A Dispersions Based Methodology for Computer Aided Simulation of Assembly Functional Requirements

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Abstract

A methodology for automating the simulation of the functional requirements and the synthesis of the functional dimensions during the design stage is modelled step by step using the dispersions method introduced by Bourdet and the minimal transfer method of Duret. This methodology is constructed around three procedures. The first procedure executes the verification of the functional requirements by automatically extracting the functional dimension chains in the mechanical sub-assembly. Then a second procedure performs an optimization of the dispersions on the basis of unknown variables. The third procedure uses the optimized values of the dispersions to compute the optimized average values and tolerances of the functional dimensions in the chains.

1. Introduction

Industrial experience shows that the allocation of tolerances to individual parts in mechanical design often requires simultaneous engineering product /process and co-operative work between design and manufacturing engineers. This tolerancing phase leads to significant choices which influence the manufacturing process and the cost of manufacturing the parts. Indeed, the decisions taken during this phase of design induce almost 70% of the total cost of producing the parts [1]. For that reason it is necessary to optimize the manufacturing means and to produce according to the functional requirements by checking the capabilities of the available means in the workshop. The objective of the optimization of design tolerances is to minimize the total cost of manufacture of all the tolerances during production. Many researchers treat only   design tolerances   [2] or manufacturing tolerances [3] separately. They often use assembly simulation by the Monte Carlo method which proved to be greedy in computing time. The synthesis of the tolerances is a more complex problem than the analysis of the tolerances. It aims at finding the values of the various tolerances taking part in the achievement of a functional requirement, by optimizing the total cost of the production. The respect of the functional requirement is taken as a constraint of the optimization problem. The objective function requires to choose a model of production   cost according to the tolerance to be produced. The models used are always empirical models which give only a rough idea of the production cost [4,5]. Some tolerance synthesis models integrate the capability parameters of the manufacturing processes in the constraints of the optimization problem in terms of statistical standard deviations [6]. But the majority of these models do not integrate these parameters in terms of manufacturing dispersions and in any case do not permit the synthesis of optimised average values of design dimensions. The method of dispersions called Δl method introduced by Bourdet [7] represents an effective method of integration of the capability parameters in term of dispersions in the process plan simulation [8,9] for the synthesis and optimisation of manufacturing dimensions and tolerances.
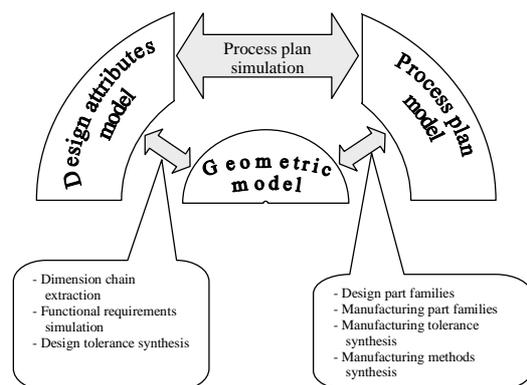

Figure 1. Simultaneous tolerance synthesis

This paper presents a model for the simulation of functional requirements of a mechanical assembly and the synthesis of optimised design dimensions and tolerances

by the Δl method. In this work, the method of dispersions is combined with the method of the minimum transfer introduced by Duret [10] to automate the procedure of the verification of the functional conditions and the extraction of the functional dimension chains by matrix algebra. The optimization of dispersions is carried out starting from a matrix of unknown dispersions [1] in place of the matrix of minimal reference dispersions. The optimized dispersions are used to calculate the tolerance intervals and the lengths of simulation are used to synthesize the average values for the functional dimensions.

2. Simultaneous tolerance design

New design processes of simultaneous engineering are under development. They are the concurrent engineering processes where all the engineering actors work simultaneously on a product. In these processes, the fact of integrating upstream the manufacturing analysis in the design stage of a product, should make it possible to optimize the tolerances and a better quality assurance of the finished products. This implies to have tools for tolerance analysis and synthesis which must integrate in their definition the functional aspects of design and the stochastic aspects of manufacture and inspection at the same time. The functional dimensioning and tolerancing tool naturally federates the technical data to ensure the functional requirements of the products. It becomes obvious that its integration in CAD/CAM systems is essential for the global definition of the product numerical model. Mastering the functional dimensioning and tolerancing tools with simultaneous engineering and co-operative work [11] as shown in figure 1 eliminates any source of incompatibility between the design activities upstream and manufacturing activities downstream. Thus, the use of such tools avoids any loss of information and minimizes scrap parts as a consequence.

