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Dynamic Asset Allocation for Oil-based Sovereign Wealth Funds: Hedging Demands and International Diversification Effects

by

Submitted by:

Brahim Ameur

Supervised by:

Dr. Noureddine Menaguer

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Jury Members

Prof. Mohamed Benbouziane	Tlemcen University	*President*
Dr. Noureddine Menaguer	Tlemcen University	*Supervisor*
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DYNAMIC ASSET ALLOCATION FOR OIL-BASED SOVEREIGN WEALTH FUNDS: HEDGING DEMANDS AND INTERNATIONAL DIVERSIFICATION EFFECTS

ABSTRACT

The goal of this study is finding the dynamic asset allocation strategy for oil-based sovereign wealth funds. We have investigated the intertemporal hedging demands for assets for SWF in the U.S., and Canada, which can invest domestically and internationally. Using an Epstein-Zin-Weil utility function, where the dynamics governing asset returns are described by a vector autoregressive process.

Our findings stress the importance of the mean intertemporal hedging demands for domestic stocks in the U.S. and to smaller extinct in Canada. A SWF in the U.S. displays small mean intertemporal hedging demands for foreign assets, while SWF in Canada exhibits sizable mean hedging demands for U.S. stocks. The international diversification seems more beneficial in Canada than U.S.

Keywords: Dynamic asset allocation, sovereign wealth funds, hedging demands,international diversification.

التوزيع الديناميكي لأصول الصناديق السيادية ذات الأصل النفطي:

الطلب التحوطي وأثرالتنويع الدولي .

تهدف هذه الدراسة إيجاد إستراتيجية للتوزيع الديناميكي الأمثل لأصول صناديق الثروة السيادية ذات الأصل النفطي. ومن أجل ذلك قمنا بدراسة الطلب التحوطي لصناديق الثروة السيادية في كل من الولايات المتحدة و كندا على الأسهم والسندات محليا ودوليا وهذا باستخدام دالة منفعة من نوع Epstein-Zin-Wei حيث تمت نمذجة عوائد الأصول بنموذج متجة الانحدار الذاتي VAR. النتائج التي توصلنا إليها تؤكد على أهمية متوسط الطلب التحوطي للأسهم المحلية في الولايات المتحدة وبنسبة أقل للأسهم المحلية في كندا. أما في حالة صندوق سيادي في الولايات المتحدة ولديه إمكانية الاستثمار في الأسهم والسندات الأجنبية يلاحظ أن متوسط الطلب التحوطي على الأصول الأجنبية يعتبر ضئيلانسبيا. أما في حالة كندا يلاحظ أن الطلب التحوطي لأسهم الولايات المتحدة يعتبر كبير ومهم. أما فيما يخص التنويع الدولي، فهي معتبرة وأكثر أهمية في حالة الصندوق السيادي الكندي منه في الولايات المتحدة.

الكلمات المفتاحية: التوزيع الديناميكي للأصول، صناديق الثروة السيادية، الطلب التحوطي، التنويع الدولي.

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Introduction

1.1 Research Problem

Sovereign wealth funds (SWFs) as a vehicle for transferring sovereign wealth from the present to the future, have been created by many different types of governments, and managed in many different structures, ranging from central banks to independent financial corporations. These SWFs have become a key player and widely recognized as a dominant force on the global financial system. According to Institute of Sovereign Wealth Funds, the total size of SWFs' estimate stands at 6.3 trillion USD as of the end of 2013, with the prospect of growing larger . Typically, there are two main types of SWFs which are sorted out by the source of their wealth:

First, commodity funds with an estimated 61% of total sovereign wealth fund asset holdings. These funds are essentially financed by the proceeds from commodity exports (e.g. oil, gas and minerals), either owned or taxed by the government. Usually, they are established for budget stabilization and/or wealth sharing across future generations to ensure that the current generation does not take full advantage of exploiting the non-renewable natural resources (Mezzacapo, 2009).

Second, non-commodity funds, these funds are created through the transfer of the excess accumulated foreign exchange reserve, which are far beyond the benchmarks of precautionary reserve adequacy (Urban, 2011). The majority of these funds are geographically situated in Asia (e.g., China, Singapore, and Korea), and they focus mainly on, a) to hedge away the impact of risk factors behind commercial surpluses, b) to generate higher rates of return and reduce the opportunity costs of holding relatively low-interest foreign debt.

Successful SWFs share some common characteristics such as an appropriate institutional and managerial framework, high levels of transparency, sound governance structure, and the accountability to the relevant legislature and the general public. Although these common features are deemed necessary, yet they are not sufficient without optimal investment strategy that reflects the objectives of the SWF (Ang, 2010). In most cases, these objectives are pure financial motives which we may summarize as achieving high returns subject to moderate risk, and limiting withdrawals to ensure long term growth. As stated in Alberta Heritage Savings Trust Fund Statement of Investment Policy and Goals: *"The investments of Heritage Fund assets must be made with the objective of maximizing long-term financial returns."*,[1] without setting this crucial objective which according to Ang (2010) taxonomy *"Benchmark of Legitimacy"* the SWF's capital will be at risk of being immediately depleted, and not preserved to benefit current and future generations. The worst outcome is that the SWF's wealth is spent aggressively to serve narrow political agendas for few members of the present generation. For instance, Alberta SWF stands as case study to see how bad management may affect the overall performance and undermine the very existence of such SWF. Despite the well-pronounced mandate to save oil revenues for future generations still Alberta Heritage Savings Trust Fund failed to save as for much of a 25-year period (Murphy & Clemens, 2013).[2]

[1] `http://www.qp.alberta.ca/documents/Acts/A23.pdf`, p.02

[2] In the period from 1987 to 2013, only two relatively small deposits were made into the fund despite the sky-high production with historically high prices over this period

Although the SWF portfolio choice is not very different from managing other institutional investors with long investment horizons such as pension funds, yet the decision of how to select asset classes and their strategic weights still remains a critical and open research avenue, especially with the sovereign character of these Funds. The asset class weights can be represented compactly in mathematical formalism as a function $F(\mathscr{M},\mathscr{I})$ with two variables $\mathscr{M}$ and $\mathscr{I}$, where $\mathscr{M}$ denotes market modeling (Return Generating Process), whereas $\mathscr{I}$ stands as the investor profile (risk tolerance, objectives, constraints and investment guidelines).

In order to maximize return for certain level of risk, the investor should choose the best model as to reduce estimation risk which has a dire consequence on the optimal choice, and diversify across asset classes and countries to take advantage of diversification effect. Since investment horizon has important impact on portfolio performance, the long-term investor should consider intertemporal risks that the short-term investor does not (Brennan, Schwartz, and Lagnado (1997); J. Campbell, Chan, and Viceira (2003)). Thus, in the case of SWF with long-term objectives, the manager would consider intertemporal hedeging demands and diversifying across time as well.

The advances in dynamic asset pricing theory and the growing empirical evidence indicating that stock and bond returns have important predictable components have revived the interest in optimal dynamic asset allocation decisions for long-term investors. As firstly considered in the seminal contributions of Samuelson (1969), and Merton (1969, 1971, 1973), the presence of stochastic opportunity set has very important implications for multi-period portfolio choice problems. In other words, return predictability can give rise to intertemporal hedging demands for assets against adverse future return shocks.

Although, three decades of empirical research have stressed the existence of asset return predictability (J. Campbell and Shiller (1988b); Fama and French (1989); Cochrane (2008)), still the subject of return predictability remains debatable as recent research has pointed out that the evidence of predictability found in-sample is washed out when out-of-sample tests are employed (Goyal & Welch, 2008).

Studying multi-period portfolio choice problems under predictability still remains difficult as exact analytical solutions are generally not available. This has led researchers to use different approaches in order to solve them, particularly, with the gain in computer power which allows them to employ computationally intensive numerical procedures to find approximate solutions. For example, the studies of (Brennan et al. (1997); Balduzzi and Lynch (1999); Barberis (2000)) use discrete-state approximations to numerically solve portfolio choice problems for investors with long horizons. Another approach uses approximate analytical methods for investors with infinite horizons in the neighborhoods of known exact solutions; see Campbell and Viceira (1999, 2001) and J. Campbell et al. (2003), henceforth CCV.

The growing empirical research dealing with the public investment strategies of sovereign wealth funds and their performance shows that SWFs tend to invest in large size foreign firms, particularly in financial and energy sectors, with low diversification and poor medium-term performance (Dyck and Morse (2011); Bortolotti, Fotak, Megginson, and Miracky (2010); Chhaochharia and Leuven (2009); S. Bernstein, Lerner, and Schoar (2013)). While other studies point out that SWFs have engaged in domestic financial markets playing the role of "investors of last resort" especially during the last financial crises (G. Clark (2010); Raymond (2010)).

On the other hand, research on the optimal asset allocation for SWFs is limited and lagging behind as it manifests in few papers which we cite them in what follows with the main contributions. Gintschel and Scherer (2008) and Scherer (2009a, 2009b) solve for the optimal allocation policy of an oil-based SWF in a static setting, and find the oil price hedging demand to be an important component of the optimal allocation decision. The shift toward dynamic models is logical and critically needed given the fact that static portfolio analysis can hardly be justified for long-term investors, and does not allow the analyses of intertemporal hedging demands in the presence of a stochastic opportunity set. A first step in that direction has been taken by Scherer (2009a) who considers a discrete-time model similar to that of CCV and finds that a hedging demand against shocks to the short-term risk-free rate is optimally required, in addition to the oil price hedging demand. The study of Martellini and Milhau (2010) have addressed the optimal allocation for an SWF in a continuous-time dynamic model by examining non-tradable commodity wealth in the SWF or exogenous liabilities set by the government and proxied by an inflation-linked investment benchmark. While the papers of Bodie and Brière (2014a, 2014b), proposed to estimate the whole sovereign economic balance sheet using the theory of contingent claims considering the joint management of all sovereign assets and liabilities in an Asset Liability Management (ALM) framework.[3]

The models which state the importance of integrating the process of asset allocation of SWF with the economic balance sheet management of the whole country as the studies of (Bodie & Brière, 2014a, 2014b) or relate it to wealth underground as in the case of Gintschel and Scherer (2008); Scherer (2009a, 2009b) entail high complexities which make them very hard to implement if not saying impossible, because the management of government resources and expenditures raises difficult issues in practice, especially with the limited tools of standard macroeconomic to estimate sovereign economic balance sheets (Bodie & Brière, 2014b). Another complexity raises from the fact that traditional macroeconomic data lack a significant dimension, namely risk (Gray, 2007). These factors make it difficult to coordinate sovereign wealth management with monetary policy, fiscal policy, and public debt management.

[3]The "sovereign" is considered in the broad sense, including all the related institutions (budgetary government, central bank, SWFs, pension funds and public entities placed under the sovereign's authority)

1.2 Research Objectives

In this thesis, the focus is mainly concentrated on the analysis of the optimal asset allocation policy for oil-based SWFs for many reasons: (i) as we have mentioned before the asset under management for commodity funds is estimated at 61% of total sovereign wealth fund assets, oil-based SWFs are the dominating force in commodity funds. Thus, they are the major player in financial markets. (ii) The peculiarity of commodity funds, in general, and oil-based SWFs in particular as they transfer non-renewable wealth into renewable wealth, the SWFs have social and ethical responsibility toward future generations.

Our aim in this study is twofold. First, we examine the predictability of stocks and bonds for three countries, namely Canada, the United States, and the United Kingdoms using in-sample and out-sample tests in the same spirit as Rapach, Wohar, and Rangvid (2005). In our study we exploit the same variables as CCV, (Rapach & Wohar, 2009), and (Engsted & Pedersen, 2012). Thus, we use nominal bill yield, dividend yield, and term spread as predictors. For the sake of comparability and estimation benefits we apply long time sample. The sample begins in 1954:06 and ends in 2004:05 for all countries.[4]

Second, in order to fill the gap between empirical studies and theoretical models, we analyze the dynamic asset allocation and evaluate the effects of intertemporal hedging demands as well as international diversification. Toward this end, we extend the CCV approach to analyze dynamic asset allocation across domestic bills, stocks, and bonds for an SWF in Canada, and the U.S., where the returns dynamics are characterized by a VAR(1) process. For a set of plausible values for the parameters relating to intertemporal preferences, we estimate the mean total, myopic, and intertemporal hedging demands for domestic bills, stocks, and bonds in each country. In addition, we consider a multi-period portfolio choice problem for SWF in the U.S. which has access to stocks and bonds from a foreign country (Canada, U.K.). The same exercise goes with SWF in Canada which can invest in a foreign country (U.K., U.S.). The sample begins in 1977:01 and ends in 2004:05 for all countries.

