The Assessment of the Wavelet Transform Theory: Application to the Electrocardiogram Signal

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Abstract
We present in this work the efficiency of the wavelet transform, in exploring non-stationary signals such as those containing transients and discontinuities, and time varying spectra signals. We examine the non-stationarity problem as well as the alternative solutions suggested to deal with like the time-frequency analysis. In this context, we have applied the continuous wavelet transform –CWT- to a set of classical types of signals showing each a particular feature and to a real signal, which is the electrocardiogram ECG signal to evaluate the CWT efficiency. The obtained results demonstrated the higher ability of the wavelet transform in localizing specified temporal and spectral features of a signal.

Keywords:
Wavelet transform, time-frequency analysis, ECG signal.

1 Introduction

By hearing the tem "signal processing" it comes to mind different ideas: signal analysis, noise removing, filtering, coding, compression... In fact, the signal processing world includes all these techniques as well as other ones. However, a major work of signal processing deals with the signal analysis. We mean by signal analysis representing a signal - more precisely the information carried by that signal - with respect of a set of well known functions or bases.

Joseph Fourier, in 1807, was the first who has demonstrated that a continuous signal f(t), with some specified conditions, can be represented by a set of coefficients with respect to a well known base. He stated that any 2π periodic signal f(t) is the summation of infinite trigonometric functions.

Since that date a new concept of mathematics and signal processing was born : signal representation or analysis with respect specified bases.

Haar, in his thesis in 1909, brought up with a new basic function called Haar basic function, a scale varying function. This basic function was in fact the first "wavelet" that was not called yet. This function was exploited by several searchers mainly in 1930's and showed the improved results compared to Fourier bases in studying some types of signals such as: Paul Levy and Littlewood-Paley technique...[10].

The physicist Gabor, in 1946, introduced the short time Fourier transform –STFT- using elementary time-frequency atoms. This transform had the ability to analyze non-stationary signal whereas it suffers from its fixed time-resolution due to the fixed window support size[12].

In early of 1980's, the theoretical physicist Alex Grossman and the engineer Jean Morlet and their collaborators introduced the wavelet function and launched a new ere of wavelet theory. In about 1986, Stéphane Mallat and Yves Meyer established relationship between the wavelet theory and the multiresolution analysis. Based on this work, Pierre-Gilles Lemarié and Yves Meyer constructed orthogonal wavelet bases in parallel with St?mberg work. In the same context, Ingrid Daubechies, in 1989, constructed a set of compact support wavelet functions with a fixed regularity [8, 12]. Since 1990, the wavelet theory finds various fields of applications in signal processing, mathematics, and engineering....

2 Time frequency problem

2.1 <u>Series expansion of signal:</u>

The series expansion of a signal, or as it has been stated in the introduction the signal decomposition, suggests that a signal f(t) of space V can be expressed as a linear combination of a set of basic complex functions $\psi_i(t)$ as follows [1]:

$$f(t) = \Sigma_i a_i \psi_i(t) \qquad (1)$$

If a set of basic functions $\{\psi_i(t)\}$ is complete, there exists a dual set $\{\varphi_i(t)\}i\in Z$ such as the expansion coefficients in equation (1) can be computed as:

$$a_i = \int \varphi_i(t)\, f(t)\, dt \qquad (2)$$

Energy computation leads to :

$$E = \int_{t2}^{t1} |f(t)|^2 .dt = \sum_{i=0}^{N} |a_i|^2 .\lambda_i \qquad (3)$$

It states that the series expansion of a signal provides a new representation of the original signal but it must preserve the total energy. This expression is the general formula of Parseval's theorem.

2.2 Fourier Analysis –FT-

Though the FT is first example of series expansion in the course of history, it remains a very powerful tool in signal processing and widely used.