3. Simulation of the functional requirements

3.1. Dispersions model

During machining the length L of a part in batch production under the same conditions and a given tool adjustment, it is always noticed a dimensional variation of the parts. The scatter given by the successive values recorded for the batch of parts between the largest and the smallest dimensions is called total dispersion Δl and is given as :

$$\Delta l = L_{max} - L_{min} \qquad (1)$$

The Δl model introduces the concept of simulation lengths L_i which makes it possible to locate, in each

reference datum of the production, the reference surface and various machined surfaces. Δl_i is the allowed dispersion of a simulation length L_i representing the variation of the location of surface i in the fixed reference system of the production machine tool. Relations are then established between the machined dimensions and the simulation lengths. Since the functional dimensions are obtained by the machined dimensions of the finish operations, the simulation lengths are used to model the average dimension values as follows:

$$(Cf_{i,j})_{moy.} = L_j - L_i \quad \text{with} \quad j > i \qquad (2)$$

where i and j are indices of the bound surfaces.

In order to model the simulation of a mechanical assembly of parts, we consider the simplified sub-assembly example of figure 2 [9]. This assembly is constructed as a matrix of dispersions assigned to surfaces of the parts. After that, matrix algebra is carried out for each functional requirement of the assembly by the minimal transfer method [10].
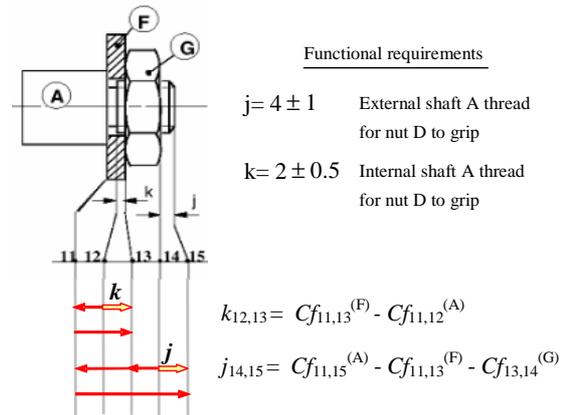

Figure 2. Sub-assembly sample example

3.2. Chain extraction and verification procedure

The assembly is constructed in the form of a matrix of Is columns and Ip lines. Is represents the index of surfaces and Ip represents the index of parts. As the first table of figure 3 shows, element $A_{Is,Ip}$ of the matrix contains a value of dispersion only when surface Is belongs to part Ip as a terminal or a contact surface, otherwise it is null. Then a verification procedure of the design functional requirements is carried out using the minimal transfer method as outlined in figure 4. When the condition of minimal transfer is satisfied, the design functional dimensions participating in the design functional requirement are those bounded by surfaces

having the two dispersions stationed on the same line (same part). Thus, we obtain all the functional dimensions in the dimension chain for every functional requirement.
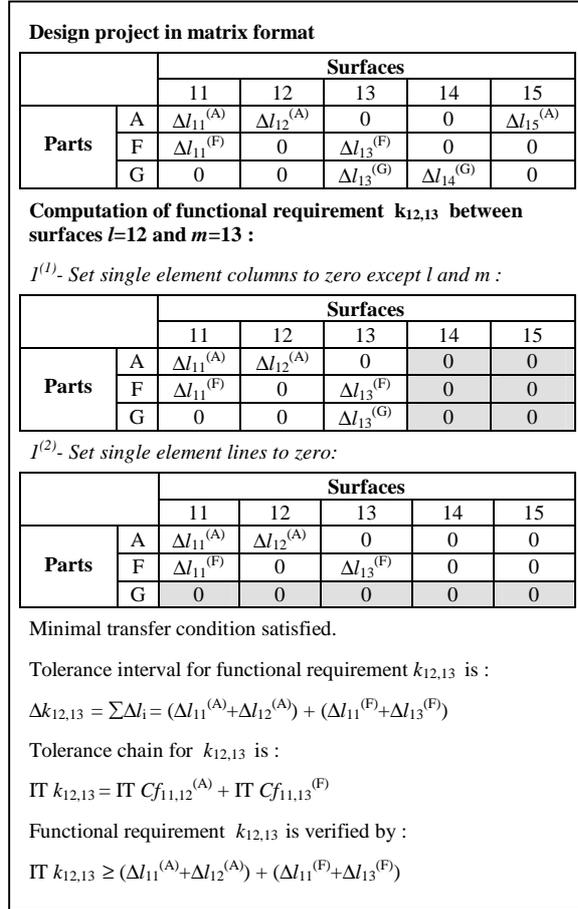
Design project in matrix format

		Surfaces				
		11	12	13	14	15
Parts	A	$\Delta l_{11}^{(A)}$	$\Delta l_{12}^{(A)}$	0	0	$\Delta l_{15}^{(A)}$
	F	$\Delta l_{11}^{(F)}$	0	$\Delta l_{13}^{(F)}$	0	0
	G	0	0	$\Delta l_{13}^{(G)}$	$\Delta l_{14}^{(G)}$	0

Computation of functional requirement $k_{12,13}$ between surfaces $l=12$ and $m=13$:

