400 + j250$



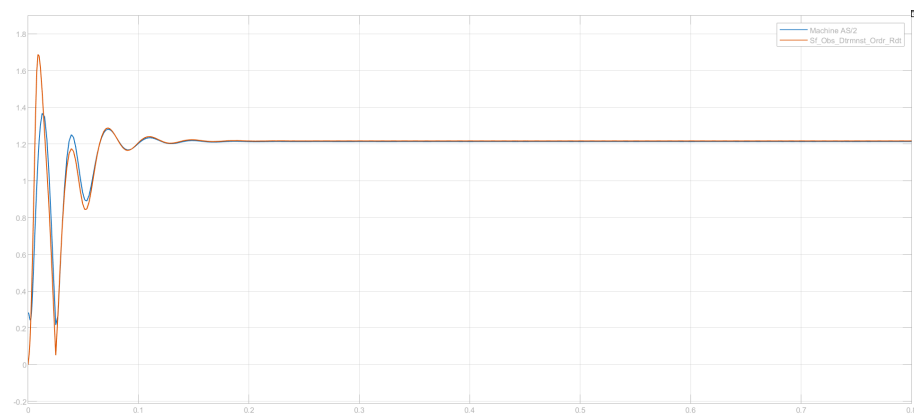
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



The graph of the rotor flux tracking for $p_1 = -250 + j10$



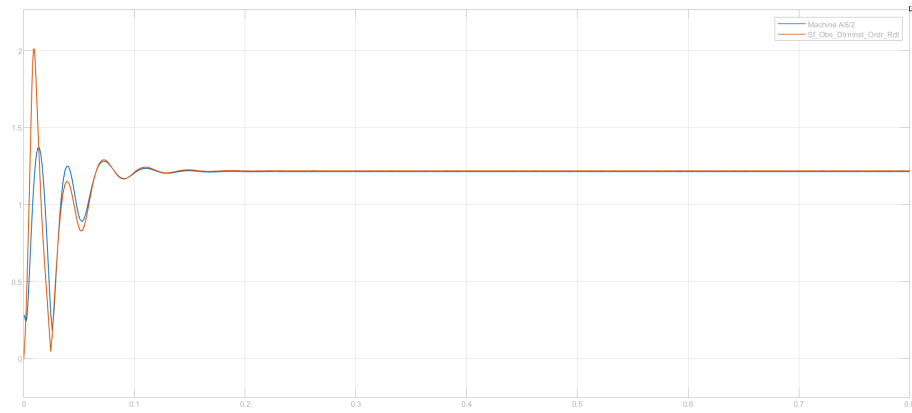
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



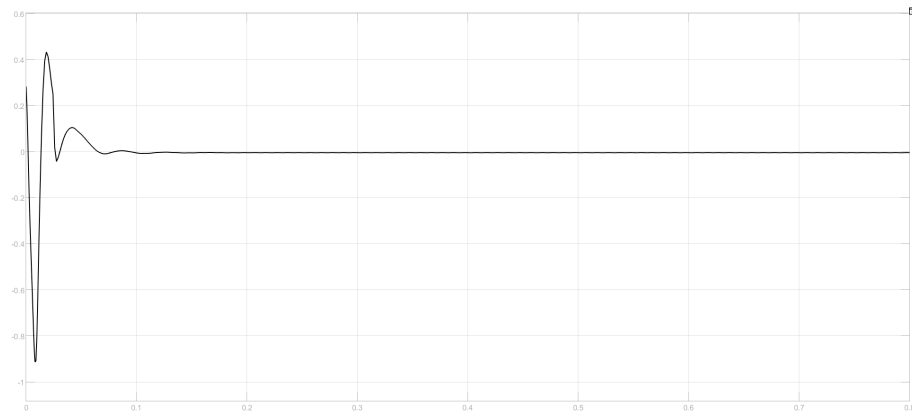
The graph of the rotor flux tracking for $p_1 = -250 + j100$