[4] Rapach and Wohar (2009) used a sample starts in 1952:04 and ends in 2004:05

The reason to choose the year 1977 is related to the date of inception of the Canadian SWF (Alberta Heritage Fund) and the U.S. SWF (Alaska Permanent Fund), both of them created in 1976.

In this thesis, we assume that (i) asset returns are exogenous, which implies (among others) that the asset demand by individual investors does not have any market impact, in other words partial equilibrium model (see e.g., CCV, and Rapach and Wohar (2009)); (ii) There is no taxes and transaction costs in investing domestically and internationally; (iii) Exchange rates are given (see e.g., Rapach and Wohar (2009); Bekaert and Hodrick (1992))

1.3 Thesis Structure

This thesis contains six chapters. In chapter 2 we try to provide a general overview describing the main characteristics of SWFs, and investigate their investment behaviors both theoretically and empirically. Section 2 discusses the main characteristics of SWFs as definition and types, the rationale to set up sovereign wealth funds, and institutional and governance structure types. Section 3 presents the process of managing SWF investments and the position of asset allocation within this process; however, our treatment tends to be with normative flavor. In the contrary, section 4 presents the main traits that characterize the investment behavior of SWFs through reviewing the empirical studies dealing with SWFs investment patterns. Section 5 concludes.

In chapter 3 we explain the basics of portfolio theory which we deem necessary to the flow and development of the more advanced framework of CCV. In section 2 we present the expected utility approach to decision making under uncertainty and how it is possible to make a ranking between all possible choices using utility functions. Various and relevant types of utility functions and risk measures are addressed. In addition, we discuss conditions under which the expected utility maximization approach is equivalent to mean-variance criterion. Section 3 explains both formally and graphically the concept of mean-variance criterion and mean-variance analysis as developed in Markowitz (1952, 1959), and Tobin's (1958) contributions.

Section 4 deals with two influential models of equilibrium prices and returns in capital markets, namely the capital asset pricing model (CAPM) and the Intertemporal Capital Asset Pricing Model (ICAPM). The CAPM or the one-factor capital asset pricing model was the first general equilibrium model developed. It is based on the most stringent set of assumptions. While the ICAPM as first developed by Merton (1973) is based on a set of more realistic assumptions. More importantly it is a dynamic model which takes into account the changes in the opportunity set.

Chapter 4 deals exclusively with asset return modeling, theoretically and empirically. Section 2 presents a short review of asset return predictability and the econometric issues related to the empirical studies; Section 3 describes Rapach et al. (2005) econometric methodology used in the in-sample and out-of-sample predictability tests, as well as vector autoregressive modeling approach; Section 4 describes our empirical approach and presents analysis results.

In chapter 5 the CCV approach to strategic asset allocation is applied to analyze international hedging demands and diversification effects on dynamic asset allocation across different asset classes and countries, where the returns dynamics are characterized by a VAR(1) process, the empirical findings of chapter 4 serve as an input to multi-period choice problem. Section 2 describes CCV framework because it is the backbone to our model; Section 3 presents the model along the empirical results. The examination period spans from January 1977 until May 2004. The data set used is an expert from the data set used in chapter 4. Chapter 6 concludes the thesis.

2

Sovereign Wealth Funds Profile

2.1 Introduction

The tremendous growth and the rapid rise of the Sovereign Wealth Funds (SWFs) in the turn of the 21st century, and according to Truman (2012), the nominal assets under management of SWFs doubled from 2005 to 2007. Such considerable growth calls to question the efficient and optimal strategy to invest these assets in order to reach the macroeconomic objectives, in which their governments have set up.

The asset allocation decision is the most influential factor driving investment performance for any portfolio in general, and for SWF in particular. The sovereign and heterogeneous nature of these types of institutional investors and the lack of transparency of their objectives make the process of asset allocation complex and critical. Thus, the decision to allocate wealth into different classes does not stand alone or operate in vacuum. It is a process that depends on several factors such as: Size, objectives, the institutional and governance structure, and risk preferences.

In this chapter we try to give a general overview describing the main characteristics of SWFs, and investment behaviors theoretically and empirically. Section 2 provides an overview of the main characteristics of SWFs as definition and types, the rationale to set up sovereign wealth funds, as well as the institutional and governance structure types. Section 3 presents the process of managing SWFs portfolios and the position of asset allocation within this process; however, our treatment tends to be with normative flavor. In the contrary, section 4 presents the main traits that characterize the investment behavior of SWFs through reviewing the empirical studies dealing with SWFs investment patterns. Section 5 concludes.

2.2 Sovereign Wealth Funds Characteristics

2.2.1 Definition and Types

Definition

The sovereign wealth fund phenomenon is relatively old in existence, recent in policy concern and academic conscience, since the word *"Sovereign Wealth Funds"* was recently coined by Andrew Rozanov (2005). Unfortunately there is no common, generally accepted definition of what a SWF actually is, due to the heterogeneous, unique, and the dynamic evolutionary structure of SWFs (IMF, 2008). However, there are several common traits that may gather these institutional investors under a broad class. Truman (2010, p.ix) refers to SWFs as *"large pools of government-owned funds that are invested in whole or in part outside their home country."* But this definition suffers from few limitations and each definition, indeed, does. To the end of getting more precise definition of SWFs, we choose the definition adopted by the International Working Group of Sovereign Wealth Funds (IWG) as:

"special purpose investment funds or arrangements, owned by the general government. Created by the general government for macroeconomic purposes, SWFs hold, manage, or administer assets to achieve financial objectives, and employ a set of investment strategies which include investing in foreign financial assets. The SWFs are commonly established out of balance of payments surpluses, official foreign currency operations, the proceeds of privatizations, fiscal surpluses, and/or receipts resulting from commodity exports." (IWG, 2008, p.27).

The IWG noted that "general government includes both central government and subnational government." According to this definition, it is worth mentioning that the following entities would be excluded from the SWF definition (Mezzacapo, 2009):

1. foreign currency reserves held by Monetary Authorities for traditional Balance of Payments/Monetary Policy purposes and needs.
2. operations of traditional State-Owned Enterprises (SOEs).
3. national pension funds with contractual liabilities disallowing their use for general macroeconomic purposes.
4. assets managed for the benefit of individuals.
5. government lending funds (i.e. mainly domestic funds).
6. government owned banks (e.g. national development banks) operating as intermediaries rather than for general economic purposes.

As we have mentioned earlier, SWFs are a broad heterogeneous set of government special purpose entities, *"differ in size, age, structure, funding sources, governance, policy objectives, risk return profiles, investment horizons, eligible asset classes and instruments"* as stated by Street (2008, p.15). Such variety that entails SWFs stands as an obstacle to the well understanding and studying of the investment patterns of these funds, unless there is a way to categorize and sort them out.

Types of SWFs

To the end of sorting SWFs out, few criterion have been used to categorize them in appropriate manner. In what follows three criterion are chosen due to their ability to clarify the nature and investment patterns of SWFs. First the source of wealth, second policy objectives, and third legal structure.

According to the source of wealth aspect, we could make a distinction between:

1. *Commodity Funds:* Historically, they were the first SWFs type to be established. These funds are essentially financed by the proceeds from commodity exports (e.g., oil, gas and minerals), either owned or taxed by the government. Moreover, the prevailing commodity source for the most powerful SWFs, are so far from oil and gas, about 60% of the total asset under management for all SWFs (Miceli, Whrmann, Wallace, & Steiner, 2015), see Figure 2.1. Usually, they are established for budget stabilization and/or wealth sharing across generations (Mezzacapo, 2009).

2. *Non-Commodity Funds:* Typically, these funds are created through the transfer of the excess accumulated foreign exchange reserve, which are far beyond the benchmarks of precautionary reserve adequacy (Urban, 2011). The majority of these funds are geographically situated in Asia (e.g., China, Singapore, Korea,)

Second, we may classify SWFs with reference to their objectives and investment policies. According to IMF taxonomy (IMF, 2008), SWFs can be broadly grouped in the following categories:

1. *Stabilization funds,* set up to insulate the economy against swings in commodity prices (macro-fiscal management).

2. *Savings funds,* which transform the income from non-renewable natural resources into a diversified portfolio of assets, accumulating savings for future generations.

3. *Reserve investment corporations,* established to increase the return on foreign exchange reserves.

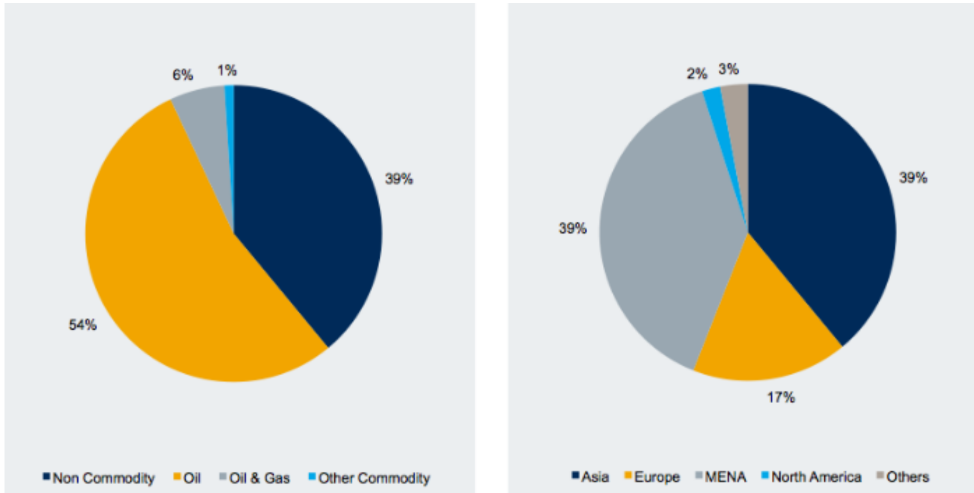


Figure 2.1: Chart of SWFs by funding source and regions
Source: Miceli et al (2015).

4. *Development funds,* which help fund infra-structural projects to increase the country's potential growth.

5. *Contingent pension reserve funds,* which complement resources from individual pension contributions to provide for pension liabilities on the government's balance sheet.

Although the classification by policy objectives is well-considered, but it is criticized for being not very practical due to some deficiencies. For example, several countries introduced SWFs with a mixture of objectives such as long-term savings (e.g., pension liabilities) and in the same time development goals, these objectives are often subject to change over time with economic and financial circumstances (see, e.g., Truman (2010); IMF (2008)).

Third, the legal framework can serve as a distinguish feature between SWFs due to its important effects not only on promoting sound institutional and governance structure, but from purely financial aspect. In practice, the legal structure may have effects on both the tax position and immunity of investments. For example, SWF investments managed by central bank will normally be protected by sovereign immunity and may also enjoy tax privileges in recipient countries[1]. Whereas taxation of corporate investments may depend on the extent to which these investments are considered part of the government's financial management (Al-Hassan, Papaioannou, Skancke, & Sung, 2013). SWFs may be classified with respect to their legal structure into three categories as mentioned in the Santiago Principles (IWG, 2008, p.27):

1. Separate legal entities, mostly constituted by a specific act of public law and providing the highest degree of operational independence. Examples include the QIA, ADIA and the Australian Future Fund.

2. State-owned corporations often governed by private law yet fully owned by the state, mostly represented by the ministry of finance. Examples are the Singaporean entities (which are so-called fifth-schedule companies with additional decision rights granted to the president of Singapore, or China's CIC.

3. Pools of assets without separate legal personality where the general rules for managing the pool are set out in specific legislation. The operational management of the asset pool may be either conferred to branches of the administration, e.g., a ministry or a parliamentary/mixed committee or to "an independent entity" such as the central bank (as in Chile or Norway) or a separate statutory agency (as with the Alberta Heritage Fund).

