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Research Article

Subharmonic Solutions of Nonautonomous Second Order Differential Equations with Singular Nonlinearities

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We discuss the existence of subharmonic solutions for nonautonomous second order differential equations with singular nonlinearities. Simple sufficient conditions are provided enable us to obtain infinitely many distinct subharmonic solutions. Our approach is based on a variational method, in particular the saddle point theorem.

1. Introduction and Main Result

In this paper we discuss the problem of the existence of infinitely many subharmonic solutions for nonautonomous second order differential equations with singular nonlinearities of the form

$$u''(t) + f(t, u(t)) = e(t), \tag{1.1}$$

where $f : \mathbb{R}^2 \to \mathbb{R}$ is continuous, is T-periodic, in its first argument with $T > 0$, and presents a singularity with respect to its second argument. Here by a subharmonic solution we mean a kT-periodic solution for any integer k if $T > 0$ is the minimal period. When the solution is not T-periodic we call it a true subharmonic. It was pointed out in [1] that singular differential equations of the form (1.1) appear in the description of many phenomena in the applied sciences, such as the Brillouin focusing system and nonlinear elasticity. Several authors have