If the signal x(t) is of finite energy, its FT X(f) is given by:

$$X(f) = \int x(t)\ e^{-2\pi f t}\, dt \qquad (4)$$

A study of the FT formula shows that :

a) The interpretation of processes that are sometimes different from the reality; switching ON and next OFF an apparatus is a good example of this wrong interpretation. After small time of switching OFF operation the signal –current or voltage- will be null 'static zero'. This nullity is well computed by FT and is interpreted by the superposition of an infinite number of sinusoidal waves 'dynamic zero'.

b) Reconstructing the original time-varying signal depends mainly on the cancellation of the high frequency Fourier coefficients, that is sensitive to high-frequency noise;

c) The FT is perfectly local in frequency whereas it is global in time. It can localize any two very adjacent frequencies, while the time occurrence of these frequency components is completely lost. This limitation arises especially when dealing with the non-stationary signal.

2.5 Duration and bandwidth of a signal

The signal transformations are based on basic functions called windows or supports. These windows must be of finite duration and bandwidth. Unfortunately, this can not be possible since a finite duration signal can not be, any way, of a finite bandwidth and vice versa. A solution consists on neglecting the amplitudes that are beyond a certain threshold of interest. The duration Tu and the bandwidth Bu are defined as the interval within which the most of energy is distributed.

The finite center Tc of a finite duration signal is given by [1, 10]:

$$T_c = \frac{1}{E}.\int t.|x(t)|^2 .dt = \frac{1}{\|x\|^2}.\int t.|x(t)|^2 .dt \qquad (5)$$

The finite duration ΔT is given by:

$$\Delta T = \frac{1}{E}.\int (t-T_c)^2 .|x(t)|^2 .dt = \frac{1}{\|x\|^2}.\int (t-T_c)^2 .|x(t)|^2 .dt \qquad (6)$$

The value Tc determines the mean position of a signal on the time axis while the quality Tu or ΔT is the measurement of the expanse of the signal around the center Tc.

Similarly for the spectral case, the frequency center Fc and the bandwidth Bu or ΔF are given by:

$$F_c = \frac{1}{E}.\int f.|X(f)|^2 .dt \qquad \text{and}$$

$$\Delta F = \frac{1}{E}.\int (f-F_c)^2 .|X(f)|^2 .dt \qquad (7)$$

The finite centers Tc and Fc are the temporal and spectral localization's of a signal while the radii ΔT and ΔF are the temporal and spectral resolutions of the transform. Obviously, a "good" transform requires simultaneous small time and frequency resolutions. Unfortunately, this in not possible due to the uncertainty principle (or Heisenberg inequality) stating that : $\Delta T * \Delta F \geq 1/(4\pi) \cong 0.08$. The only solution, using a particular window support, is to trade time resolution for the frequency resolution or vice versa.

2.6 Windowed Fourier transform

The drawbacks of the FT stated in paragraph 2.2 have arisen new concepts such as finite time localization rather than the global time localization of the FT. Gabor, in 1945, introduced a translated modulated time-varying window g(t), with a finite temporal support ΔT, given by [12]:

$$g_{p,s}(t) = g(t-p)e^{-ist} \qquad (8)$$

Its spectrum expression is :

$$G_{p,s}(w) = G(w-s).\ e^{-ip(w-s)} \qquad (9)$$

The windowed Fourier transform, known as short time Fourier transform, defined by Gabor is the inner product of the time-varying signal f(t) and the set of modulated shifted windows $g_{p,s}(t)$, given by :

$$STFTf(p,s) = \langle f(t), g_{p,s}(t)\rangle = \int f(t)\, g^*_{p,s}(t)\, dt$$
$$= \int f(t)\, g(t-p)e^{-ist}\, dt \qquad (10)$$

This is the FT of the truncated portion of the original signal f(t) - f(t) g(t-p) –centered at the time occurrence p+kΔT with k is an integer number.