[1](e.g., Norway has negotiated tax exemptions for its SWF investments in several bilateral tax treaties)

2.2.2 The Rationale to Set up Sovereign wealth Funds

According to Miceli et al. (2015) there are 69 SWFs with estimated assets under management (AuM) amounting to 6.3 trillion USD as of the end of 2013. Funds originating from oil and gas revenues account for around 60% of the total. The table 2.1 shows that the top 11 SWFs manage more than 100 billion USD each, and account for about 82% of the total AuM by SWFs. These relative huge amounts call for questioning the motives behind SWFs creation. The incentives for setting up SWFs by governments are closely related to the very nature of the economic structure of these countries. Thus, we may summarize the important incentives in the following:

Stabilization

There are economies characterized mainly by the production of one or two commodities, for example, oil and gas. Actually, the income of these economies and their wealth are tightly correlated to the market situation (boom and bust). Therefore the cyclical nature of the commodity market and the price volatility affect the revenue, which in turn affects the fiscal stability. As consequence, most oil and gas exporters such as GCC, Algeria, Iran and Libya have primary set up their SWFs to stabilize their economies in cases when prices go down.

Theoretically, the excess funds are removed from government budget when revenues are high, and put back when the revenue shrinks. However, in practice it is somehow complicated due to the legal right to withdraw, liquidity and the marketability of the SWF assets. To illustrate the process, we take the case example of the Norway's Government Pension Fund Global (GPFG) which is considered so far the best model for SWFs management, apart from being the biggest SWF in the world. In fact, the Norwegian government has set up its SWF with a good mechanism design, so to align this fund with fiscal policy, the chart 2.2 exhibits the integration of the fiscal policy with the Fund management. To ensure the expenditure stability and the future sustainability, the government has set well-defined operation rules, for example the size of any transfer from the SWF into state budget

Table 2.1: Largest 20 SWFs by the end 2013

Country	Sovereign Fund Name	Asset$ Billion	Inception	Origin
Norway	Government Pension Fund - Global	839	1990	Oil
UAE - Abu Dhabi	Abu Dhabi Investment Authority	773	1976	Oil
Saudi Arabia	SAMA Foreign Holdings	676	NA	Oil
China	China Investment Corporation	575	2007	Non-commodity
China	SAFE Investment Company	568	1997	Non-commodity
Kuwait	Kuwait Investment Authority	410	1953	Oil
China - Hong Kong	Hong Kong Monetary Authority Investment Portfolio	327	1993	Non-commodity
Singapore	Government of Singapore Investment Corporation	320	1981	Non-commodity
Singapore	Temasek Holdings	171	1974	Non-commodity
Qatar	Qatar Investment Authority	170	2005	Oil
China	National Social Security Fund	161	2000	Non-commodity
Australia	Australian Future Fund	89	2006	Non-commodity
Russia	National Wealth Fund	89	2008	Oil
Russia	Reserve Fund	87	2008	Oil
Kazakhstan	Samruk-Kazyna Jsc	84	2008	Non-commodity
Algeria	Revenue Regulation Fund	77.2	2000	Oil
UAE - Dubai	Investment Corporation of Dubai	70	2006	Oil
Kazakhstan	Kazakhstan National Fund	69	2000	Oil
UAE - Abu Dhabi	International Petroleum Investment Company	63	1984	Oil
Libya	Libyan Investment Authority	60	2006	Oil

Source: adapted from Miceli et al. (2015).

must not exceed 4% of SWF return (Chambers, Dimson, & Ilmanen, 2012). Therefore the 4% withdrawal rule ensures the expenditure stability in bad economic situation, as it allows in the same time the fund to grow because it corresponds to the fund's anticipated long-run annualized real return.

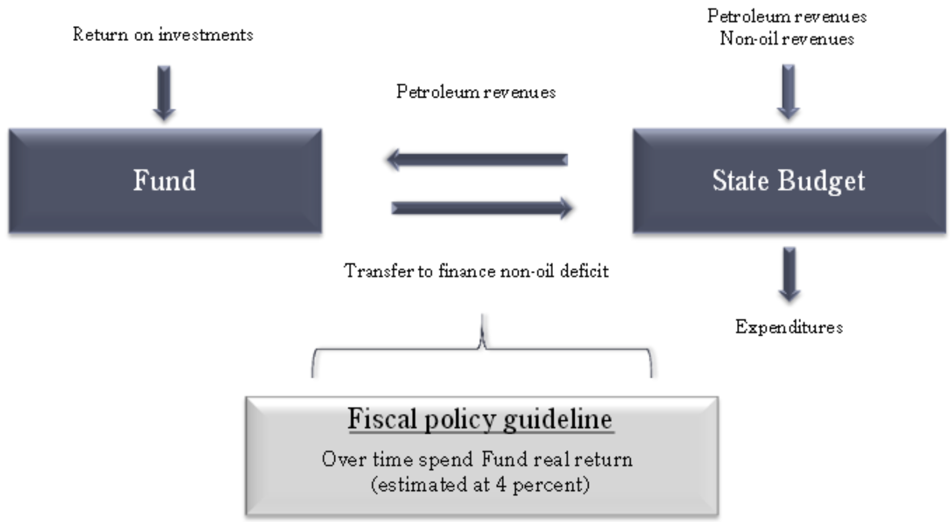


Figure 2.2: Integration of GPFG with government budget policy

Source: Retrieved from, https://www.regjeringen.no/contentassets/453856f5908e47f6a870f19723fc0b32/columbia_sj_april2015_ppt.pdf

Diversification

Economies that depend solely upon non-renewable resources are likely to be exposed to a sudden increase in their wealth, which usually pushes the domestic prices higher and the currency appreciation, such changes may cause a decrease in international competitiveness, and manifest into much de-industrialization (The Dutch disease) (Steigum, 2012). Furthermore, the resource curse does not only harm the industrialization in its materialistic aspects, but human characters as well. Diversifying the economy away from the dependence upon one commodity was and still remains an open challenge, not because of non-conscious of the problem, but mainly in how to get a solution in a highly specialized global market.

Therefore, the creation of SWF stands as an ideal starting point and an intermediate state into more diversification. Actually, the Norwegian SWF(GPFG) experience proved its usefulness in the diversification process, given the relative short life of the Norwegian fund and the acquired results. To illustrate that, the GPFG was established in 1990 and started operating by 1996 when the fund received its first allocation of US$ 0.3 billion (Chambers et al., 2012). Recently, it grows to US$ 839 billion (Miceli et al., 2015). In addition to this extraordinary growth in size, the return of the GPFG assets was 13.4% in 2012 (GPFG, 2012).

The accumulation of 'excess' foreign exchange reserves

The world witnesses unprecedented sustained increase in the level of foreign exchange reserves. We may essentially identify three factors that can explain this surge:

1. Countries with abundant natural resources, particularly the oil producers have accumulated huge foreign exchange reserves due to the increase of oil prices, from around US$20 in 2000 to reach its peak in 2008 with US$145 and since 2011 it has been fluctuating around US$100.

2. The permanent structural surplus in trade exchange for most East Asia economies, which implement the policy of keeping their exchange rates below its market value, In order to be able to stabilize their currencies into a certain level, the central bank usually intervenes as a buyer of foreign currencies in a step to stop the domestic currency from appreciation. China is considered so far the most pronounced case since its foreign exchange reserves reach US$ 3.44 trillion at the end of March 2013 (IMF, 2013).

3. Some developing countries motivated by self-insuring their economies from the adverse outflow and the risk of sudden stops have started to accumulate foreign exchange reserves, especially after Asian crisis in 1997.

These huge reserves are not only far beyond any benchmark precautionary purposes, but they have heavy costs as well. According to Rodrik (2006) estimates, China loss due to the opportunity cost of reserve holding is about US$ 100 billion, which explains the creation of China Investment Corporation (CIC) in 2007. Huge foreign exchange reserves and the desire to get higher yield are considered to be the most important factors for setting up SWFs. And as result, China has created 5 SWFs with total asset under management US$ 1542.2 billion, which is about 28% of the entire asset under management of SWFs, see (SWFInstitute, 2013).

Fighting inflation

It is well-known that fighting inflation is among many economic policy objectives in general, and the main objective of any effective monetary policy. Trade surpluses tend to increase domestic prices, through the increase of money quantity. Thus, the creation of SWF has the ability to soak up excess liquidity, and in the same time invest it to get a return above the risk-free. Even though SWFs are not a tool in the hand of the monetary authority, yet they are very effective in the reduction of inflationary pressures (Sen, 2010).

2.2.3 Institutional and Governance Structure

Institutional Frameworks

In practice there are a variety of institutional frameworks across SWFs, but we may consider the two dominant forms of institutional structure for SWFs (Al-Hassan et al., 2013). The manager model and the investment company model, the models are illustrated in 2.3. In the manager model, the principal (the Ministry of Finance) which is the legal owner of the fund delegates the asset management to an appointed manager (agent) under precise investment mandate. Within this model, there are three main sub-categories (IWG, 2008):

1. The ministry of finance gives an investment mandate to the central bank to manage the SWF assets (e.g., Norway, and Chile). Under this case, the central bank may seek advice and consultations from one or more external investment companies in the process of portfolio management.

2. In this case, such as Government Investment Corporation (GIC) of Singapore, the ministry of finance gives an investment mandate to a separate management entity owned by the government.

3. The ministry of finance gives a direct mandate to an external (private) fund manager.

In the investment company model, such as Temasek Singapore, the government as owner sets up an investment company which in turn owns the assets of the fund. Typically, this model is employed when the investment strategy implies more concentrated investments and active ownership in individual companies, or the fund has a development objective in addition to a financial return objective.

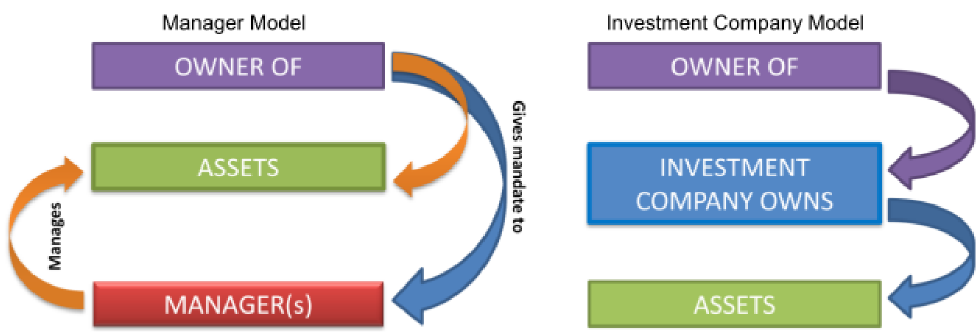


Figure 2.3: SWFs Institutional Framework
Source: (Al-Hassan et al., 2013)

Governance Framework

Funds that function operationally as separate legal entities (e.g., New Zealand (NZ Super Fund), Alaska) usually have a governance structure that differentiates an owner, a board, and the operational management of the SWF. Where the fund is a unit within the central bank (e.g., Saudi Arabia and Norway) operational independence could be embedded in a clear legal foundation and internal governance structure in which the decision making framework and oversight functions are clear and the relationship between the principal (owner) and its agent (central bank) is well established. The decision to adopt either approach may depend to some extinct on two important factors: Costs, and SWF objectives. Setting up a fund as a separate legal entity has costs, while a unit in the central bank makes use of existing infrastructure and human resources. Therefore, it could be more cost-efficient if a small size fund were to be managed within an existing institution(see e.g., (IWG, 2008; Al-Hassan et al., 2013)).

The investment strategy of SWF with stabilization objectives tends to be relatively less aggressive with short term horizon which shares some common traits with the investment strategy of central bank foreign reserves. Thus, the central bank seems an ideal choice for countries with stabilization objectives (see Figure 2.4).

The governance structure must be commensurate with the risks and complexities of the investment strategy. Whenever funds get into complex investment strategies and riskier asset classes, the governance and risk management must be strengthened. This approach is not only applicable to SWFs, but also to large institutional investors as they have been moving toward adopting a risk factor based approach to portfolio construction.

