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NONHOMOGENEOUS ELLIPTIC EQUATIONS WITH DECAYING CYLINDRICAL POTENTIAL AND CRITICAL EXPONENT

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Abstract. We prove the existence and multiplicity of solutions for a nonhomogeneous elliptic equation involving decaying cylindrical potential and critical exponent.

1. Introduction

In this article, we consider the problem

$$\begin{gathered} -\operatorname{div}(|y|^{-2a}\nabla u) - \mu|y|^{-2(a+1)}u = h|y|^{-2_*b}|u|^{2_*-2}u + \lambda g \quad \text{in } \mathbb{R}^N, \quad y \neq 0 \\ u \in \mathcal{D}_0^{1,2}, \end{gathered} \tag{1.1}$$

where each point in $\mathbb{R}^N$ is written as a pair $(y,z) \in \mathbb{R}^k \times \mathbb{R}^{N-k}$, k and N are integers such that $N \geq 3$ and k belongs to $\{1,\dots,N\}$; $-\infty < a < (k-2)/2$; $a \leq b < a+1$; $2_* = 2N/(N-2+2(b-a))$; $-\infty < \mu < \bar{\mu}_{a,k} := ((k-2(a+1))/2)^2$; $g \in \mathcal{H}'_\mu \cap C(\mathbb{R}^N)$; h is a bounded positive function on $\mathbb{R}^k$ and λ is real parameter. Here $\mathcal{H}'_\mu$ is the dual of $\mathcal{H}_\mu$, where $\mathcal{H}_\mu$ and $\mathcal{D}_0^{1,2}$ will be defined later.

Some results are already available for (1.1) in the case $k = N$; see for example [10, 11] and the references therein. Wang and Zhou [10] proved that there exist at least two solutions for (1.1) with $a = 0$, $0 < \mu \leq \bar{\mu}_{0,N} = ((N-2)/2)^2$ and $h \equiv 1$, under certain conditions on g. Bouchekif and Matallah [2] showed the existence of two solutions of (1.1) under certain conditions on functions g and h, when $0 < \mu \leq \bar{\mu}_{0,N}$, $\lambda \in (0, \Lambda_*)$, $-\infty < a < (N-2)/2$ and $a \leq b < a+1$, with Λ_* a positive constant.

Concerning existence results in the case $k < N$, we cite [6, 7] and the references therein. Musina [7] considered (1.1) with $-a/2$ instead of a and $\lambda = 0$, also (1.1) with $a = 0$, $b = 0$, $\lambda = 0$, with $h \equiv 1$ and $a \neq 2-k$. She established the existence of a ground state solution when $2 < k \leq N$ and $0 < \mu < \bar{\mu}_{a,k} = ((k-2+a)/2)^2$ for (1.1) with $-a/2$ instead of a and $\lambda = 0$. She also showed that (1.1) with $a = 0$, $b = 0$, $\lambda = 0$ does not admit ground state solutions. Badiale et al [1] studied (1.1) with $a = 0$, $b = 0$, $\lambda = 0$ and $h \equiv 1$. They proved the existence of at least a nonzero nonnegative weak solution u, satisfying $u(y,z) = u(|y|,z)$ when $2 \leq k < N$ and

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