The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$

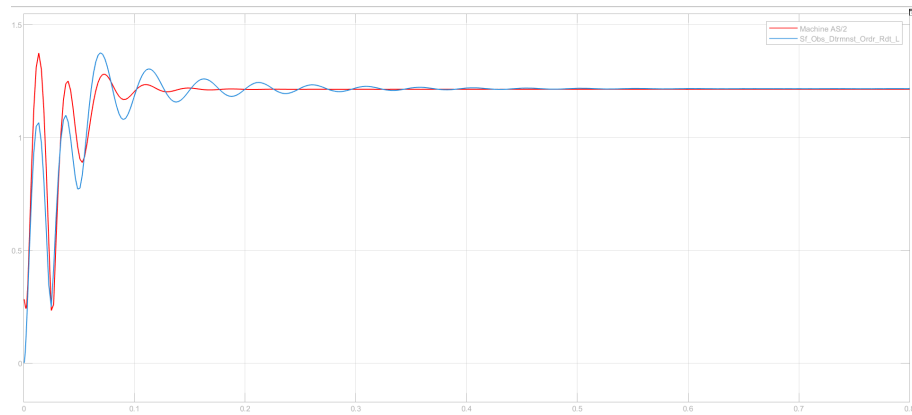


The graph of the rotor flux tracking for $p_1 = -250 + j400$

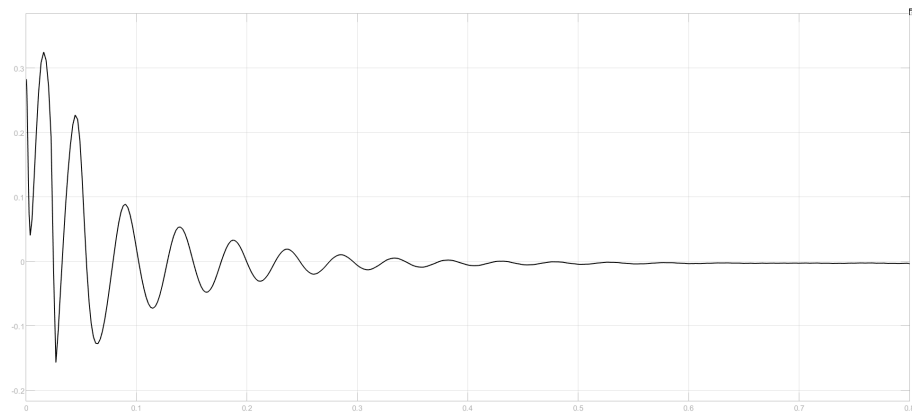


The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$

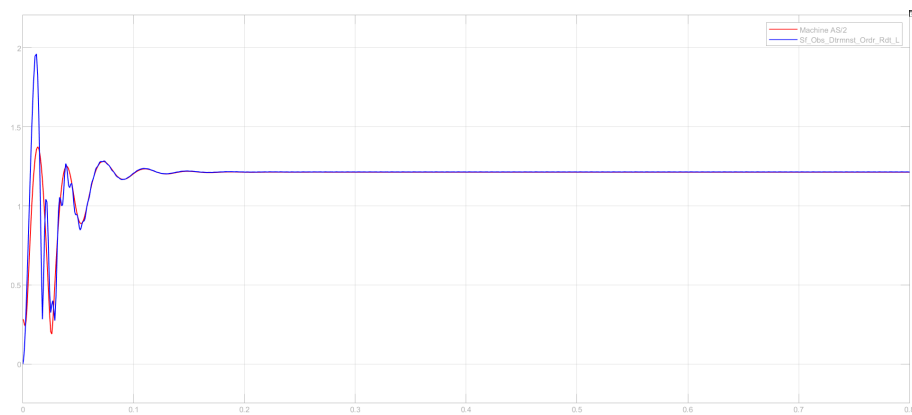
Reduced-order deterministic observer with a diagonal gain matrix fixing the real part