Conversely, by applying the Fourier Parseval formula, STFTf(p,s) can be expanded as :

$$STFTf(p,s) = (1/2\pi) \langle F(w), G_{p,s}(w)\rangle$$
$$= (1/2\pi) \int F(w)\, G^*_{p,s}(w)\, dw \qquad (11)$$

This leads, by using equation 12, to:

$$STFTf(p,s) = (1/2\pi)\ .\ e^{-ips} \int F(w)\, G_{p,s}(w-s)\, e^{-ipw}\, dw \qquad (12)$$

This expression can be viewed as the inverse FT of the truncated portion of the original signal spectrum F(w)

–F(w) $G_{p,s}$(w-s)- centered at the frequency localization s + k.ΔF with k is an integer number.
These two remarks yields to state that the STFT is a simultaneous time-frequency of a signal whose basic functions or atoms, as it is shown on figure 1, are rectangles of length of ΔT and height of ΔF related to the basic function window g(t) sizes.

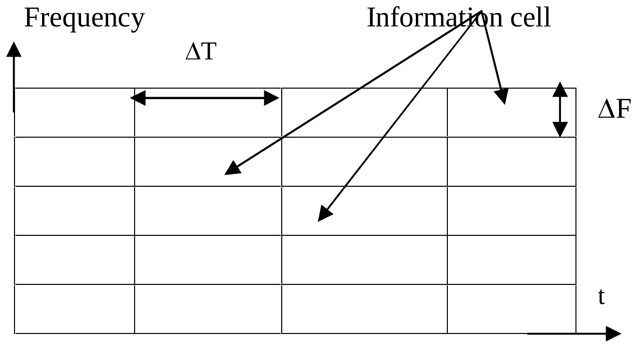

Figure 1 : time-frequency view of the signal –STFT-.

STFT provides time localization of a signal while at the other hand it degrades the frequency localization due to the convolution process. The two parameters (ΔT and ΔF) depend firmly on the window and its spectrum sizes. They mean that a signal can not be represented as point in the time-frequency plan; its position can be only determined within a rectangle of ΔT*ΔF [16].
The STFT is the FT analysis of a non-stationary time-varying signal that is divided into a sequence of fixed size segments in which the signal is assumed to be quasi-stationary. Obviously, the major limitation of the STFT is the "fixed" time-frequency resolution. This obligated the signal analysts to bring up with a more "flexible" approach to overcome this limitation.

3 Wavelet analysis

3.1 <u>Wavelet function</u>

Morlet and Grossman introduced a finite support function called the wavelet ψ(t) or the "mother" wavelet. Scaling and shifting, the mother wavelet, respectively, with a factor of s and τ generates a family of functions called wavelets given by [8, 16]:

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\psi(\frac{t-\tau}{s}) \qquad \text{with } s>0 \qquad (13)$$

As a basic constraint on the wavelet function, it has to be localized both in time and frequency domains, i. e. it has to be of finite temporal and spectral supports. This is given by:

$$\int |\psi(t)| \, dt < \infty \quad \text{and} \quad \int |\Psi(f)| \, df < \infty \qquad (14)$$

where Ψ(f) is the FT of ψ(t).
The term $1/(s)^{1/2}$ guarantees the energy normalization:

$$\int |\psi_{s,\tau}(t)|^2 \, dt = \int |\psi(t)|^2 \, dt = 1 \qquad (15)$$

so all the wavelets -$\psi_{s,\tau}(t)$- of the same family -ψ(t)- have the same energy and the same shape.
In contrast to the CFT, that requires the trigonometric functions as bases, the wavelet function has not a standard basis function. The constructed wavelet must satisfy two additional properties that are : the admissibility and the regularity. The admissibility condition allows the signal reconstruction while the regularity condition implies a quick decrease of wavelet coefficients with decrease of scale [16].

3.2 <u>The Continuous Wavelet transform CWT</u>

The CWT is the inner product of the time-varying signal x(t) and the set of wavelets $\psi_{s,\tau}(t)$, it is given by:

$$Wx(s,\tau) = < x, \psi_{s,\tau} > = \frac{1}{\sqrt{s}} \int x(t)\psi^*(\frac{t-\tau}{s})dt \qquad (16)$$

using Parseval's identity, CWT can be written, also, as:

$$Wx(s,\tau)=(1/2\pi).<X(w), \Psi(w)> \qquad (17)$$

Where X(f) and Ψ(f) are the FT of x(t) and ψ(t) respectively.
Applying FT properties:

$$Wx(s,\tau) = (s)^{1/2} \int X(w)\, \Psi^*(sw)\, e^{iw\tau} \, dw \qquad (18)$$

The CWT is a simultaneous temporal and spectral analysis of the time varying signal. In fact, the CWT is a time-scale representation of a signal however it can be considered as time-frequency analysis due the inverse proportionality between the scale and the frequency.
We have mentioned that the wavelet temporal function and its spectrum are of finite support. This yields to finite temporal and spectral centers and radii. Mathematical computations lead to that the CWT with specified scale s_0 and shift τ_0 picks up the information about x(t) within the time interval [τ_0 +(s_0.T_c)-(s_0.ΔT), τ_0 +(s_0.T_c)+(s_0.ΔT)] and the frequency –scale- interval [(Fc-ΔF)/s_0, (Fc+ΔF)/s_0]. These two intervals determine the time-frequency window (or the information cell) sizes having temporal support of (2.s_0.ΔT) and spectral support of (2.ΔF/ s_0) and centered at temporal occurrence (τ_0 + (s_0.T_c)) and frequency position (ΔF/ s_0). It is obvious that the information cell sizes of the CWT are governed by the scale s_0 and shift τ_0 values. In summary, the CWT is simulated to microscope where the scale and the shift values represent the zoom (in, out) and the position of the picked image respectively. This zooming (in and out) procedure permits analyzing specified portions of the signal with different scales.

3.3 <u>Wavelet applications</u>

To evaluate the wavelet transform theory we have applied the CWT to two cases of signals: the first one representing a slight discontinuity and a sharp transient; and the second case is a superposition of two

linear chirp functions of the form f(t)= $\exp(j\pi\propto_1 t^2) + \exp(j\pi\propto_2 t^2)$ with α_1=250 and α_2=50.
Figures 2 and 3 show: (a) the temporal representation of the studied signal, (b) the time-scale analysis of the signals by applying the CWT Matlab command [13].

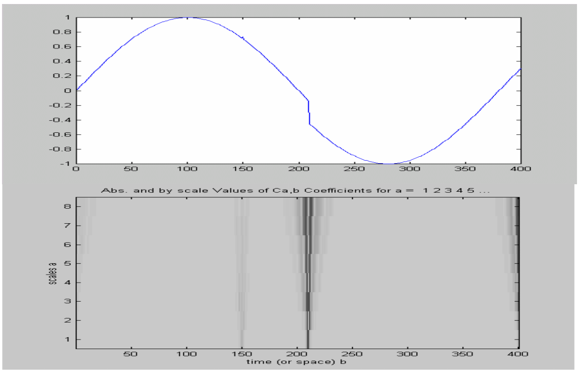

Figure 2 : Application of the CFT (middle) and CWT (bottom) to transient signal (top)

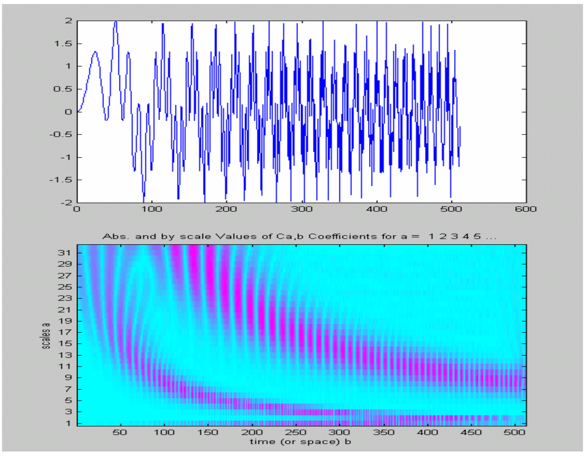

Figure 3 : Application of the CFT (middle) and the CWT (middle) to the chirp signal (top)

For the first case, the CWT picks up obviously the discontinuity at the temporal sample 150 and the sharp transient at the temporal sample 210. For the second case, by using the Morlet wavelet function, the CWT can distinguish obviously the two slopes α_1 and $\alpha_{2.}$