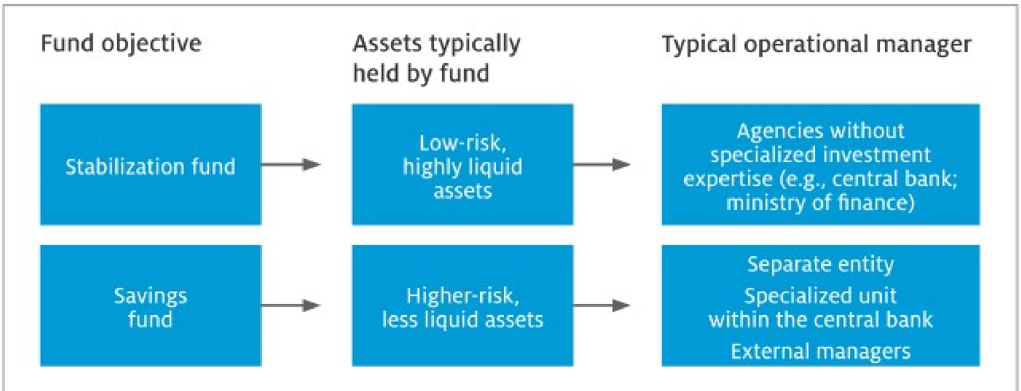


Figure 2.4: Typical relation between SWF objectives and management model
Source: (Al-Hassan et al., 2013)

2.3 SWF Portfolio Management

The aim of this section is to show the importance of the asset allocation decision in the investment process for investors in general and for SWFs in particular. In addition, we present the main steps to construct a portfolio for SWF. In order to understand and visual the process we choose to mention few cases of transparent SWF experiences.

The decision of asset allocation as defined by Reilly and Keith (2012, p.33) *"is the process of deciding how to distribute an investor's wealth among different countries and asset classes for investment purposes.* and they define the asset class as *" An asset class is comprised of securities that have similar characteristics, attributes, and risk/return relationships.* In a reply to an asked question about the importance of asset allocation decision to an investor, Reilly and Keith (2012, p.50) state:
" In a word, very. Several studies by Ibbotson and Kaplan (2000); Brinson, Hood, and Beebower (1986); and Brinson, Singer, and Beebower (1991) have examined the effect of the normal policy weights on investment performance, ... The studies all found similar results: About 90 percent of a fund's returns over time can be explained by its target asset allocation policy."

Actually, the asset allocation decision for SWF is as important as for any institutional investor. The SWF of New Zealand (NZ Super Fund) states in its Investment Risk Allocation Policy (p.3): *"We have an investment belief that asset allocation is the key investment decision. Undertaking it effectively is a key part of maximising return over the long term, without undue risk."*[2]

This decision, however, is not an isolated choice; but, a dynamic and integrated component of a structured four-step portfolio management process that we present in this section[3]. The first step is the development of investment policy statement that will guide all future decisions. The second, the portfolio manager assesses the current financial and economic conditions and forecasts future trends. Along with investment policy, the manager designs an asset allocation strategy. The third step, is the portfolio. Finally, the fourth step is the evaluation of the actual portfolio performance, and assessment of the investor's needs and capital market conditions and, when necessary, updating the policy statement. For instance, Alaska Permanent Fund Corporation considers the entirety of its investment policies at least in bi-annual basis, and may be modified at any time by Board action.[4]

2.3.1 Investment Policy Statement

It is the owner of the SWF responsibility, or any delegated party to develop investment policy, after seeking an advice from the investment committee (or board). In many instances, the consultations with the fund's stakeholders (i.e., the parliament, general public, non government organizations, etc.) deem necessary to reduce the risk of unilateral decisions by any single party (Al-Hassan et al., 2013). For example, in regards Alberta Heritage Savings Trust Fund. The investment policy of the Fund is the responsibility of the Minister who on return delegates to the Department of Finance and Enterprise the task of developing the investment policy, which is prepared in collaboration with Alberta Investment Management

[2] `https://www.nzsuperfund.co.nz/sites/default/files/documents-sys/Investment\%20Risk\%20Allocation\%20Policy.pdf`

[3] I followed the same approach of Reilly and Keith (2012, Chapter 2) to construct these steps

[4] See Investment Policy of Alaska Permanent Fund Corporation, `http://www.apfc.org/home/Media/investments/Investment\%20Policy\%20072015b.pdf`

Corporation (AIMCo)[5].

The policy statement is invaluable planning tool that helps the SWF's owners understanding their needs, as well as assists portfolio manager in carrying on the duty of managing funds. While it does not guarantee investment success, a policy statement will provide discipline for the investment process and reduce the possibility of making risky, inappropriate decisions. There are mainly two reasons for constructing a policy statement: First, it helps planning given a well-defined investment goals after studying and evaluating the financial markets and the risks of investing; second, it sets a benchmark by which to evaluate the performance of the portfolio manager.

The Policy Statement Ojectives

As just mentioned, there are two main objectives of developing a policy statement the first understanding and articulating realistic goals, the second evaluating the performance through setting standard and benchmark.

An important purpose of developing a policy statement is to help the SWFs understand their own needs, objectives, and investment constraints. As part of this, SWFs need to learn about financial markets and the risks of investing. This background will guard them against making inappropriate investment decisions in the future based on unrealistic expectations, and increase the possibility that they will satisfy their specific, measurable financial goals. Thus, the policy statement helps the investor to specify realistic goals and become more informed about the risks and costs of investing. According to Santiago Principles(Santiago Principles GAAPs 18) (IWG, 2008, p.20): *The SWFs investment policy should be clear and consistent with its defined objectives, risk tolerance, and investment strategy, as set by the owner or the governing body(ies), and be based on sound portfolio management.*

[5]See the Statement of Investment Policy and Goals of Alberta Heritage Savings Trust Fund, p.6. `http://www.finance.alberta.ca/business/ahstf/heritage-fund-statement-investment-policy-and-goals.pdf`

The policy statement also assists in judging the performance of the portfolio manager. Performance cannot be judged without an objective standard; the policy statement provides that objective standard. The portfolio's performance should be compared to guidelines specified in the policy statement, not on the portfolio's overall return. For example, if an investor has a low tolerance for risky investments, the portfolio manager should not be fired simply because the portfolio does not perform as well as the risky S&P 500 stock index. The point is, because risk drives returns, the investor's lower-risk investments, as specified in the policy statement, will probably earn lower returns than if all the investor's funds were placed in the aggregate stock market Reilly and Keith (2012, p.39-40).

The policy statement should include a benchmark portfolio, or comparison standard. The risk of the benchmark, and the assets included in the benchmark, should agree with the SWF's risk preferences and investment needs. Typically, there are two models used in setting benchmark portfolio: The Strategic Asset Allocation (SAA) approach, and the Reference Portfolio approach (Al-Hassan et al., 2013). NZ Super Fund compares the two approaches as follows

"Reference Portfolio differs from the SAA in two key aspects. First, the Reference Portfolio is a simple, low-cost and passive portfolio that contains only traditional asset classes.Second, while the Reference Portfolio is static, it acts as a benchmark for the Fund's actual portfolio. The actual portfolio can deviate substantially from, and is more dynamic in nature than, the allocations in the Reference Portfolio. The decisions to deviate from the Reference Portfolio are delegated to the Fund's management, subject to a clear set of risk limits and guidelines."[6]

[6] https://www.nzsuperfund.co.nz/sites/default/files/documents-sys/2015_Reference_Portfolio_white_paper.pdf

Thus, under reference portfolio approach the SWF manager enjoys more freedom in constructing the actual portfolio given the investment policy guidance. For example, the reference portfolio of NZ Super Fund in 2015 is divided 80:20 split between growth and fixed-income investments and its foreign currency exposures are 100% hedged to the New Zealand dollar[7]. The Figure displays the asset classes and the target weights under two variants of SAA model used by Alaska Permanent Fund Corporation (APFC), namely the risk factors method and asset classes method.

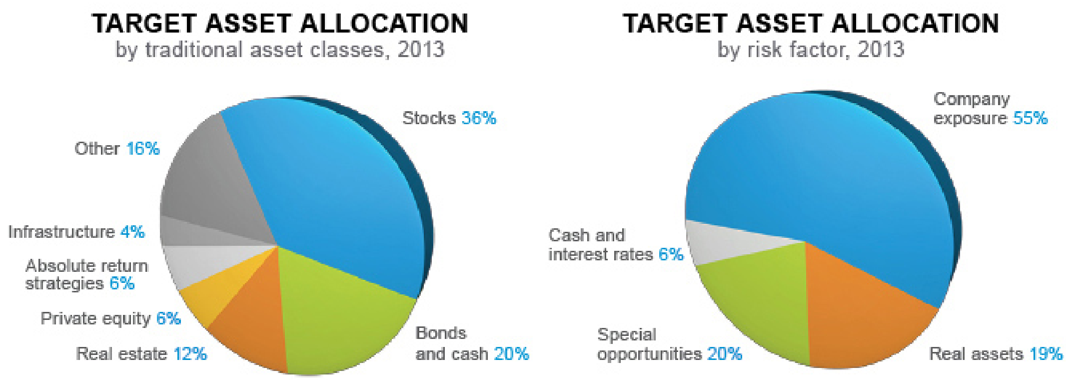


Figure 2.5: APFC benchmark portfolio.

Source: Retrieved from,http://www.apfc.org/home/Content/investments/assetAllocation2009.cfm

Notably, both the SWF owner (Board) and the portfolio manager must agree that the Reference Portfolio reflects the risk preferences and the appropriate return requirements for the SWF. In some instances, like NZ Super Fund the portfolio manager team proposes the reference portfolio then the Board approves it. In turn, the investment performance of the actual portfolio should be compared to this reference portfolio. For example, a SWF which specifies low risk investments in the policy statement should compare the portfolio manager's performance against a low-risk benchmark portfolio. Managers should mainly be judged by whether they consistently followed the SWF's policy guidelines.

[7] https://www.nzsuperfund.co.nz/sites/default/files/documents-sys/2015\%20Reference\%20Portfolio\%20Review.pdf

The portfolio manager who makes unilateral deviations from policy is not working in the best interests of the SWF (Al-Hassan et al., 2013).

A tightly related issue with the benchmark portfolio is the deviation limits from the target weights and the rebalancing policy. The policy statement should set limits to which the day-to-day allocations to asset classes in a SWF can differ from the chosen reference[8], and to what extinct the deviations from the reference portfolio should be treated.

The rebalancing policy is the mean by which the manager is guided to determine when and how the actual portfolio weights will be brought back to the target weights. Rebalancing can add value to the portfolio by systematic buying of assets that have fallen in value and sale of assets that have increased in value especially in volatile markets. However, frequent rebalancing increases transaction costs. For this reason, the rebalancing policy should set guiding rules to guide to optimize the benefits and costs of rebalancing (Al-Hassan et al., 2013).

Policy Statement Inputs

a- Investment objectives

The SWF's objectives are its investment goals expressed in terms of both risk and return. APFC states its financial objective as:

"The Board's investment allocation will be equity-dominant given its long-term investment horizon and goal, but will include multiple asset classes having varying risk and correlation assumptions. Based on the Consultant's financial models for a 5% real return objective, the Fund's long-term expected standard deviation is approximately 12%.[9]"

[8] In the case, of APFC very detailed rules can be found in the investment policy statemen (p.12), http://www.apfc.org/home/Media/investments/Investment\%20Policy\%20072015b.pdf

[9] See Investment Policy, p.7. http://www.apfc.org/home/Media/investments/Investment\%20Policy\%20072015b.pdf

Expressing goals only in terms of returns can motivate the manager to take on more aggressive approach as to maximize returns. Thus, a careful analysis of the SWF's risk tolerance should precede any discussion of return objectives. It makes little sense for a SWF which is risk averse to have its funds invested in high-risk assets.

First of all, risk is a complex concept especially if we consider the sovereign character of these funds. Moreover, measuring the riskiness of an investment strategy cannot be fully captured by one single number or indicator. It is necessary to have a broader approach, and to use several indicators to assess the riskiness of a particular strategy.