The graph of the rotor flux tracking for $p_1 = -10$



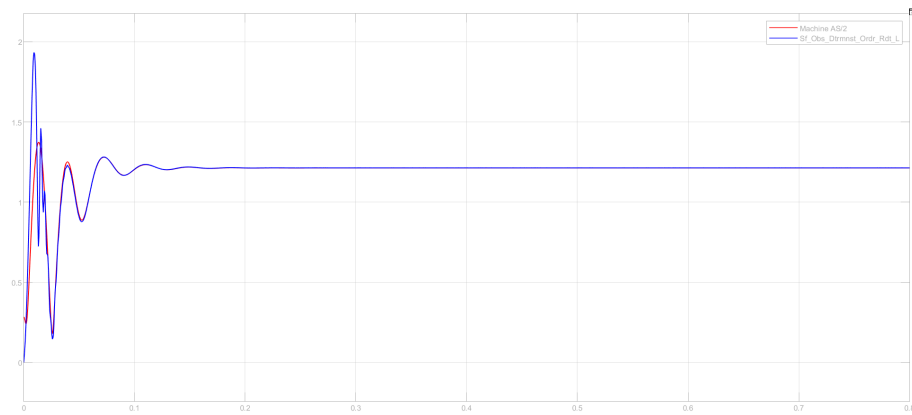
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



The graph of the rotor flux tracking for $p_1 = -100$



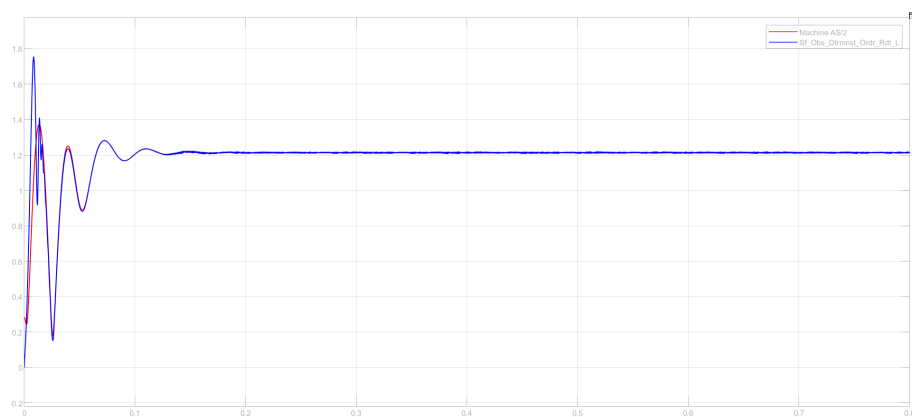
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



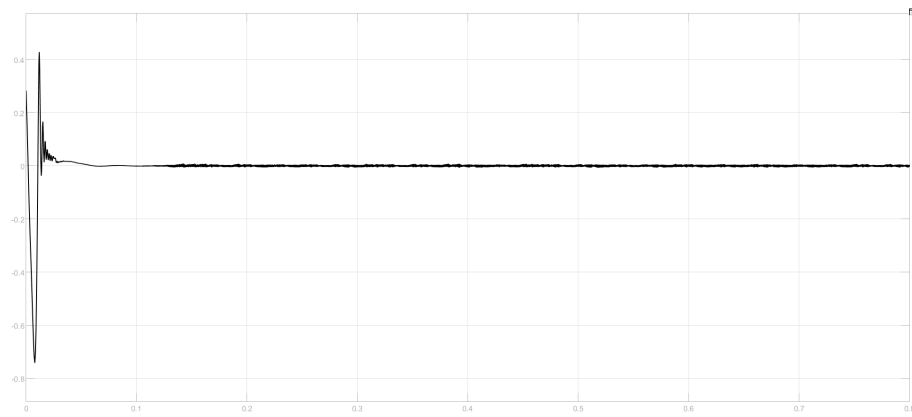
The graph of the rotor flux tracking for $p_1 = -250$