$I^{(1)}$- *Set single element columns to zero except l and m :*

		Surfaces				
		11	12	13	14	15
Parts	A	$\Delta l_{11}^{(A)}$	$\Delta l_{12}^{(A)}$	0	0	0
	F	$\Delta l_{11}^{(F)}$	0	$\Delta l_{13}^{(F)}$	0	0
	G	0	0	$\Delta l_{13}^{(G)}$	0	0

$I^{(2)}$- *Set single element lines to zero:*

		Surfaces				
		11	12	13	14	15
Parts	A	$\Delta l_{11}^{(A)}$	$\Delta l_{12}^{(A)}$	0	0	0
	F	$\Delta l_{11}^{(F)}$	0	$\Delta l_{13}^{(F)}$	0	0
	G	0	0	0	0	0

Minimal transfer condition satisfied.

Tolerance interval for functional requirement $k_{12,13}$ is :

$\Delta k_{12,13} = \sum \Delta l_i = (\Delta l_{11}^{(A)} + \Delta l_{12}^{(A)}) + (\Delta l_{11}^{(F)} + \Delta l_{13}^{(F)})$

Tolerance chain for $k_{12,13}$ is :

IT $k_{12,13}$ = IT $Cf_{11,12}^{(A)}$ + IT $Cf_{11,13}^{(F)}$

Functional requirement $k_{12,13}$ is verified by :

IT $k_{12,13} \geq (\Delta l_{11}^{(A)} + \Delta l_{12}^{(A)}) + (\Delta l_{11}^{(F)} + \Delta l_{13}^{(F)})$

Figure 3. Verification and chain extraction for $k_{12,13}$

3.2.1. Minimal transfer method. As figures 3 and 4 illustrate, the principle of the method is to recognize first the two surfaces which delimit a functional requirement. They are noted *l* and *m*. The verification procedure is carried out for each functional requirement (CC) by successive elimination of the dispersions from single element columns and lines, except for columns *l* and *m*. The elimination process is repeated until the minimal transfer condition given by zero or two Δl per column is reached. The functional dimensions of the parts participating in the chain are then extracted from the surfaces (columns) containing the dispersions present on the corresponding line of the matrix.

3.2.2. Functional requirements verification based on minimal dispersions. The verification of the functional requirements is carried out by checking the feasibility of the assembly design in regards to the capabilities of the available manufacturing processes in the workshop in terms of minimal dispersions [12]. This condition is fulfilled when the manufacturing means can produce the design set dimensions imposed by the design office. In technical terms this condition is satisfied when the tolerance interval (*IT*) of the functional requirement CC is greater or equals the manufacturing stack up tolerance due to the summation of all dispersions Δl_i with reference minimal values and given by equation (3) as :
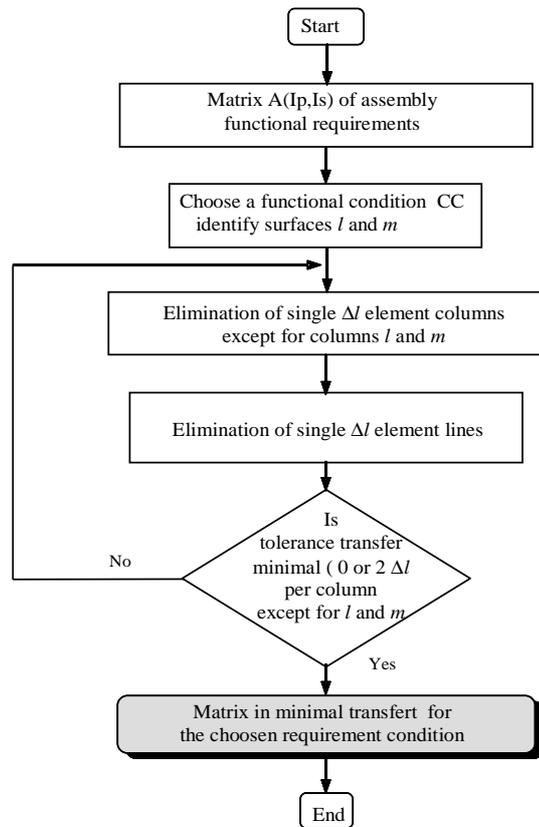
$$IT_{CC} \geq \sum \Delta l_i \qquad (3)$$