The risk tolerance is influenced by many factors such as, time horizon, expected income, and liabilities. SWFs with higher incomes are more inclined to undertake risk because their incomes can help cover any shortfall. In practice, the risk tolerance has to be inferred from the fund's approved investment universe and its current investment strategy. In general, the long investment horizon signals a higher capacity to take on investment risks, see Figure 2.6 shows the relation between objectives and time horizon with asset classes. Funds with intergenerational savings objectives (e.g., Norway, Alaska, and New Zealand) tend to ride through market downturns as a key competitive advantage given their higher tolerance for risk (Al-Hassan et al., 2013). Contrarily, when the SWFs have stabilization objectives they follow more prudent investment strategies as the case of Kazakhstan National Fund (Miceli et al., 2015).

Overall, there are several common ways in which an explicit risk tolerance can be specified, including stress; shortfall probability; and limit on the fund's value-at-risk[10]. For a public fund, the formal risk tolerance chosen not only needs to be analytically robust but must also be readily understood by the stakeholders.

[10] APFC in its investment policy statement sets full and helpful details about how risk tolerance is specified

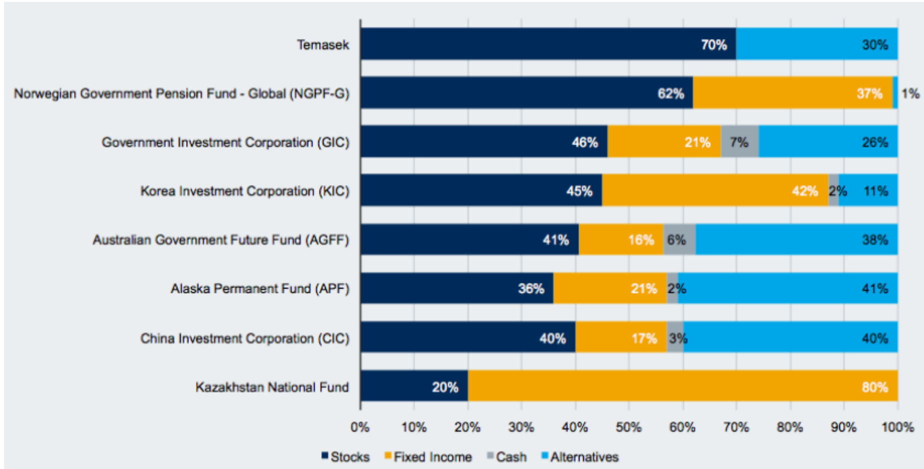


Figure 2.6: Portfolio asset allocation for SWFs (2013)
Source: (Miceli et al., 2015)

b-Investment Constraints

In addition to the investment objective that sets limits on risk and return, certain other constraints also affect the investment plan. Investment constraints include liquidity needs, an investment time horizon, legal and regulatory constraints (Reilly & Keith, 2012, p.45).

- Liquidity needs: are related to asset classes, objectives, and time horizon. For an asset to be liquid, that means it can be quickly converted to cash at a price close to fair market value. SWFs may have liquidity needs that the investment plan must consider. For example, even though a SWF may have a primary long-term objectives (saving Funds), yet there are several near-term liabilities may require available funds. Thus, the manager should consider investing part of the SWF funds in liquid assets. As we have discussed in subsection (2.2.2) Norway has set a 4% withdrawal rule, therefore the Fund must provide these funds when needed. Notably, liquidity needs are more pronounced in stabilization Funds than with saving Funds.

- Time horizon: is tightly related with the stated objectives. Hence, there exists a relationship between an investor's time horizon, liquidity needs, and ability to handle risk. Generally, SWFs with long investment horizons require less liquidity and can tolerate greater portfolio risk, because any shortfalls or losses can be overcome by earnings and returns in subsequent years. Contrarily, SWFs with shorter time horizons generally favor more liquid and less risky investments because losses are harder to overcome during a short time frame.

- Legal and Regulatory Factors: the investment process and the financial markets are highly regulated and subject to numerous laws. At times, these legal and regulatory factors constrain the investment strategies. Regulations can also limit the size and the investment choices available. Some SWFs may want to exclude certain investments from their portfolio on the basis of social consciousness reasons or environmentally harmful products (e.g., Norway and New Zealand).

2.3.2 Investment Implementation and Evaluation

As discussed above, the policy statement does not indicate which specific securities to purchase and when they should be sold, it should provide guidelines as to the asset classes to include in terms of a reference portfolio. How the manger divides funds into different asset classes is the process of asset allocation. Much of an asset allocation strategy depends on the SWF's policy statement, which includes the objectives, constraints, and investment guidelines.

The process of investing involves analyzing the current market situation, predicting future trends, and deriving strategies that offer the best possibility of meeting the policy statement guidelines. Thus, the SWF's needs, as reflected in the policy statement and financial market expectations will jointly determine the investment strategy.

Portfolio Construction

The third step of the portfolio management process is the construction of the actual portfolio. With the SWF's policy statement and financial market forecasts as input, the manager implements the investment strategy and determines how to allocate the available funds in order to minimize risks while meeting the target financial objectives specified in the policy statement. Financial theory frequently assists portfolio construction, which will be discussed in Chapter 3 of this thesis.

The management of SWF assets in accordance with the reference portfolio can take two forms: (i) exposure to various asset classes deviating from the target weights; and (ii) active management of assets within an asset class (Al-Hassan et al., 2013). Deviations from target weights can be made actively or can occur passively. For example, in the case of New Zealand the portfolio managers seek to improve the performance of the Actual Portfolio by adding value to the Reference Portfolio in three ways[11]:

- by temporarily adjusting (tilting) the Fund's market exposures in response to changes in expected returns (Strategic Tilting).
- through accessing return premia (whether market or skill based) not available in the Reference Portfolio (Active Returns).
- by gaining access to the desired risk exposures, rebalancing the Fund, and managing liquidity risk in the most cost effective manner possible (Portfolio Completion).

The chart 2.7 shows the main steps taken as to construct the actual portfolio of NZ Super Fund.

[11] Further details can be found in Reference Portfolio of NZ Super Fund

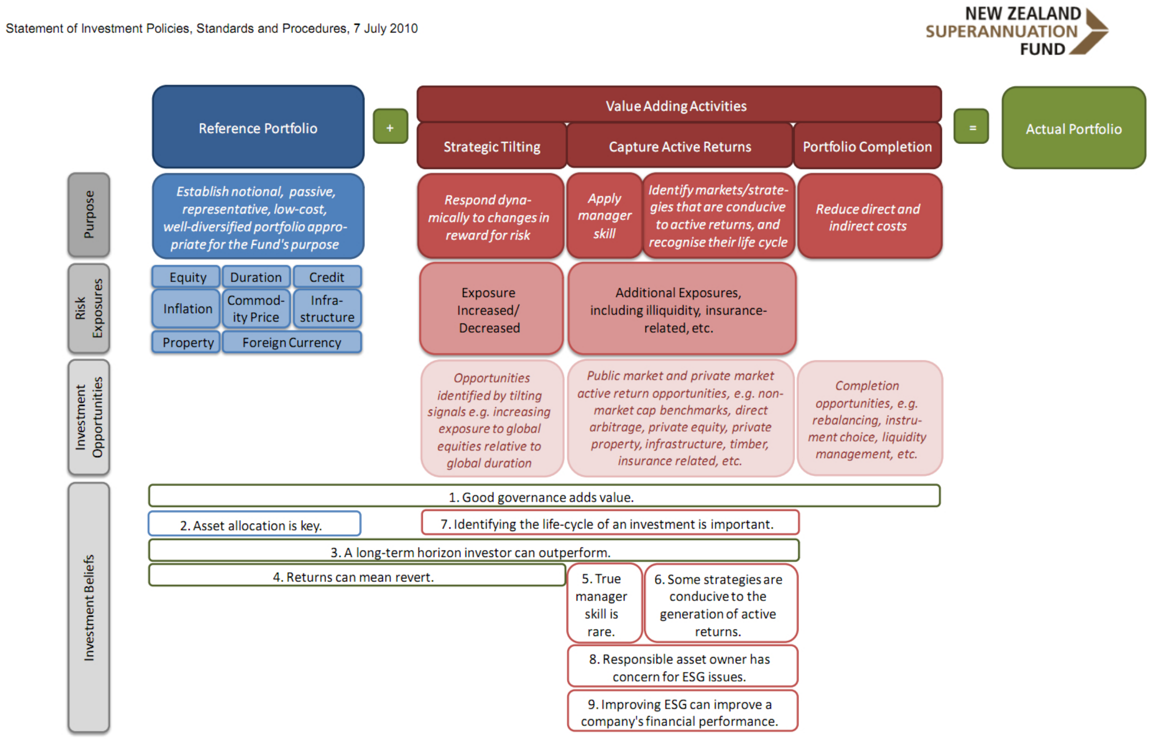


Figure 2.7: Asset allocation process in NZ Super Fund

Portfolio Evaluation

The fourth step in the portfolio management process is the continual monitoring of the SWF's needs and capital market conditions and, when necessary, updating the policy statement. Based upon all of this, the investment strategy is modified accordingly. An important component of the monitoring process is to evaluate portfolio's performance and compare the relative results to the expectations and the requirements listed in the policy statement. The benchmark portfolio serves as a standard to assess the performance of portfolio managers.

2.4 Investment Behavior of Sovereign Wealth Funds

The global financial system has witnessed, in the first decade of the 21st century a growing number of SWFs with a sizable asset under management. Although SWFs are by no means a recent phenomenon, as we may trace back their root to 1950s, since the first sovereign wealth fund Kuwait Investment Authority was established in 1953 alongside the initial oil strikes in the Persian Gulf.

The systematic growth, coupled with sovereign character and lack of transparency have provoked a variety of policy concerns among the host countries (countries that receive the investments of SWFs), particularly after the bids that have aimed to takeover some key strategic sectors in the United States. For instance, the most notable two cases were attempted by foreign corporations, state-owned or controlled entities, the first one took place in 2005 when a Chinese company - China National Offshore Oil Company (CNOOC) - tried to purchase Unocal, a U.S. oil producer; whereas, the second unaccomplished bid case happened a year after, when Dubai Ports World tried to acquire the British-based Peninsula and Oriental Steam Navigation Company (P&O) (Cohen, 2009).

With these two cases in mind, debates have boiled upon the real investment objectives of SWFs and whether they serve as instruments to reach non-commercial ends. The reaction in host countries has taken many facades, which ranges from the legitimate fears and concerns to extreme actions.

Actually, concerns were not all with political flavor. Therefore, some researchers have raised concerns about the negative impact on firms and market stability as well. For example, Jen and Miles (2007) believed that the investment behaviors of SWFs would lower international risk aversion index, raise US treasury bonds return and P/E ratio (price to earning), thus they got to the conclusion that SWFs would exacerbate market volatility. Some scholars indeed went so far to claim that SWFs were a threat to the whole capitalistic system.

Summers (2007) wrote in the July edition of *Financial Times* that the concerns raised over sovereign wealth funds are *"profound and go to the nature of global capitalism."* However, some economists held different views among them (Truman, 2007) who argued that SWFs should not be considered a threat, as long as every country tries its best to normalize its behaviors and improve the transparency. He insisted as well, that all these concerns remain *"largely in the realm of the hypothetical."* (Truman, 2008, p.4) .

To deal with such concerns, two approaches have been taken. In one hand policy responses, while the second was empirical studies aiming to get robust responses to what extent these concerns were real. Policy responses, among the host countries aiming to overcome the potential misuse of SWFs, manifested in two broad ways:

First, individual policy actions have taken place among the G7 countries. Since most enacted policy measures were justified on the basis of defending national security and/or protecting strategic industries. For example, in the United States the Congress passed the Foreign Investment and National Security Act (FINSA) of 2007 (Cohen, 2009).

Second, given the heterogeneous, somehow conflicting concepts about what stands as strategic sector, and from fears of disintegration of financial markets through growing protectionism collective approaches have been initiated. The key initiative came formally after the call of the G7 the IMF and OECD to take up the issue and dispatch the roles as follows: the IMF would work with SWFs home countries to develop principles which guide the SWF behaviors, whereas the OECD would deal with the host countries side (Truman, 2011) .