The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$

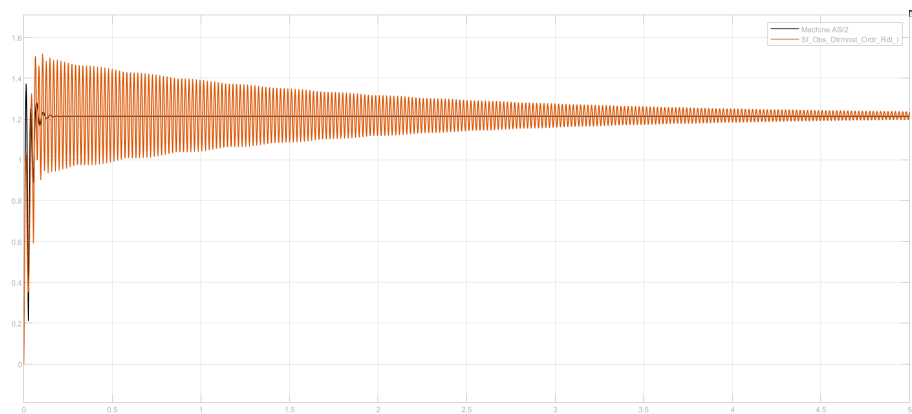


The graph of the rotor flux tracking for $p_1 = -400$

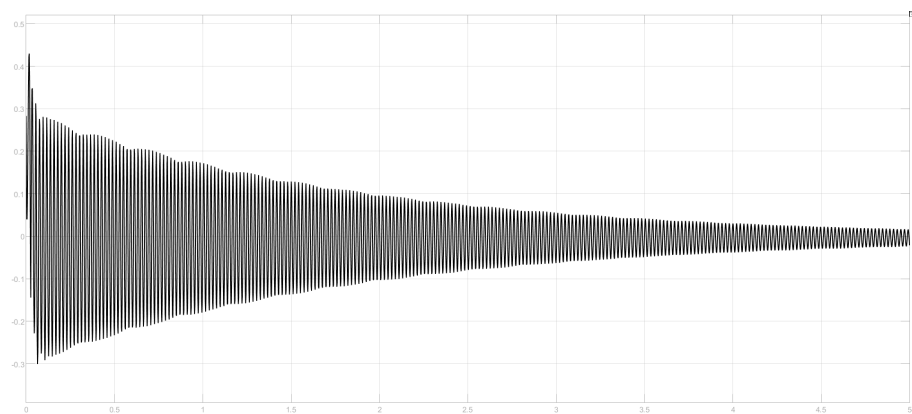


The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$

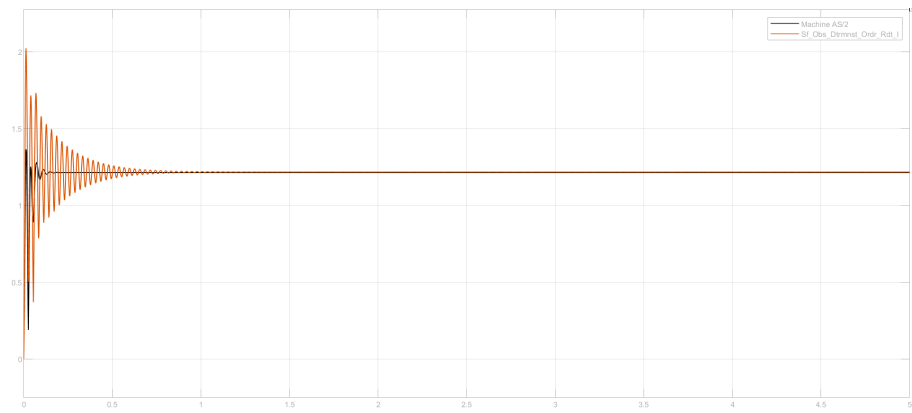
Reduced-order deterministic observer with a diagonal gain matrix fixing the imaginary part