Figure 4. Minimal transfer flowchart

3.3. Procedure for the optimization of dispersions

Using the extracted tolerance chains, we build an optimisation matrix of *Ic* lines and *Id* columns where each line represents a functional requirement and each column represents a dispersion. We can notice that each line corresponds to a tolerance chain associated to a functional dimension chain. When a dispersion is present in a chain a variable *x* is affected to the corresponding position in the matrix. Otherwise the position takes the value zero [13].
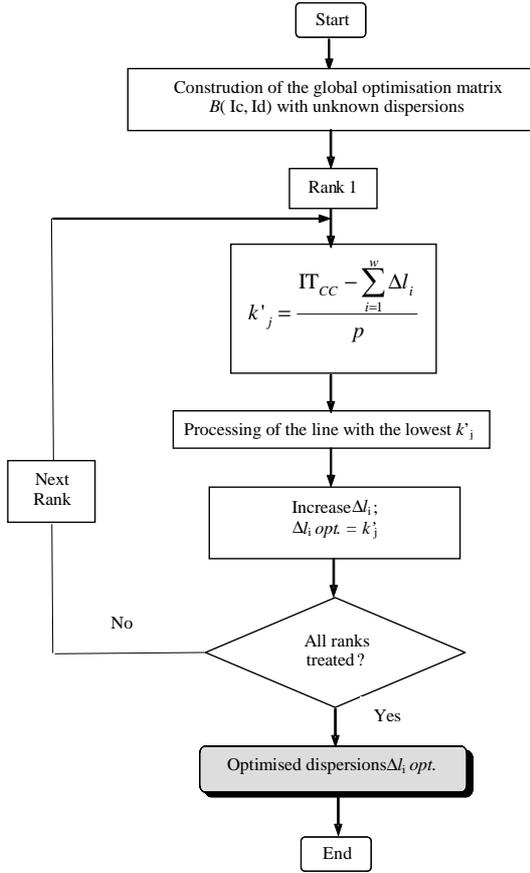
Figure 5. Flowchart of the optimisation

As the flowchart of figure 5 explains, we start by computing a distribution coefficient k'_j for all the lines using equation (4); where w is the number of known dispersions, p the number of unknown dispersions and j the rank number.

$$\left(\Lambda l_i\right)_{\text{opt.}} = k'_j = \frac{\text{IT}_{CC} - \sum_{i=1}^{w}\Delta l_i}{p} \tag{4}$$

Figure 6 explains the optimisation process for the sample example which is carried out by equal distribution for the line with the lowest k'_j. The computed dispersions of the line are then introduced in all the columns where they appear. The process is repeated for the remaining lines using the new Δl_i values.

3.3. Procedure for design dimensions synthesis

The designer generally determines, through classical design computations (material strength, weight,…), the limit values for design dimensions not to be exceeded.

The dispersions method can be used to simulate and determine the optimal values of the functional dimensions which fulfil the functional requirements. Based on the fundamental model developed by Bourdet [7] and on the matrix format of the mechanical assembly, we can calculate the average lengths L_i limited by two Δl_i. For each part, the origin of basic average lengths are taken on the leftmost surface (surface 1, $L_1=0$). Using the functional requirements (CC) and the standard dimensions (CS) we build a system of equations to determine the basic average lengths L_i using relations (5) and (6) as follows :

$$(CC_{i,j})_{\text{moy.}} = L_j - L_i \tag{5}$$
$$(CS_{i,j})_{\text{moy.}} = L_j - L_i \tag{6}$$

Figure 6. Optimisation for the sample example

The CS dimensions are selected to supplement the system among the functional dimensions of standard parts. In the case of unbounded dimensions, CCmoy and CSmoy are calculated by using the optimized dispersions by the following relations :

$$CC_{\text{moy}} = \frac{CC_{\text{min}} + \left(CC_{\text{min}} + \sum\left(\Delta l_i\right)_{\text{opt.}}\right)}{2} \tag{7}$$

$$CS_{\text{moy}} = \frac{CS_{\text{min}} + \left(CS_{\text{min}} + \sum\left(\Delta l_i\right)_{\text{opt.}}\right)}{2} \tag{8}$$

Global design data matrix with optimised dispersions

N°	CC	Δl_{11}(A)	Δl_{11}(F)	Δl_{12}(A)	Δl_{13}(F)	Δl_{13}(G)	Δl_{14}(G)	Δl_{15}(A)	IT	$\Sigma\Delta l_i$
1	$k_{12,13}$	**0.25**	**0.25**	**0.25**	**0.25**	0	0	0	1	1
2	$j_{14,15}$	**0.25**	**0.25**	0	**0.25**	**0.416**	**0.416**	**0.416**	2	2
3	$CS_{11,13}$(F)	0	**0.25**	0	**0.25**	0	0	0	-	0.50
4	$CS_{13,14}$(G)	0	0	0	0	**0.416**	**0.416**	0	-	0.832

Computation of simulation average lengths

5 surfaces $\rightarrow$ 5 lengths to compute $\rightarrow$ 5 equations to solve with $L_{11} = 0$ (origin).