At the IMF front, series of meetings, which are best known as the International Working Group of Sovereign Wealth Fund (IWG-SWF), have taken place discussing how should SWFs be structured, governed, and managed. By October 2008, the IWG reached an agreement which is called by the name Generally Accepted Principles and Practices (GAPP). In fact, the aim of GAPP was to dispel fears and anxiety toward the practice of SWFs (Cohen, 2009).

On the other front, the OECD efforts were less fruitful resulting in no change in the already existent framework applied to cross-border investments (Truman, 2011).

There has been a growing amount of empirical studies addressing different aspects of SWFs' investment behaviors, we may classify the existent literature into three categories: Studies under this category analyze the impact of SWF investments on firm's asset prices. Thus, most studies use event-study methodology to assess the reaction of prices after the announcement of SWFs investment. All these studies document a significant positive mean abnormal return around the announcement date, followed by negative long-run returns (Fotak, Bortolotti, & Megginson, 2008; Kotter & Lel, 2011; Johan, Knill, & Mauck, 2013; Bortolotti et al., 2010). While Fernandes (2011) uses a different methodology (Tobin's Q) in which he focuses on SWFs holdings, rather than transactions. His results are in contrast with the above studies, he finds that firms with higher SWF ownership have better firm valuations and operating performance.

This category deals mainly with the determinants of SWF investments, since the SWFs' motives were the debate fuel. The study of S. Bernstein et al. (2013) examines the direct private equity investments of SWFs across different geographies with reference to governance structure. The authors find that where politicians are involved in governance, the investment decision leans toward investing in home country, which is not considered a problem in itself given the issue is well-known under the home bias puzzle, but the fact that these SWFs are likely to invest at home even with high equity prices and low P/E rates, signals that SWF investments are politically influenced. Furthermore, the results on the SWFs' investment behaviors given the involvement of external managers go in contrast with the result when politicians are involved, which is another sign strengthening the claim of political influence in the investment decisions.

Chhaochharia and Leuven (2009) analyze how SWFs make investment decisions, and they find that SWFs exhibit a home bias and tend to diversify away from industries at home into countries share the same culture. However, such behavior is fully in agreement with other institutional investors. Candelon, Kerkoury, and Lecourtz (2011) carry a research to examine whether macroeconomic factors are determinants of SWFs investment decision. They find indeed that SWFs take into consideration macroeconomic factors. For instance, when the targeted country is in Europe or North America the exchange rate stability matters much. While structural factors such as democracy, governance and crude oil prices turn out to be determinant of the investment decision in the rest of the world.

Dyck and Morse (2011) study the allocation of SWFs' assets into different regions and industries. The findings suggest that allocations are very home-region and industries biased particularly toward finance, transportation, telecommunication and energy. In addition, SWFs invest actively (with control right) which they explain with industrial planning objective. Authors find evidence as well for financial portfolio investor benchmarking and hedging of income covariance risk.

We choose in this category studies treating the investment decisions of SWFs and their impact on market stability. As documented in the above studies the acquisitions of SWFs have a little to do in changing prices due to the relative size of the asset under management of SWFs compared to other institutional investors such as pension funds. Especially if we take diversification into account.

Johan et al. (2013) document a lower volatility for target acquisitions in a step to assess whether SWFs add instability to market as claimed by the press. While Fernandes (2011) goes further to say that SWFs have a stabilizing effect on financial markets. In fact, the role played during the subprime crisis attests that SWFs have a stabilizing effect, since SWFs rescued the western banking system by purchasing about $60 billion of stocks in the largest American and European banks (Bortolotti et al., 2010). Actually, the study of Miceli (2013) goes beyond claiming that SWFs have the stabilizing effect, but documents that SWFs do not herd in equity markets across industries, this feature about SWFs investment behavior make them differ to some extent from other institutional investors. Moreover, there are indeed studies giving implicit explanations why SWFs do not herd and bear much unrealized losses. They explain that with long horizon effect and non-liability in short and medium terms.

2.5 Conclusion

In this chapter, we have discussed the main features of SWFs which we deem important in influencing the asset allocation process from the institutional and governance structure to the management of their investments.

We conclude that SWFs are in general long-term institutional investors with no liabilities or with long-term expected liabilities, usually with no leverage. They are characterized of being risk taker compared to other institutional investors. These features grant SWFs the ability to act as a stabilizer force in the face of down turn times. Although SWFs are not considered homogeneous in every single aspect yet the just-mentioned features are common traits. Furthermore, the more transparent the SWFs they are, the more they seek to maximize risk-adjusted return. Whereas SWFs with less transparency and politician involvement in management are more likely to mix objectives.

3

Portfolio Theory Background

3.1 Introduction

One of the most critical decisions for financial investors is the selection of asset classes and their weights, especially in securities markets characterized with high volatility and low returns. To optimally allocate wealth across financial assets, an investor requires a model that describes his preferences and represent asset returns dynamics. In practice investors can employ a variety of models, including the classical models which will be the subject of our discussion in this chapter. The aim of this chapter is to intuitively explain the fundamental theory that will help understand the theoretical framework applied in our research.

We will discuss in second section topics from the financial theory that outline the investment decision of an investor within the expected utility theory and how the attitude of financial agents differ towards risk.

Section 3 comprehensively describes the “Mean-Variance” approach and how to determine the properties of risky portfolios given the properties of the individual assets, delineating the characteristics of portfolios that make them preferable to others. In addition, we show how the composition of the optimal portfolios can be determined.

Section 4 deals with models of equilibrium prices and returns in the capital markets. If investors behave as portfolio theory suggests they should, then their actions can be aggregated to determine prices at which securities will sell. General equilibrium models help to determine the relevant measure of risk for any asset and the relationship between expected return and risk for any asset when markets are in equilibrium. Furthermore, though the equilibrium models are derived from models of how portfolios should be constructed, the models themselves have major implications for the characteristics of optimum portfolios. The subject of equilibrium models is not only important from theoretical point of view, but it is closely related and relevant to our study. In this section we restrict our discussion to the main two equilibrium models, the capital asset pricing model (CAPM), or the one-factor capital asset pricing model which was the first general equilibrium model developed, and its amelioration ,the Intertemporal Capital Asset Pricing Model (ICAPM), as first developed by Merton (1973).

3.2 Expected Utility and Risk Aversion

Under uncertainty, investors are often required to make investment decisions, more or less risky portfolio. These decisions are obviously determined partly by the market data (prices, interest rates) and partly by some objective variables characterizing these agents (wealth, age in particular). But intuitively, we understand that these choices also depend fundamentally on subjective parameters such as preferences, tastes and attitude towards risk.

The aim of this section is to present the theoretical concepts that have been developed particularly in the context of expected utility which we judge necessary but not sufficient condition to go further into the analysis of financial asset demand under risk.

3.2.1 The Expected Utility Hypothesis

It is worthwhile to understand the development of expected utility hypothesis in its "historical" perspective, and how it became a dominant paradigm to deal with decisions under uncertainty.

Bernoulli's St. Petersburg Paradox

It is now widely acknowledged that the analysis of decision under risk was born when the great Swiss mathematician Daniel Bernoulli discussed a puzzle that was suggested by his cousin Nicolas. Consider the following gambles as stated by Lengwiler (2004, p.69):

"I have a fair coin here. I'll flip it, and if it's tails I pay you $1 and the gamble is over. If it's heads, I'll flip again; if it's tails then I pay you $2, if not I'll flip again. With every round, I double the amount I will pay to you if it's tails." If we value this gamble based on the expected payoffs, which was the prevailing valuation concept, as follows

$$E(payoff) = \sum_{t=1}^{\infty} \underbrace{(\frac{1}{2})^t}_{probability} \underbrace{2^{t-1}}_{payoff} = \frac{1}{2}\sum_{t=1}^{\infty}(1)^t = \infty$$

Certainly this gamble seems interesting since we are not going to lose anything. However, in practice, no one is prepared to pay high to purchase the right to take this gamble (Lengwiler, 2004, p.69). In a clever attempt to solve the "St. Petersburg paradox", Bernoulli explicitly introduced utility function concept, expected utility hypothesis, and the presumption of diminishing marginal utility of money (Bernoulli, 1954). In other words, Bernoulli's idea was that utility increments of large payoffs are smaller than utility increments of small payoffs, and that these utilities should be weighted with their probabilities.

He suggested the logarithmic function as a natural measure of utility that any given payoff provides to the receiver. Thus, the true value of the gamble, according to this idea, is then given by

$$E(utility) = \sum_{t=1}^{\infty} \underbrace{(\frac{1}{2})^t}_{probability} \underbrace{\ln(2)^{t-1}}_{utility} = \ln 2 < \infty$$

So, Bernoulli would have paid at most $\ln 2$ for the right to participate in this gamble (Lengwiler, 2004, p.70). Bernoulli's theory of expected utility maximization has fallen from circulation until the renewal of game theory literature, especially, after the publication of *"Theory of Games and Economic Behavior"* by von Neumann and Morgenstern (1944), where complete set of axioms was first given to Bernoulli theory. In what follows we present the concept of expected utility.

Preferences under Uncertainty

Lotteries: We may represent any risky framework as a lottery. For example, in a coin flipping gamble where heads means that you win an amount x of money, tails that you lose x. Such gamble or risky situation can be represented by the possible outcomes and their respective probabilities as $[+x, 0.5; -x, 0.5]$; More generally, we say that

$$[x_1, \pi_1; ...; x_s, \pi_s], \; with \, \pi_s \geq 0 \, and \, \sum_{s=1}^{S} \pi_s = 1,$$

is a lottery, where $x_1, ..., x_s$ are real numbers denote prizes' amount. For the sake of simplicity we only consider situations with a finite set of possible outcomes (S is some finite number).

Axioms of Preference: Let $\mathscr{L}$ denote the lotteries set. We assume that a rational agent has a preference relation $\succ$ on $\mathscr{L}$ that satisfies the usual axioms of ordinal utility theory [1]:

1. Completeness For any two lotteries, L and L' either $L \succ L'$ or $L' \succ L$ or $L \sim L'$, where $\succ$ means stick preference and $\sim$ indifference between choices.

2. Transitivity If $L'' \succeq L' \, and \, L' \succeq L, \, then \, L'' \succeq L$

3. Continuity IF $L'' \succeq L' \succeq L$, there exists some $\lambda \in [0,1]$ such that $L' \sim \lambda L'' + (1-\lambda)L$, where $\lambda L'' + (1-\lambda)L$ denotes a "compound lottery," namely with probability λ one receives the lottery L'' and with probability $(1-\lambda)$ one receives the lottery L. Stated differently, small variation in probabilities do not affect the ordering between two lotteries (Gollier, 2001, p.4).

These standard axioms[2] imply that we can represent preferences such that for all $L, L' \in \mathscr{L}$ with a continuous utility function $V : \mathscr{L} \to \mathbb{R}$, so that

$$L \succ L' \iff V(L) > V(L').$$

Under this axiomatic structure the utility function V is no different from the usual ordinal utility theory. Therefore, preferences are monotonic in the prizes with positive probability. Note that, this utility function is not unique; as any monotonic transformation would rank these lotteries (Gollier, 2001, p.6). In order to express the utility function through a particular monotonic transformation in the expected utility form additional axiom is needed (Varian, 2010, p.173). This additional structure originates from the independence axiom. von Neumann and Morgenstern (1944, p.23-30) add the independence axiom, so that the preferences of a rational agent may be represented in the expected utility form rather than utility function.

[1] For a thorough presentation of utility and preferences, see e.g., (Varian, 2010) ch 3, and 4.