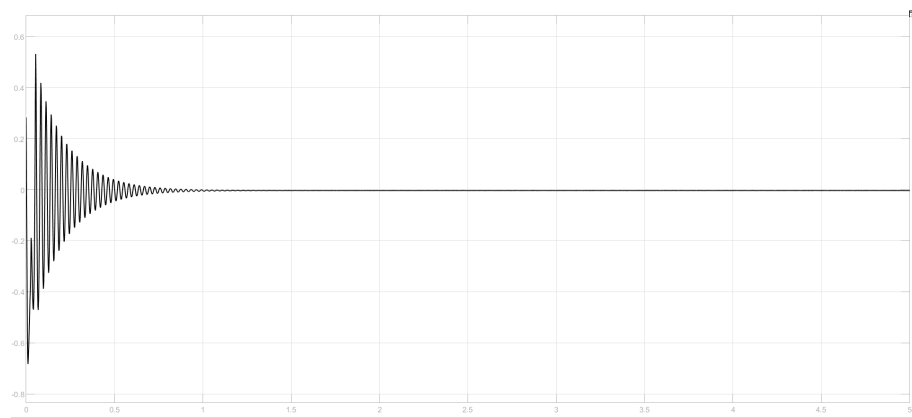
The graph of the rotor flux tracking for $p_1 = -10$



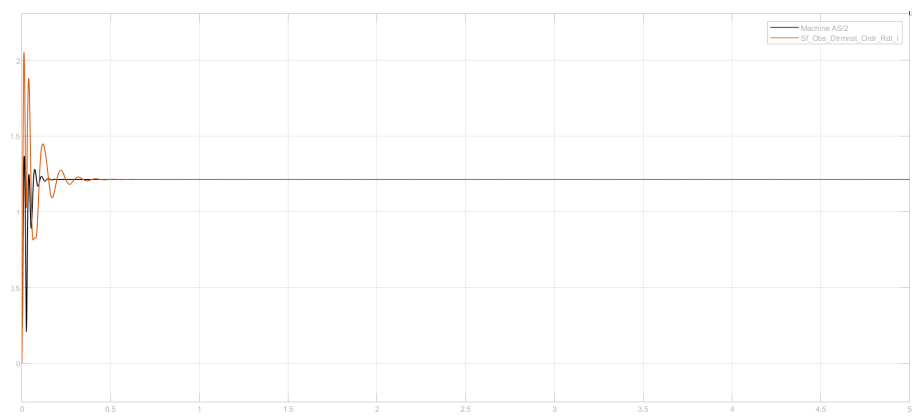
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



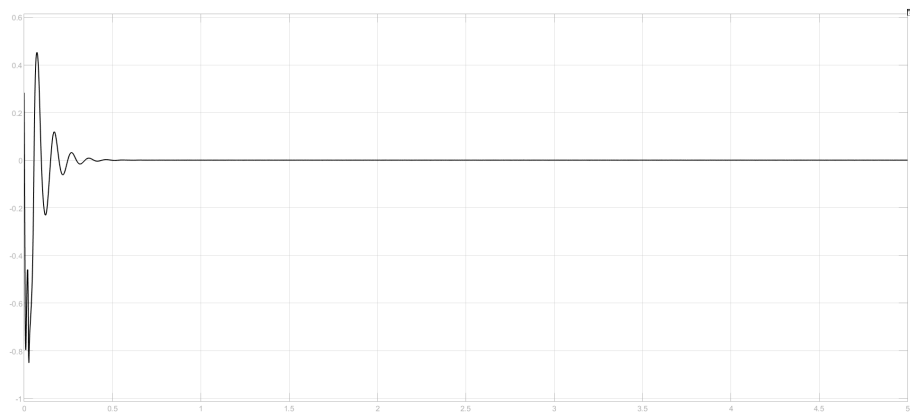
The graph of the rotor flux tracking for $p_1 = -100$



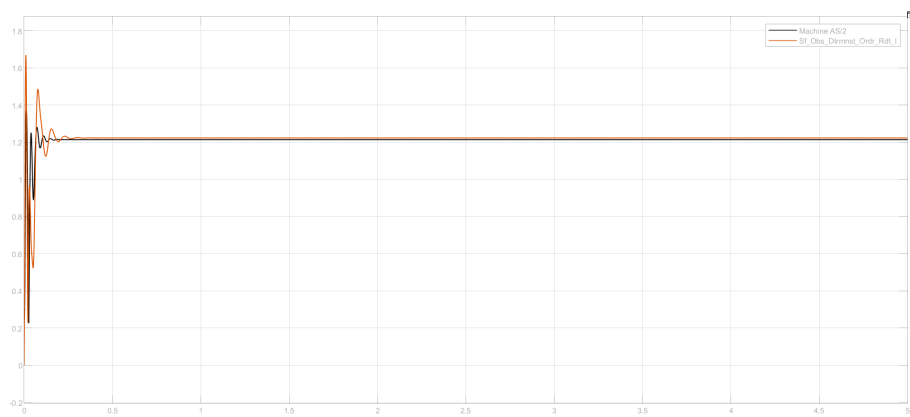
The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