Equations due to bounded functional requirements

Equation 1 : $k_{12,13} = L_{13} - L_{12} = 2$
Equation 2 : $j_{14,15} = L_{15} - L_{14} = 4$

Equations due to single bound conditions

Equation 3 : $CS_{11,13} = L_{13} - L_{11} = 4.25$
Equation 4 : $CS_{13,14} = L_{14} - L_{13} = 10.416$

Equation 5 : $L_{11} = 0$

The following system of equation is constructed :

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} L_{11} \\ L_{12} \\ L_{13} \\ L_{14} \\ L_{15} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 4.25 \\ 10.416 \end{bmatrix}$$

After resolution of the system we get the following lengths:

L_{11}	L_{12}	L_{13}	L_{14}	L_{15}
0	2.25	4.25	14.666	18. 666

Computation of average functional design dimensions :

$Cf_{11,15mov}$	$Cf_{11,12mov}$	$Cf_{11,13mov}$	$Cf_{13,14mov}$
$L_{15} - L_{11}$	$L_{12} - L_{11}$	$L_{13} - L_{11}$	$L_{14} - L_{13}$
18.666	2.25	4.25	10.416

Computation of tolerances for functional design dimensions :

Parts	Functional dimensions	$\sum\Delta l_i$	I T
A	$Cf_{11,12}$(A)	Δl_{11}(A) $+ \Delta l_{12}$(A)	0.5
	$Cf_{11,15}$(A)	Δl_{11}(A) $+ \Delta l_{15}$(A)	0.67
F	$Cf_{11,13}$(F)	Δl_{11}(F) $+ \Delta l_{13}$(F)	0.5
G	$Cf_{13,14}$(G)	Δl_{13}(G) $+ \Delta l_{14}$(G)	0.83

Synthesied functional design dimensions :

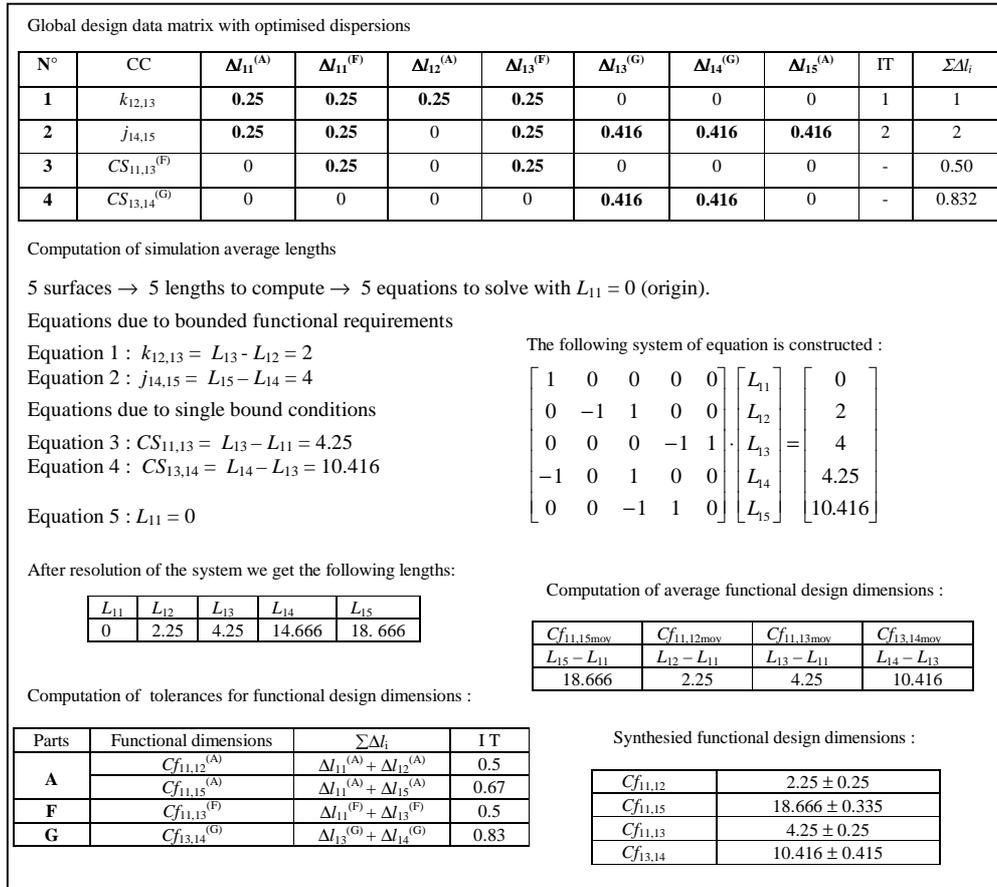| $Cf_{11,12}$	2.25 ± 0.25
$Cf_{11,15}$	18.666 ± 0.335
$Cf_{11,13}$	4.25 ± 0.25
$Cf_{13,14}$	10.416 ± 0.415

Figure 7. Optimal design dimension and tolerance synthesis

Equations (5) to (8) give a system of *n* equations and *n* unknown L_i. When the simulation lengths L_i are computed, we calculate the average functional dimensions using equation (2). Figure 7 illustrates step by step this procedure for the sample example. This procedure is automated in the flowchart of figure 8. For the sample example we have five surfaces so we need five functional requirements. We complete the two functional conditions *k* and *j* with two conditions as minimal dimensions in the dispersions matrix. These are given by the standard parts in the assembly. In our case, we take the nut D ($CS_{13,14}$(G) = 10 mini) and the disc F ($CS_{11,13}$(F) = 4 mini). The global matrix after the optimisation of dispersions using the tolerance intervals of the bounded functional conditions is given for this example in the first table of figure.