[2] Varian (1992, p.95) includes the reflexivity axiom. However, some consider it trivial and a straight consequence of completeness see e.g. (Mas-Colell, Whinston, & Green, 1995, p.6)

The independence axiom states that if we mix two different lotteries with a third one, then the ranking of the two resulting mixtures is independent of the particular third lottery. Technically (Gollier, 2001, p.6),

Independence Axiom: *The preference relation* $\succeq$ *on the space of simple lotteries* $\mathscr{L}$ *such that for all* $L'', L', L \in \mathscr{L}$ *and for all* $\lambda \in [0,1]$*:*

$$L \succeq L' \iff \lambda L + (1-\lambda)L'' \succeq \lambda L' + (1-\lambda)L''$$

Under the axiomatic structure we have just mentioned, we can present the expected utility theorem, which is due to von Neumann and Morgenstern (1944)

The von Neumann-Morgenstern Representation:

In addition to the usual preference axioms under ordinary utility theory, von Neumann and Morgenstern (1944) add the axiom of independence, so that the preferences can be represented by evaluating the expected utility of a lottery. More precisely, to find a function v that maps a single outcome x_s to some real number $v(x_s)$, and will then compute the expected value of v. Formally, the utility function V has an expected utility representation v such that

$$V([x_1, \pi_1; \ldots; x_s, \pi_s]) = \sum_{s=1}^{S} \pi_s v(x_s),$$

Since the ordinal utility function on the space of lotteries,V, represents preference relation between lotteries, in a sense of ranking the lotteries, e.g. with $V(L_1) < V(L_2)$ meaning L_2 is better than L_1. Any monotonic transformation of V is equivalent because it does not change the ranking and thus represents the same preferences. However, the von Neumann-Morgenstern utility function v is unique up to positive affine transformations, meaning that $\tilde{v}$ is equivalent to v if and only if

$$\exists a \, \exists b > 0 \, \forall x \; \tilde{v} = a + bv(x).$$

Thus far, our presentation of expected utility theory has said little regarding the utility function specification, and the relation between risk and function form. In what follows, we turn to the discussion of relation between utility function and agents' risk preferences. The basic reference for the next subsection is chapter 4 of Lengwiler (2004) book.

3.2.2 Risk Aversion

Certainty equivalent and risk premium

To keep the argument simple, Let us consider only the case where the lottery space consists solely of gambles with money prizes. Moreover, we assume that agents dislike risk. Let $E(L)$ denote the expected value of the prize of lottery L,

$$E(L) = \sum_{s=1}^{S} \pi_s x_s.$$

Consider the degenerate lottery $[E(L),1]$ which pays $E(L)$ with certainty. We say that an agent is:

Definition: *Let V be some utility function on $\mathscr{L}$, and let L be some lottery with expected prize $E(L)$. The certainty equivalent of L under V is defined as*

$$V([CE(L),1]) = V(L)$$

In words, $CE(L)$ is the level of (non-random) wealth that yields the same utility as the lottery L, i.e. the amount of payoff that an agent would have to receive to be indifferent between that payoff and a given gamble. The risk premium is the difference between the expected prize of the lottery, and its certainty equivalent, $RP(L) = E(L) - CE(L)$. The definitions of risk aversion and certainty equivalent discuss the aspects in general, without going into the details of the utility function specification or the relation between the function form and risk aversion.

1. risk neutral if $CE(L) = E(L)$; that is, the risk in L is irrelevant to the agent.

2. risk averse if $CE(l) < E(L)$.

3. risk loving if $CE(l) > E(L)$.

Risk Aversion and Concavity

Let v be a NM utility function, consider a binary lottery $[x_{low}, \pi; x_{high}, 1-\pi]$. Let us evaluate v at the two prizes, $v(x_{low})$ and $v(x_{high})$. Expected utility is $E(v(x)) = \pi v(x_{low}) + (1-\pi)v(x_{high})$. The points $(x_{low}, v(x_{low})$, $(E(x), E(v(x)))$, and $(x_{high}, v(x_{high}))$ lie on one straight line, by definition (See Figure 3.1). As seen before, the certainty equivalent is the level of wealth that gives the same utility as the lottery gives on average.

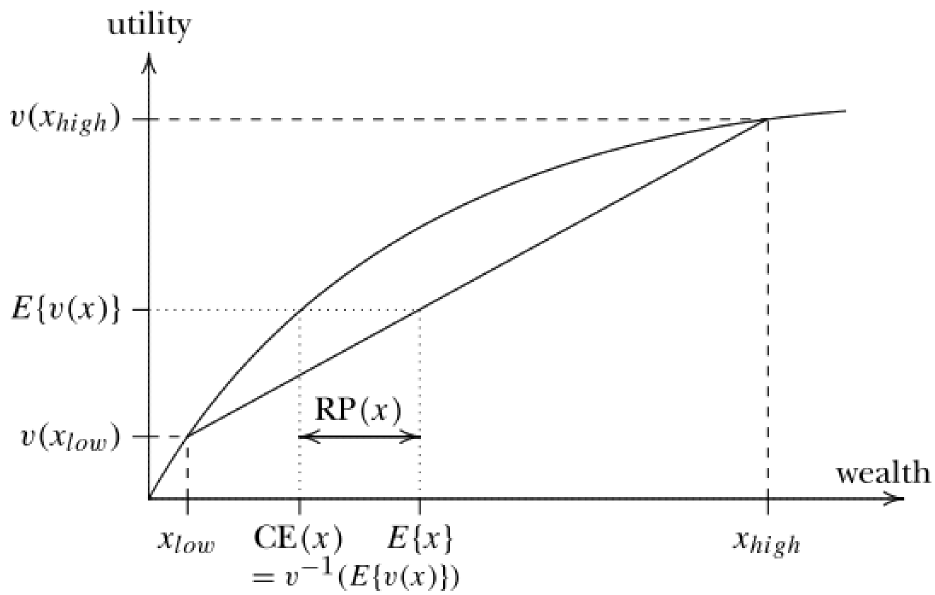


Figure 3.1: The relation of risk-averse and the concavity of NM utility function
Source: (Lengwiler, 2004, p.80)

Formally, $v(CE(x)) = E(v(x))$.. Hence, solving for certainty equivalent gives $CE(x) = v^{-1}(E(v(x)))$. Using Jensen's inequality [3] the Figure 3.1 shows that the agent is risk averse if and only if v is a concave function. For this reason, the risk premium is positive if v is strictly concave. Therefore, the curvature of v is a measure of risk aversion.

[3]Jensen's inequality states that a strict convex combination of two values of a function is strictly below the graph if the function is concave,

Measures of Risk Aversion

Human beings in general and investors in particular exhibit heterogeneous preferences. For example, some may prefer safe investments to risk and invest significantly on “riskless” assets such as treasury bonds, whereas others do not and allocate high proportion of their portfolios to stocks. Since utility functions are defined up to affine linear transformations, the concavity itself is not sufficient to characterize risk aversion’s degree. For this reason, Pratt (1964) and Arrow (1964) come out with another approach to relate the concavity with risk premia. Consider the following results due to Arrow (1971):

Definition: Let u and v be two utility functions representing preferences over wealth. The preference u has more risk-aversion than v if the risk-premia satisfy: $RP_u(x) \geq RP_v(x)$, for all random real-valued variables x

Definition: The term $A(x) = -v''(x)/v'(x)$ is called *The Arrow-Pratt Measure of Absolute Risk Aversion (ARA)*. Another measure allows us to take account of the level of wealth: the ratio $R(x) = -xv''(x)/v'(x)$ which is called *the Arrow-Pratt Measure of Relative Risk Aversion (RRA)*.

3.2.3 Standard Utility Functions

In what follows we highlight some standard utility functions that are of great importance in finance, particularly in portfolio selection, because they offer simplicity and tractability. Most of these functions belong to the class of harmonic absolute risk aversion (HARA). The definitions are taken from Prigent (2007, Chapter 1).

HARA Utility Functions

Definition: *“A utility function v is said to have harmonic absolute risk aversion (HARA) if the inverse of its absolute risk aversion is linear in wealth.”* The class of HARA utility functions take the following form:

$$v(x) = a(b + \frac{x}{c})^{1-c}, \tag{3.1}$$

with v defined on the domain $b + \frac{x}{c} > 0$. The constant parameters a, b and c satisfy the condition: $a(1-c)/c > 0$.

The *ARA* is given by:

$$A(x) = (b + \frac{x}{c})^{-1}, \tag{3.2}$$

which clearly has an inverse linear in wealth x. To ensure that $v' > 0$ and $v'' < 0$, it is assumed that $a(1-c)/c > 0$.

Usually, three main subclasses among HARA can be distinguished:

- **Constant Absolute Risk Aversion CARA**

  $A(x) = -v''(x)/v'(x) = b$, a positive constant independent of wealth, which means no wealth effects

- **Constant Relative Risk Aversion CRRA**

  This set has been by far the most used category, where functions exhibit a constant relative risk aversion and a decreasing absolute risk aversion (DARA). These functions are defined as:

  $$v(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma} & \text{for } \gamma \geq 0, \gamma \neq 1 \\ \ln(x) & \text{for } \gamma = 1 \end{cases}$$

- **Quadratic Utility Function**

  The quadratic utility functions take the following form: $v(x) = ax - bx^2$,

  This type of utility functions implies that both of the absolute risk aversion and relative risk aversion are increasing in wealth, which neither agrees with financial theory nor experimental side. For this reason research excluded them from current practice.

Time-Separable Utility Function

In dynamic settings, the preferences and behaviors of long-term investors[4] differ in some aspects from short-term investors. In short-terms the utility is derived from maximizing over wealth. However, in long term the utility is derived from consumption stream J. Campbell and Viceira (2002, Chapter 2).

In literature the intertemporal behavior is mainly presented by a time separable (additive) utility function, which can take the following form

$$E[u_1(c_1, c_2, ..., c_t)] = E_t[\sum_{t=1}^{\infty} \delta^{t-1} u(c_t)] = u(c_t) + E_1[\delta^{t-1} u_2(c_2, c_3, ..., c_t)] \tag{3.3}$$

where δ is the time discount factor, which has a value between 0 and 1, and c the consumption. Given the nice properties of CRRA specification the utility presentation in equation (3.3) can be written as

$$E[u_1(c_1, c_2, ..., c_t)] = E_t[\sum_{t=1}^{\infty} \beta^{t-1} \frac{c_t^{1-\gamma}}{1-\gamma}] = \frac{c_1^{1-\gamma}}{1-\gamma} + E_1[\beta \frac{c_2^{1-\gamma}}{1-\gamma} + ... + \beta^{t-1} \frac{c_t^{1-\gamma}}{1-\gamma}] \tag{3.4}$$

The maximization of the time separable CRRA/power utility problem can be defined intertemporal as:

$$\max E_t \sum_{i=0}^{\infty} \delta^i U(c_{t+i}) = E_t \sum_{i=0}^{\infty} \delta^i \frac{c_{t+i}^{1-\gamma} - 1}{1-\gamma} \tag{3.5}$$

Under the intertemporal condition,

$$W_{t+1} = (1 + R_{p,t+1})(W_t - c_t) \tag{3.6}$$

Where δ is the time discount factor, which has a value between 0 and 1 , $R_{p,t+1}$ is the simple net return on the portfolio.

The constraint for this optimization problem is quite straightforward. The investor's

[4]Pension funds that manage people savings for retirement, university endowments, sovereign wealth funds with saving objectives, ...

wealth next period is a function of how much wealth it can reinvest after consumption, and the return it realizes on its reinvested wealth.

3.2.4 Mean-Variance Preferences

Most of financial theory describes investor preferences under uncertainty in terms of mean and variance. However, mean-variance analysis (MV) is not always a sensible or useful description of agent's behavior in face of risky investments. In this subsection we deal with cases under which MV analysis is equivalent to the expected utility approach.[5]

- **Case 1:** v is quadratic. Therefore, the expected utility theory get simplified to a MV approach to decision-making under uncertainty. Suppose that the VNM utility takes a quadratic form, $v(x) = ax - bx^2$. Note that the domain of wealth on which v is defined comes from the necessary requirement that v increases monotonically in wealth, which is true only if $x < a/2b$. Let's consider the expected utility representation

$$\begin{aligned} E(v(x)) &= \sum_{s=1}^{S} \pi_s v(x_s) \\ &= aE(x) - bE(x^2) \\ &= aE(x) - b(E(x)^2 + \text{var}(x)) \\ &= aE(x) - bE(x)^2 - b\,\text{var}(x) \end{aligned}$$

  This function increases monotonically in the mean as long as $E(x) < a/2b$, and it decreases monotonically in the variance. The quadratic utility functions, as we have mentioned before, have a questionable property. Namely, they exhibit increasing absolute risk aversion.