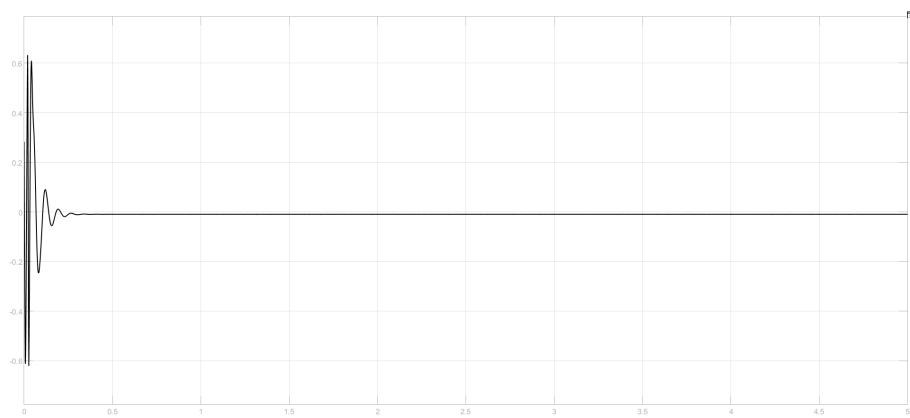
The graph of the rotor flux tracking for $p_1 = -250$



The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$



The graph of the rotor flux tracking for $p_1 = -400$



The graph of the Rotor Flux Estimation Error $\Delta(\psi_r)$

Appendix B: Simulation Results of the Induction Machine

The following simulation results were obtained using the mathematical model of the induction machine and the parameter set identified experimentally using the classical identification method. The identified values of the electrical and mechanical parameters were introduced into the Simulink model in order to reproduce the actual behavior of the machine as accurately as possible. The figure below represents the Simulink model used for the simulation:

The presented figures illustrate the evolution of the main electrical and mechanical variables and serve as complementary results to those discussed in the main body of the thesis.

Figure 17: Stator Current I_{sa}

The stator current I_{sa} exhibits a high-amplitude transient at startup due to the establishment of the magnetic flux and rotor acceleration. The oscillations are progressively damped, and the current reaches a steady sinusoidal waveform after approximately 0.125 s. This behavior confirms the stable operation of the induction machine under the considered operating conditions.

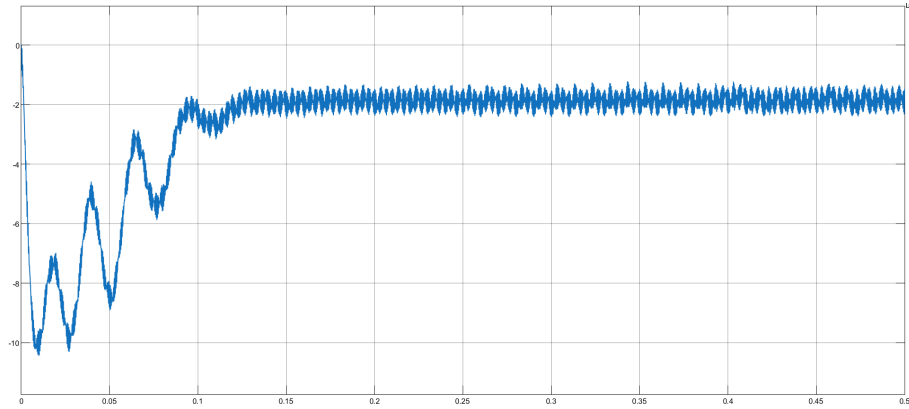


Figure 18: Stator Current I_{sa}

The direct-axis stator current I_{sd} exhibits a transient response at startup, characterized by damped oscillations caused by the establishment of the machine fluxes. The current gradually converges to a steady-state value of approximately -2 A after about 0.15 s, with only small residual ripples. This behavior indicates stable operation and proper flux establishment in the induction machine.