4 ECG and Wavelet theory

As its name indicates, the electrocardiogram –ECG- signal is the electrical picked-up signal of the heart activity. A normal ECG signal consists of a set of well known waves: P, Q, R, S, and T waves (see figure 4). The P wave is associated with the atria depolarization –or activity-; it is characterized by its lower amplitude value and a typical duration from 0.05 to 0.08 sec. The QRS complex is associated to the ventricle depolarization, it is the most prominent feature of the ECG signal due to the important amplitude of the R wave. The T wave is associated to the ventricle repolarization –the rest state-, it is characterized by its largest duration typically from 0.12 to 0.24 sec.
A satisfactory analysis of the ECG signal implies accurate measurements of waves boundaries in addition to some important intervals that are RR, PQ, and QT intervals and the ST segment, [2, 17].
In practice, the real ECG signal is corrupted by several types of noise such as: the EMG, muscle artifacts, 50 Hz powerline interference, the base line wandering... The non-stationary behavior of the ECG signal, that becomes severe in the cardiac anomaly case, obligated biomedical engineers to analyze the ECG in both time and frequency simultaneously. The ability of the wavelet transform to explore signals into different frequency bands with adjustable time-frequency resolution makes it a suitable tool for the ECG signal analysis and processing. Two major areas of wavelet theory applications to ECG signal are distinguished: ECG signal analysis with no requiring signal synthesis and wavelet coefficients treatment requiring signal reconstruction [15]. We state in the first case : ECG analysis [11], ECG waves detection [17], ventricular late potentials detection [14], and ST segment analysis in ischemic [6]. In the second case, we state mainly ECG compression [9], and ECG denoising [5].

5 Results and discussion

The main purpose of this work is to study the effect of wavelet transform application to the ECG signal. In this context, we have applied the CWT to a set of ECG records each with different signal-to-noise ratio SNR level. We are organized our work as follows: first, we applied the CWT to a high SNR ECG signal and next to ECG signals with weak SNR levels and base-line wandering. The first step is considered as a “learning” phase that allows the ECG waves localization; this procedure permits discriminating the useful ECG information from the corrupted noise discrimination.
The analyzed ECG signals are sampled at the rate of 200 HZ with non a-priori filtering. Figures 5, 6, and 7 show the CWT, obtained by applying the CWT Matlab command, of the ECG signals; parts (a) show the real ECG signal while parts (b) show the CWT coefficients.
The CWT coefficients are gray-scale values, i.e. darker area indicates a high correlation of the ECG portion and the wavelet with the specified scale and shift parameters and vice versa.

5.1 CWT of high SNR ECG signal

Figure 5 shows the CWT coefficients of a high SNR ECG signal. The resulting figure shows mainly the time occurrence of the R wave characterized by a set of parallel darker spikes along the different scales due to its sharp edges, whereas the T waves start to be visible at scale 10 due to its larger duration whereas detecting the P wave is detected hardly due to its low amplitude.

