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Nonlinear elliptic fourth order equations existence and multiplicity results

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Abstract. This paper deals with the existence of solutions to a class of fourth order nonlinear elliptic equations. The technique used relies on critical points theory. The solutions appeared as critical points of a functional restricted to a suitable manifold. In the case of constant coefficients we obtain the existence of three distinct solutions.

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1. Introduction

Let (M, g) be a Riemannian compact smooth n-manifold, $n \geq 5$, with metric g and scalar curvature S_g, we let $H_2^2(M)$ be the standard Sobolev space which is the completion of the space

$$C_2^2(M) = \{u \in C^{\infty}(M) \colon \|u\|_{2,2} < +\infty\}$$

with respect to the norm $\|u\|_{2,2} = \sum_{l=0}^{2} \|\nabla^l u\|_2$.

In this paper, we investigate solutions of a class of fourth order elliptic equations, on compact n-dimensional Riemannian manifolds, of the form

$$\Delta^2 u + \nabla^i (a(x) \nabla_i u) + b(x) u = f(x) |u|^{N-2} u + \lambda |u|^{q-2} u \tag{1.1}$$

where a, b, and f are smooth functions on M, $N = \frac{2n}{n-4}$ is the critical exponent, $1 < q < 2$ a real number, $\lambda > 0$ a real parameter.

Consideration for such problem comes from conformal geometry: indeed, in 1983, Paneitz [11] introduced a conformal fourth order operator defined on 4-dimensional Riemannian manifolds which was generalized by Branson [6] to higher dimensions.

$$PB_g(u) = \Delta^2 u + div\left(-\frac{(n-2)^2 + 4}{2(n-1)(n-2)} S_g . g + \frac{4}{n-2} Ric\right) du + \frac{n-4}{2} Q^n u$$

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