Numerical Approach for Local Bifurcation Analysis of Nonlinear Physical System

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Abstract:
In this paper, we propose an study that combines classical linearization method with the Routh Hurwitz criterion theory of complex nonlinear systems to compute local stability boundaries and visualize such bifurcation surfaces of nonlinear dynamical systems as function of parameters set (Analytical Search for Bifurcation Surfaces in Parameter Space).
Therefore, we proposed a numerical method for the bifurcations analyses. Our goal is to applied the optimal derivative (based on the minimization in the least-square sense) as introduced by O. Arino and T. Benouaz. In order to gain some progress with this procedure in the term of bifurcation analysis (detection of the local bifurcation in the neighborhood of the bifurcation parameters with respect to an initial condition), this algorithm allows us to compare the results obtained with those found by the classical linearization (Fréchet derivative (jacobian matrix) in the equilibrums points).

Keywords: Nonlinear ordinary differential equation– optimal derivative– Classical linearization (Fréchet derivative in the equilibrium point) - asymptotic stability - bifurcation analysis.

1. Introduction
Nonlinear models arise for most systems and their presence in one form or another is generally the rule. The source of nonlinearity in structural and mechanical systems may a result of interaction with surrounding, forces can arise due to interacting media or fields. The resulting nonlinear models exhibit a rich variety of phenomena of interest to scientists and engineer. The presence of a bifurcation is of great importance in many physical, chemical and biological systems [1-2].

In the study of nonlinear ordinary differential equations, this is an approximation procedure based on the minimization of a certain functional with respect to a curve starting from an initial value x₀ and going to zero as t goes to infinity.

The localization of critical parameter sets called bifurcations point is often a central task of the analysis of a nonlinear dynamical system. Bifurcations of codimension 1 that can be directly observed in nature and experiments form surfaces in three-dimensional parameter spaces. In this paper, we propose an application of the optimal derivative as introduced by O. Arino and T. Benouaz enables us to compare the results obtained with those found by the classical linearization (Fréchet derivative in the equilibrium point).

In the 1960s and 1970s the mathematical theory of dynamical systems experienced much development, with the introduction of new ideas by Smale, Arnold, Lorenz, Yorke, and Feigenbaum, to name just a few of the contributors [8, 9, 10]. With these theoretical developments there came a renewed interest in the dynamics of electronic circuits in the early 1980s, when new ideas and methods were introduced to the study of (periodic or non-periodic) oscillations generated by nonlinear electronic circuits of low dimension. However, the theory usually only provides a framework for different phenomena that one may find in a given circuit. To perform an effective study of the actual dynamics it is necessary to resort to numerical methods. In order to