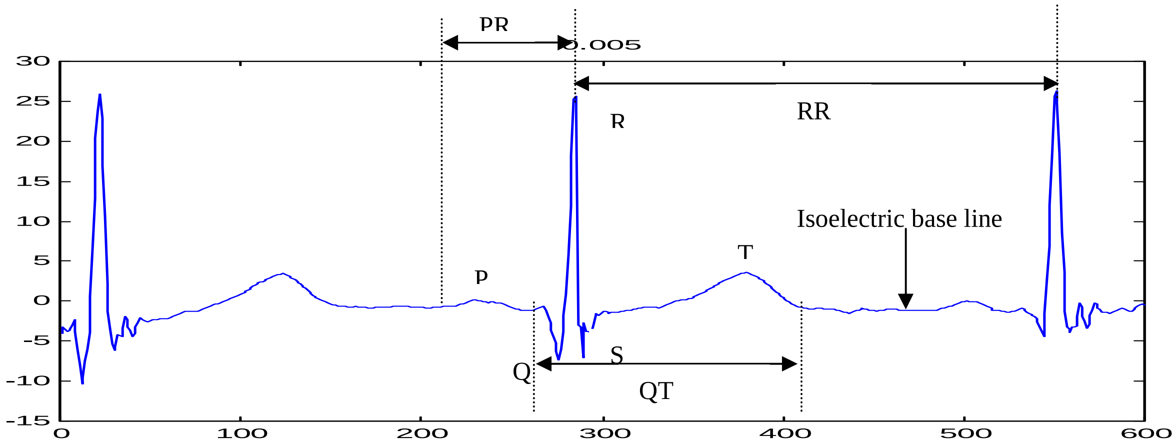

Figure 4: A real ECG signal with its features

This information is of a great importance when we deal with noisy ECG signals. Since we are interested in this work to detect the ECG waves, different experiments show that a lower duration wavelet, which is 'db3', is well suitable for this task.

5.2 CWT of medium SNR ECG signal:

Figure 6 shows the CWT of an ECG signal with two different SNR levels: a medium SNR portion of ECG signal from the 1st sample to around the 1200th sample, and a high SNR portion of ECG signal from around 1200th sample to the end of the record. Comparing the CWT of the two portions indicates clearly the noise that is captured by the CWT as low scale –high frequency- signal mainly from scale 2 to scale 5

-in fact the frequency band [50, 20] Hz. At the other hands, the R and T waves remain detectable even in the case of low amplitude 5th and 7th R waves (from left to right of the record).

5. 4 CWT of ECG with BLW

figure 7 shows the CWT of an ECG signal with a great BLW noise. Referring to the characteristics of a normal ECG, the base-line is iso-electric and flat. Unfortunately, one of the major problems encountered in real ECG signal analysis is the presence of the base line wandering noise where the base-line will be no more flat and stable, as it is shown on the figure below, which is due to respiratory and patient movement.

Different approaches are established to remove this noise, we state mainly the high pass filtering and the derivative process [2] whereas the major drawback of these techniques is that the overall ECG shape will be changed that prevents the adequate ECG components extraction [4]. The figure below shows obviously that the CWT cancels this noise, BLW, which allows accurate localization of R and T waves. This can be explained by the fact that the BLW is a DC offset component where referring to the admissibility property of the wavelet function this DC component is rejected.

Conclusion

We have presented in this work the assessment of the wavelet transform theory in the signal processing world mainly in the electrocardiography field. We have shown the efficiency of the CWT to explore the complicated signals and detect the hidden features that are invisible using different techniques such as the FT and band pass filtering. To evaluate practically the wavelet transform, we have applied the CWT to real ECG signals with different SNR levels. The obtained results show obviously the powerful ability of the wavelet transform in discriminating the useful ECG signal information from the undesired corrupted noise. As conclusion, the obtained results can be exploited in ECG waves localization in the presence of huge noise or the BLW and in designing filters with required cut-off frequencies.

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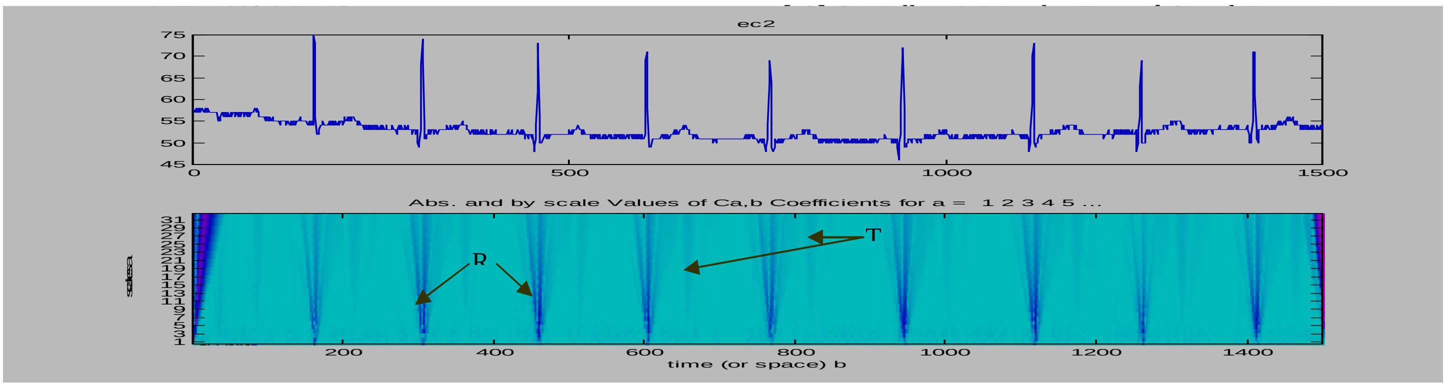