4. Applications of the simulation

A simulation module of design mechanical assemblies based on the method of dispersions was programmed on the basis of the methodology previously outlined by the flowcharts of figures 4, 5 and 8. The final program named CAFT (*Computer Aided Functional Tolerancing*) consists of four functions. The first one consists of the preparation of the simulation data of the assembly in matrix form. The three other functions correspond to the three procedures composing simulation. The program was tested and validated on several examples giving results agreeing with the manual simulation of the simple examples and very satisfactory results for the simulation of complex assemblies [14]. The program can be easily integrated into a CAD/CAM system.

Figure 9 gives the output results of the CAFT program run as a separate module on the illustrative example on the basis of an optimisation with minimal dispersions. The program is in a phase of refinement under Delphi for an optimization with unknown dispersions as explained previously in the optimisation section. On the other hand, a communication interface with CAD/CAM systems is under development in order

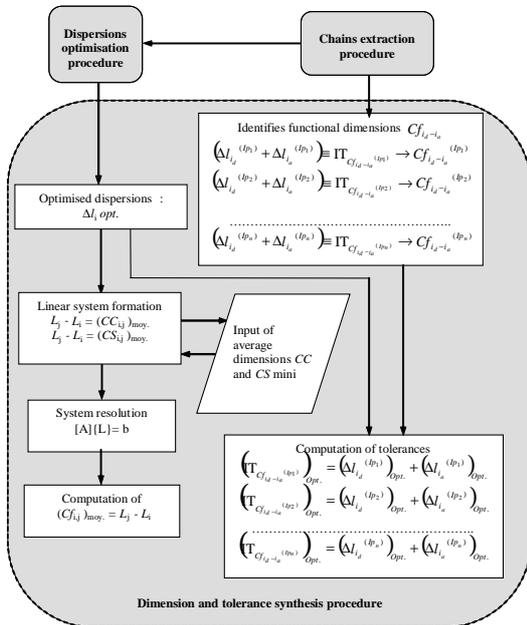to allow the module to extract the simulation data from the mechanical assembly model built in these systems.

Figure 8. Flowchart of the optimisation

5. Conclusions

The objective of this work which is based on the approach of simultaneous engineering is to contribute to the development of a CAD/CAM tool for functional design dimensions synthesis expressed by the average values and the tolerance intervals. The dispersions method known as the Δl method which proved reliable in the simulation of manufacturing dimensions wass used to develop a methodology for the automation of the simulation. This methodology was constructed and tested manually step by step using a simplified example of mechanical assembly. Afterwards, it was automated by the realization of a computer program named CAFT. The program was tested thereafter on the simpler example as well as on complex assemblies which are difficult or impossible to treat manually. The automatic treatment gave results agreeing with the manual treatment for the illustrative example and showed the effectiveness of the automated simulation by solving the complicated examples. The final program can thus be used at will in order to simulate the functional requirements of design projects and to make it possible to choose adequate and optimal average values and tolerance intervals for the functional dimensions among several possible solutions.

6. References

[1] C. Marty and J. M Linares, *Industrialisation des Produits Mécaniques*, Conception et Industrialisation, Publication Hermes, Tome 1, Paris, 1999.

[2] K. W. Chase , and A. R Parkinson, "A Survey of Research in the Application of Tolerance Analysis to the design of Mechanical Assemblies" , *ADCATS*, Report No. 91-1, Brigham Young University, 1991.

[3] G. R. Tang, R. Kung, and J. Y. Chen, "Optimal Allocation of Process Tolerances and Stock Removals", *International Journal of Production Research*, 32(1), 1994, pp. 23-35.

[4] K.W. Chase, W.H. Greenwood, B.G. Loosli, and L.F. Hauglund, "Least Cost Allocation for Mechanical Assemblies with Automated Process Selection", *Manufacturing Review*, 3 (1), 1990, pp.49-59.

[5] G. Zhang, and M. Porchet, "Tolerance Design in CIM, the State of the Art", *Proceeding of the International Symposium on Robotics, Mechatronics and Manufacturing*, IMACS/SICE, 1992, pp1137-114.

[6] O. Bjorke, *Computer Aided Tolerancing*, 2nd Ed., ASME Press, NY, 1989.