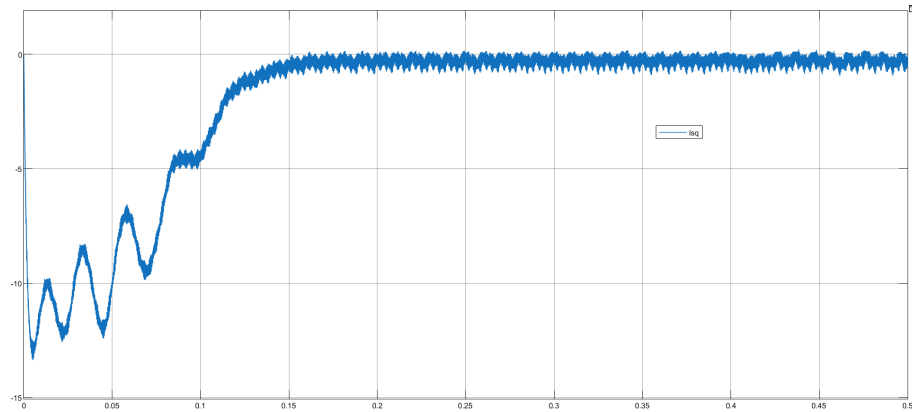


Figure 19: Stator Current I_{sq}

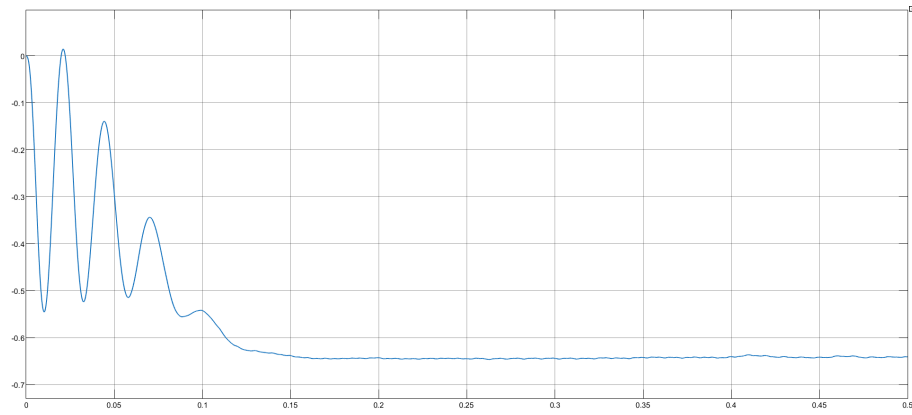
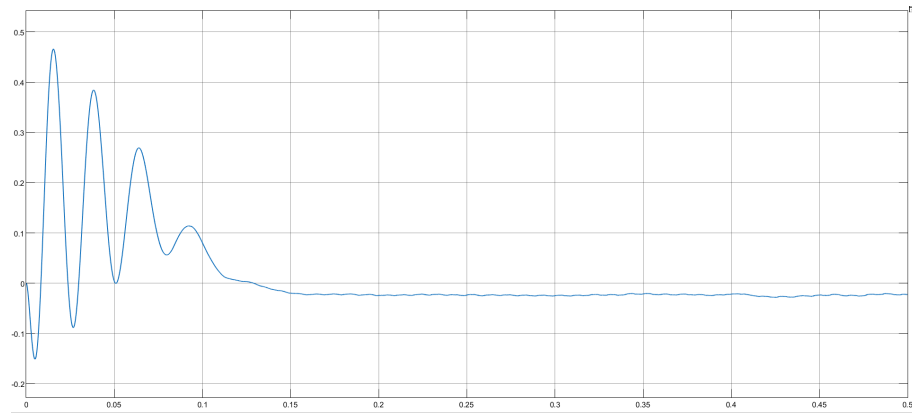
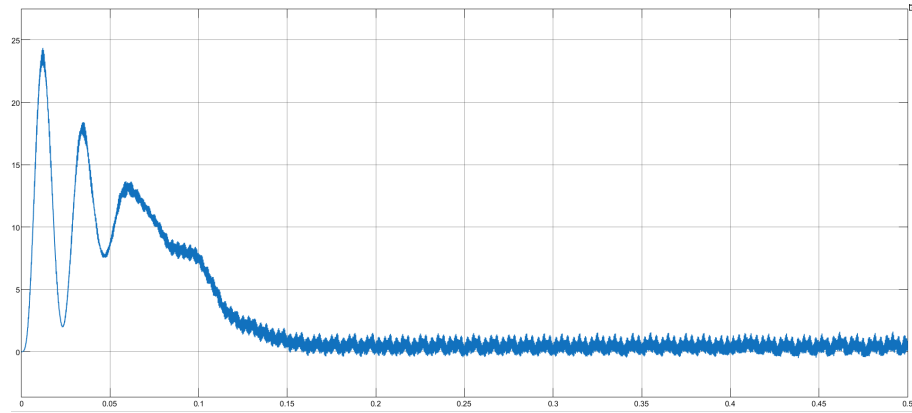
Figure 20: Rotor Flux ψ_{rd}