Figure 5 : The CWT of a high SNR ECG signal

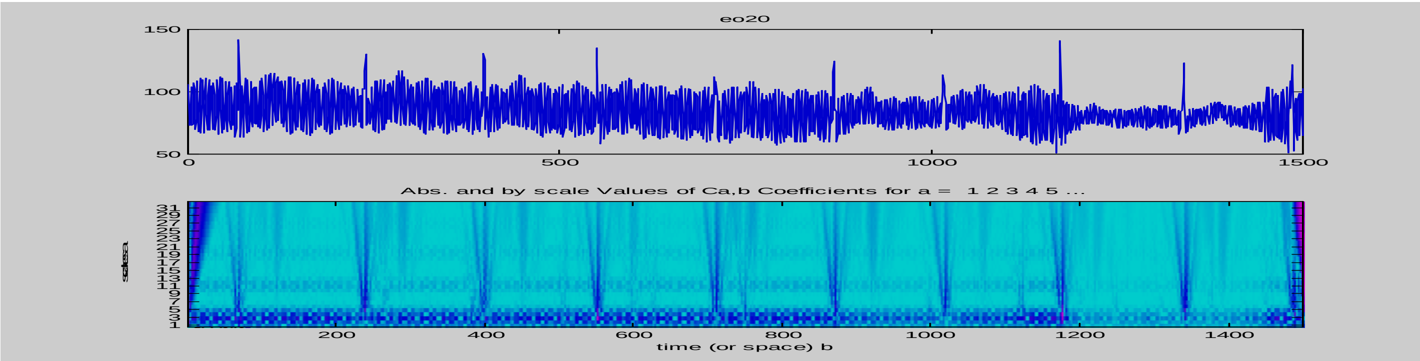

Figure 6: The CWT of real ECG signal with medium and high SNR

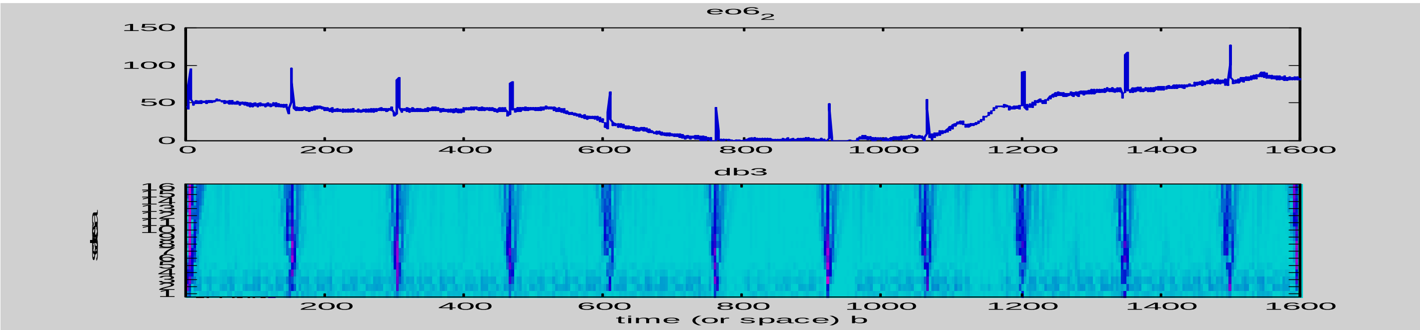

Figure 7 : The CWT of an ECG signal with BLW noise

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