[7] P. Bourdet, "Chaînes de Cotes de Fabrication", *Revue de l'Ingénieur et le Technicien de l'Enseignement Technique*, N°191, 1973, pp. 15-23.

[8] B. Anselmetti , *Tolerancement : Cotation de Fabrication et Métrologie*, Edition Lavoisier, Tome 3, Paris, 2003.

[9] P. Bourdet, *Chaînes de Cotes Unidirectionnelles*, ENS Cachan, Paris, 2003.

[10] D. Duret, "Simulation de Gamme d'Usinage", *Revue de l'Ingénieur et le Technicien de l'Enseignement Technique*, 229, 1981, pp. 34-37.

[11] N. Cheikh, A. Cheikh, and S. Hamou, "Modélisation Orientée Produit Fini des Tolérances en CFAO Mécanique", *Conférence Internationale de Mécanique Avancée*, Alger, 2004.

[12] S. Hamou, 2004, *Contribution à l'Optimisation de la Spécification des Etats Intermédiaires dans le Processus de Fabrication*, Thèse de doctorat, Université Abou Bekr Belkaid, Tlemcen, 2004.

[13] S. Hamou, A. Cheikh, J. M. Linares, and A. Benamar, "Machining Dispersions Based Procedures for Computer Aided Process Plan Simulation", *Int. J. Computer Integrated Manufacturing*, 17(2), 2004, pp. 141-150.

[14] N. Cheikh, *Modélisation de l'Optimisation des Tolérances Fonctionnelles par la Méthode des Dispersions*, Master Thesis, Abou Bekr Belkaid University, Tlemcen, 2005.

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          SIMULATION DE L'ASSEMBLAGE Nø2

LES DONNEES DE L'AVANT PROJET

Nombre de Pièces          : 3
Nombre de Surfaces        : 5
Nombre de cotes conditions : 4

Representation matricielle de l'avant projet :
   0.05   0.20   0.00   0.00   0.10
   0.10   0.00   0.10   0.00   0.00
   0.00   0.00   0.20   0.20   0.00

Cotes conditions representees de la maniere suivante :
Numero S.Source S.Cible IT  Bil(1/0)  Ct.Min  Ct.Max
1        2        3      1.00    1      1.50    2.50
2        4        5      2.00    1      3.00    5.00
3        3        4      0.00    0     10.00    0.00
4        1        3      0.00    0      4.00    0.00

Nature des dispersions (Brute ou Non Brute(1/0):) :
0,  0,  0,  0,  0,  0,  0,

VERIFICATION DE L'AVANT PROJET

CALCUL DE L'ECART DE FABRICATION CONCERNANT LA COTE CONDITION
1

Comprise entre les surfaces l= 2  et  m= 3

------------------------------------------------
La matrice apres l'annulation des colonnes à un seul element
sauf l et m
 0.05 0.20 0.00 0.00 0.00
 0.10 0.00 0.10 0.00 0.00
 0.00 0.00 0.20 0.00 0.00
La matrice apres l'annulation des lignes a un seul element
 0.05 0.20 0.00 0.00 0.00
 0.10 0.00 0.10 0.00 0.00
 0.00 0.00 0.20 0.00 0.00
Somme [ 1]=0.45 IT[ 1]=1.00  Reliquat[ 1]=0.55
L avant-projet est verifie pour la cote condition 1

CALCUL DE L'ECART DE FABRICATION CONCERNANT LA COTE CONDITION
2

Comprise entre les surfaces l= 4  et  m= 5
------------------------------------------------
La matrice apres l'annulation des colonnes à un seul element
sauf l et m
 0.05 0.00 0.00 0.00 0.10
 0.10 0.00 0.10 0.00 0.00
 0.00 0.00 0.20 0.20 0.00
La matrice apres l'annulation des lignes a un seul element
 0.05 0.00 0.00 0.00 0.10
 0.10 0.00 0.10 0.00 0.00
 0.00 0.00 0.20 0.20 0.00
Somme [ 2]=0.75  IT[ 2]=2.00 Reliquat[ 2]=1.25
L avant-projet est verifie pour la cote condition 2

CALCUL DE L'ECART DE FABRICATION CONCERNANT LA COTE CONDITION
3

Comprise entre les surfaces l= 3  et  m= 4
------------------------------------------------
La matrice apres l'annulation des colonnes à un seul element
sauf l et m
 0.05 0.00 0.00 0.00 0.00
 0.10 0.00 0.10 0.00 0.00
 0.00 0.00 0.20 0.20 0.00
La matrice apres l'annulation des lignes a un seul element
 0.00 0.00 0.00 0.00 0.00
 0.10 0.00 0.10 0.00 0.00
 0.00 0.00 0.20 0.20 0.00
La matrice apres l'annulation des colonnes à un seul element
sauf l et m
 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.10 0.00 0.00
 0.00 0.00 0.20 0.20 0.00
La matrice apres l'annulation des lignes a un seul element

