GLOBAL EXISTENCE FOR NONLINEAR PARABOLIC PROBLEMS WITH MEASURE DATA. APPLICATIONS TO NON-UNIQUENESS FOR PARABOLIC PROBLEMS WITH CRITICAL GRADIENT TERMS

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Abstract. In the present article we study global existence for a nonlinear parabolic equation having a reaction term and a Radon measure datum:

\[
\begin{cases}
(\varphi(v))_t - \Delta_p v &= f(x,t)(1 + \varphi(v)) + \mu \quad \text{in } \Omega \times (0, +\infty), \\
v(x,t) &= 0 \quad \text{on } \partial\Omega \times (0, +\infty), \\
v(x,0) &= v_0(x) \quad \text{in } \Omega,
\end{cases}
\]

where \(1 < p < N\), \(\Omega\) is a bounded open set of \(\mathbb{R}^N\) (\(N \geq 2\)), \(\Delta_p u = \text{div}(|\nabla u|^{p-2}\nabla u)\) is the so called \(p\)-Laplacian operator, \(\varphi(s) = (1 + \frac{|s|}{p-1})^{p-1} - 1\) \(\text{sign } s\), \(\varphi(v_0) \in L^1(\Omega)\) and \(\mu\) is a finite Radon measure and \(f \in L^\infty(\Omega \times (0, T))\) for every \(T > 0\). Then we apply this existence result to show wild nonuniqueness for a connected nonlinear parabolic problem having a gradient term with natural growth.

1. Introduction and Statement of Main Results

In this paper we will consider two related problems. The first one is a doubly nonlinear parabolic equation having a reaction term and a measure datum:

\[
\begin{cases}
(\varphi(v))_t - \Delta_p v &= f(x,t)(1 + \varphi(v)) + \mu \quad \text{in } \Omega \times (0, +\infty), \\
v(x,t) &= 0 \quad \text{on } \partial\Omega \times (0, +\infty), \\
v(x,0) &= v_0(x) \quad \text{in } \Omega,
\end{cases}
\]

where \(f \in L^\infty(\Omega \times (0, T))\) for every \(T > 0\), \(\Delta_p v = \text{div}(|\nabla v|^{p-2}\nabla v)\), with \(1 < p < N\), \(\mu\) is a Radon measure whose total variation is finite in \(\Omega \times (0, T)\) for every \(T > 0\), and \(\varphi(v_0) \in L^1(\Omega)\); here and in what follows

\[
\varphi(s) = \left(1 + \frac{|s|}{p-1}\right)^{p-1} - 1 \quad \text{sign } s.
\]