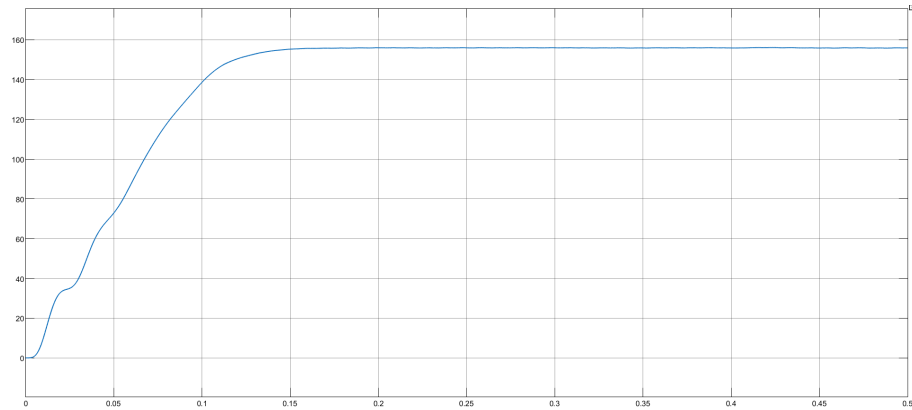
Figure above shows the evolution of the rotor flux ψ_{rd} . At startup, the flux exhibits damped oscillations due to the transient operating conditions of the induction machine. The oscillation amplitude gradually decreases, and the flux converges to a steady-state value of approximately -0.64 Wb. This behavior indicates a stable response and good convergence of the system.

Figure 21: Rotor Flux ψ_{rq}

The figure abovej) shows the evolution of the quadrature-axis rotor flux ψ_{rq} . The signal exhibits damped oscillations during the transient phase before converging rapidly to a value close to zero. The decrease in oscillation amplitude confirms the stability of the system and the good dynamic behavior of the rotor flux.

Figure 22: Electromagnetic Torque T_e/C_{em}

The figure above presents the evolution of the electromagnetic torque during the startup of the induction machine under no-load conditions. A high transient torque is produced at startup in order to accelerate the rotor from standstill. As the rotor speed increases, the electromagnetic torque gradually decreases and exhibits damped oscillations before reaching a steady-state value close to zero. This behavior is expected since the load torque is null ($C_r = 0$) and the machine only compensates for internal losses and friction.

Figure 23: Rotor Angular Speed ω

The figure above presents the evolution of the rotor angular speed during no-load startup. The speed increases progressively from zero and converges toward the synchronous speed of approximately 157 rad/s. The transient response is characterized by a settling time of about 0.15 s and negligible overshoot, indicating good dynamic performance and stability of the developed induction motor model. The small residual oscillations observed in steady state are attributed to inverter switching effects and numerical discretization.