 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.00 0.00 0.00
 0.00 0.00 0.20 0.20 0.00
Somme [ 3] = 0.40

CALCUL DE L'ECART DE FABRICATION CONCERNANT LA COTE CONDITION
4

Comprise entre les surfaces l= 1  et  m= 3
------------------------------------------------
La matrice apres l'annulation des colonnes à un seul element
sauf l et m
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```
 0.05 0.00 0.00 0.00 0.00
 0.10 0.00 0.10 0.00 0.00
 0.00 0.00 0.20 0.00 0.00
La matrice apres l'annulation des lignes a un seul element
 0.00 0.00 0.00 0.00 0.00
 0.10 0.00 0.10 0.00 0.00
 0.00 0.00 0.00 0.00 0.00
Somme [ 4] =  0.20

TABLEAU D'OPTIMISATION ET DE REPARTITION DES DISPERSIONS
   0.05   0.10   0.20   0.10   0.00   0.00   0.00
   0.05   0.10   0.00   0.10   0.20   0.20   0.10
   0.00   0.00   0.00   0.00   0.20   0.20   0.00
   0.00   0.10   0.00   0.10   0.00   0.00   0.00

OPTIMISATION DES DISPERSIONS
S[1]=  0.450   IT[1]=  1.000   Reliquat[1]=  0.550
S[2]=  0.750   IT[2]=  2.000   Reliquat[2]=  1.250
Rang       Reliquat        Ordre de traitement
----       --------        -------------------
 1          0.550           1
 2          1.250           2

Rang : 1

Avant l'optimisation des dispersions de la cote 1
 0.050 0.100 0.200 0.100 0.000 0.000 0.000
IT de la cote condition ---------------------->=  1.000
Ecart de fabrication concernant la cote condition -----------
->=  0.450
Reliquat concernant la cote condition -------->=  0.550
Nombre de dispersions optimisable concernant la cote
condition ->=  4
La repartition du reliquat -------------------->=  0.137
Apres l'optimisation des dispersions de la cote condition 1
 0.188 0.237 0.338 0.237 0.000 0.000 0.000
IT de la cote condition---------------------->=  1.000
Ecart de fabrication concernant la cote condition ->=  1.000
Reliquat concernant la cote condition -------->=  0.000
Rang : 2

Avant l'optimisation des dispersions de la cote 2
 0.188 0.237 0.000 0.237 0.200 0.200 0.100
IT de la cote condition ---------------------->=  2.000
Ecart de fabrication concernant la cote condition -----------
->=  1.163
Reliquat concernant la cote condition -------->=  0.837
Nombre de dispersions optimisable concernant la cote
condition ->=  3
La repartition du reliquat -------------------->=  0.279

Apres l'optimisation des dispersions de la cote condition 2
 0.188 0.237 0.000 0.237 0.479 0.479 0.379
IT de la cote condition ---------------------->=  2.000
Ecart de fabrication concernant la cote condition -----------
->=  2.000
Reliquat concernant la cote condition -------->=  0.000

Dispersions optimisees
 0.188  0.237  0.338  0.237  0.479  0.479  0.379

CALCUL DES COTES FONCTIONNELLES

Les longueurs moyennes de base:

Equation[1]:      L 1 = 0,000
(On va  prendre l'origine sur la surface nø1)

Equation[ 1]:      0.000 = L 1
Equation[ 2]:      2.000 = L 3 - L 2
Equation[ 3]:      4.000 = L 5 - L 4
Equation[ 4]:     10.479 = L 4 - L 3
Equation[ 5]:      4.237 = L 3 - L 1

Systeme d'equations :                    Solution L :

  1L1+ 0L2+ 0L3+ 0L4+ 0L5 =    0.000     L[ 1]=  0.000
  0L1- 1L2+ 1L3+ 0L4+ 0L5 =    2.000     L[ 2]=  2.238
  0L1+ 0L2+ 0L3- 1L4+ 1L5 =    4.000     L[ 3]=  4.238
  0L1+ 0L2- 1L3+ 1L4+ 0L5 =   10.479     L[ 4]= 14.716
 -1L1+ 0L2+ 1L3+ 0L4+ 0L5 =    4.237     L[ 5]= 18.716

Cotes fonctionnelles moyennes :

Cf moy [ 1  2] =   2.238
Cf moy [ 1  3] =   4.238
Cf moy [ 1  5] =  18.716
Cf moy [ 3  4] =  10.479

COTES FONCTIONNELLES :
Cf [ 1  2] =  2.238  +0.263 -0.263
Cf [ 1  3] =  4.238  +0.237 -0.237
Cf [ 1  5] = 18.716  +0.283 -0.283
Cf [ 3  4] = 10.479  +0.479 -0.479
```

Figure 9. Output of the CAFT module for the sample example