- **Case 2:** Asset returns are jointly normal, if the returns are normally distributed the probability distribution is fully characterized by its mean and variance. Thus, we

[5]For more details see (Lengwiler, 2004) chapter 4.

may find an equivalent utility function representation v, taking only the mean and the variance of the returns as arguments,$f(\mu_x, \sigma_x) = E(v(x))$. However, it is generally not appropriate to use a normal distribution for wealth or asset returns.

- **Case 3:** Any smooth function $f : \mathbb{R} \to \mathbb{R}$ can be represented as an infinite polynomial using Taylor expansion. let's consider a utility function v, initial wealth w, and a zero mean risk (i.e. some x with $E(x) = 0$). For small risks, expected utility is close to $v(w)$. Taking the second-order Taylor approximation of expected utility around w gives

$$\begin{aligned} E(v(w+x)) &\approx v(w) + v^{'}(w)E(x) + \frac{1}{2}v^{''}(w)E(x^2) \\ &= v(w)\frac{1}{2}v^{''}(w)\,\mathrm{var}(x) \end{aligned}$$

Let $c := CE(w+x)$ be the certainty equivalent. For small risks, c is close to w. Consider the first-order Taylor approximation of c around w,

$$v(c) \approx v(w) + v'(w)(c-w)$$

By definition $v(c) = E[v(w+x)]$ hence, for small risks,

$$v(w) + v''(w)\frac{var(x)}{2} \approx v(w) + v'(w)(c-w),$$

which simplifies to

$$w - c \approx A(w)\frac{var(x)}{2} \tag{3.7}$$

$w - c$ is the risk premium and therefore a measure of the utility cost of the small risk. This means that small risks can be evaluated approximately just by their variance.

3.3 Mean-Variance Analysis

Markowitz states in the foreward of (Guerard, 2010, p.v)
"Many ascribe assumptions underlying mean-variance analysis to me; they are, in fact, credited to Tobin(1958) and eschewed by Markowitz(1959)." Thus, this section is devoted to the mean-variance analysis as introduced by Markowitz (1952, 1959), with the extensions of Tobins's two fund separation theorem (1958), which has very important implications not just for the mean-variance analysis, but on the Capital Asset Pricing Model as well.

3.3.1 Mean-variance Criterion:

The use of utility functions is too complex in general, because even by using the Arrow-Pratt measure of risk aversion still the utility function has to be known in order to determine the expected utility derivatives. To overcome this complexity and the shortcomings of expected utility approach, many criteria have been proposed.[6] Markowitz's mean-variance criterion (MV) is by far the most pronounced alternative, despite its inability to determine solely the optimal choice. Due to some nice properties of MV criterion it became the corner stone of portfolio choice theory of Markowitz (1959) and Tobin (1958). Moreover, the subsequent asset pricing models are more or less based on MV criterion such as Sharpe-Lintner CAPM (Sharpe, 1964; Lintner, 1965) which is fully based on it, and to some extinct the Arbitrage Pricing Theory of Ross (1976).

Markowitz in his seminal article of (1952) introduced the optimality rule of MV, or in another words expectation-variance in Markowitz's terminology(E,V), into the process of portfolio selection as he noted (Markowitz, 1952, p.82):
"The E-V rule states that the investor would (or should) want to select one of those port-

[6]For example, Friedman and Savage, developed in their "Utility Analysis of Choices Involving risk", (Friedman & Savage, 1948) 1948, their own utility function known as the Friedman-Savage utility function. They argued that a single individual could have different utility functions depending on their initial wealth. The implication of an individual being, at the same time, risk-loving and averse, implies that its utility function has different curvatures

folios which give rise to the (E,V) combinations ... those with minimum V for given E or more and maximum E for given V or less."

MV approach considers only two moments of asset return distribution, the expected return and the variance as a measure of risk. The MV criterion can be defined formally as:

$$a_i \succeq a_j \Leftrightarrow \begin{cases} var(a_i) < var(a_j) \,\text{and}\, E(a_i) \geq E(a_j) \\ \text{or} \\ var(a_i) \leq var(a_j) \,\text{and}\, E(a_i) > E(a_j) \end{cases}$$

It is a necessary, but not sufficient condition to prefer a_i over a_j when $var(a_i) \leq var(a_j)$ and $E(a_i) \geq E(a_j)$. It is easy to choose between alternatives when one has a smaller risk (variance) and a larger expected return. However, nothing can in general be said if $var(a_i) > var(a_j)$ and $E(a_i) > E(a_j)$, which calls for other decision rules to be applied (Levy & Sarnat, 1972, p.308).

3.3.2 Diversification

Diversification is by no means a new notion, but an age-old concept "don't put your eggs in one basket," obviously predates economic theory. However, there was no formal model to take full advantage of this aspect until Markowitz genuinely succeeded to quantify the diversification concept through the use of statistical notion of covariance between individual assets and the overall variance of a portfolio (Markowitz, 1952). The common security selection methods used prior to Markowitz's breakthrough focused mainly on the returns generated by investment opportunities.

The standard practice was to identify securities with high expected return with low risk and compile from them a portfolio. Thus, it would be highly possible to construct a portfolio from the same industry securities.

There are three propositions in which Markowitz's MV approach may affect diversification (Moraux, 2010, p.12):

1. The addition of an extra asset into a portfolio reduces the risk of the latter, when the asset is not perfectly and positively correlated to the portfolio.

2. There is a limit to the reduction of diversified risk (unsystematic) of a portfolio. The residual risk can be called the undiversified risk (systematic risk).

3. The volatility of the most diversified portfolio is the square root of the average covariance.

The first case is consistent with what could be expected from diversification. More formally, it teaches us that the portfolio volatility is less than the weighted average of individual volatilities. It is possible to see the intuition behind diversification through the interpretation of the probable covariance between the individual asset R_i and Portfolio return R_p

$$\sigma_p = \sum_{i=1}^{N} \alpha_i \frac{cov(R_i, R_p)}{\sigma_p} \tag{3.8}$$

The term $cov(R_i, R_p) / \sigma_p$ may be interpreted as the contribution of asset i on the overall portfolio volatility. Proposition 2 is an important result which states that there is a limit to diversification process, whereas the third Proposition characterizes it. The existence of a limit (non-zero) suggests that the effectiveness of the diversification by $N+1$th asset decreases when the number of portfolio assets N is already large. The cost generated by diversification opens the question of optimizing this procedure. How many assets are required to significantly diversify a portfolio, without offsetting the diversification benefits with extra costs?

The figure 3.2 shows that the total risk has two components one can be diversified whereas the other is undiversifiable or market risk because it affects the whole market. The diversification has the effect to reduce risk whenever new securities are added, but in decreasing manner and tends towards a horizontal level (market risk).

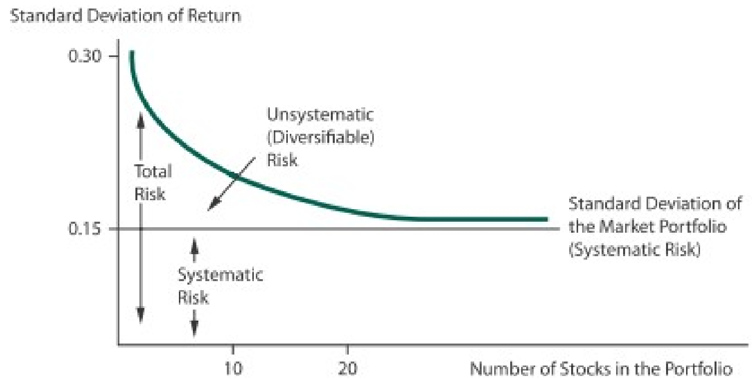


Figure 3.2: illustrates the diversification effect on portfolio risk as a function of stocks number

Source: (Reilly & Keith, 2012, p.213)

3.3.3 Risky Efficient Frontier

The decisive advantage of MV formalization, in addition to its simplicity, is its ability to be explained graphically in a two dimensional space only. Since portfolios can be constructed using any number of risky assets with variety of weight proportions of each asset, there is a wide range of risk-return combinations. If these portfolios are plotted in $(0, \sigma, E(R))$ plan, the universe of all these risk-return alternatives is called the investment opportunity set or feasible portfolios. These alternatives lie in a compact convex set. See Figuer 3.3.

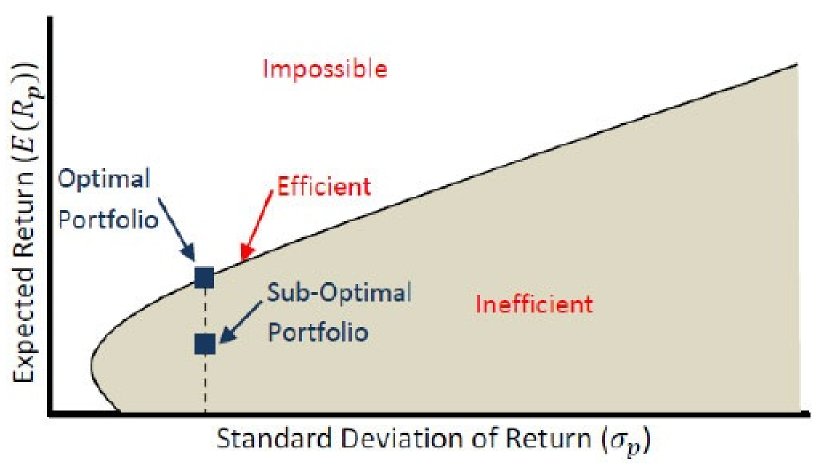


Figure 3.3: illustrates the efficient frontier diagram

The alternatives that belong to the investment opportunity set but they are not dominated by another alternative given the MV criterion are situated on the upper left of the opportunity set, and they are called the efficient frontier. To sum up, efficient frontier consists of the set of all efficient portfolios that yield the highest return for each level of risk. These are the only portfolios that a rational investor will hold on the market. There are three approaches to get the efficient frontier (Moraux, 2010, p.16-17).

- The first approach is to find minimum variance portfolios frontier with fixed expected return, then we identify the minimum variance portfolio $(\sigma_{\min}, E(R_{\min}))$, after that we retain only the portfolios from the frontier which have the expected return higher than $E(R_{\min})$.

- The second approach is to solve the optimization program that accurately translates the definition of an efficient portfolio:

$$\left.\begin{array}{c} \max_{\alpha_1,\dots,\alpha_N}\ E(R_p) \\ s.c \begin{cases} \sigma_p^2 & = A_{\sigma^2} \\ \sum_{i=1}^{N} \alpha_i & = 1 \end{cases} \end{array}\right\} \Leftrightarrow \left\{\begin{array}{c} \max_{\alpha}\ \mathbf{R}'\alpha \\ s.c \begin{cases} \alpha'\Sigma\alpha & = A_{\sigma^2} \\ \mathbf{e}'\alpha & = 1 \end{cases} \end{array}\right.$$

  The right system is a vector representation of the efficient portfolios, where A is a constant and Σ is the covariance matrix.

- The third approach is to construct the set of optimal portfolios of all investors. However, it is necessary to know first the optimal portfolio of each of them.

3.3.4 Optimal Risky Portfolio

Two definitions are possible to characterize the optimal portfolio of a particular investor. Both rely on the concept of utility function.

1. The optimal portfolio is a portfolio from the efficient frontier that gives the investor the greatest utility.

2. The optimal portfolio is a portfolio from the investor feasible set that maximizes his utility function.

Suppose that the investor utility is well described by a quadratic function U defined on space coordinates $(0,\sigma,E(R))$ as follows:

$$U(p)=U(\sigma_p,E(R_p))=E(R_p)-\frac{\gamma}{2}\sigma_p^2=\mathbf{R}'\alpha-\frac{\gamma}{2}\alpha'\Sigma\alpha$$

where γ is a positive number that captures the risk aversion of the investor. Portfolios satisfy $U(p)=A$, provide the investor the same utility. Their coordinates verify $E(R_p)=A+\frac{\gamma}{2}\sigma_p^2$. This equation describes the investor indifference curves.

The first definition suggests a three-step procedure that involves a) construct the efficient frontier, b) identify the indifference curves (the set of portfolios providing the same level of satisfaction), and c) deduce the most efficient portfolio. The optimal portfolio is located both on the efficient frontier and the indifference of greatest utility curve. This is the portfolio that is tangent to the efficient frontier and belongs to one of the investor's indifference curves as in Figure 3.4.

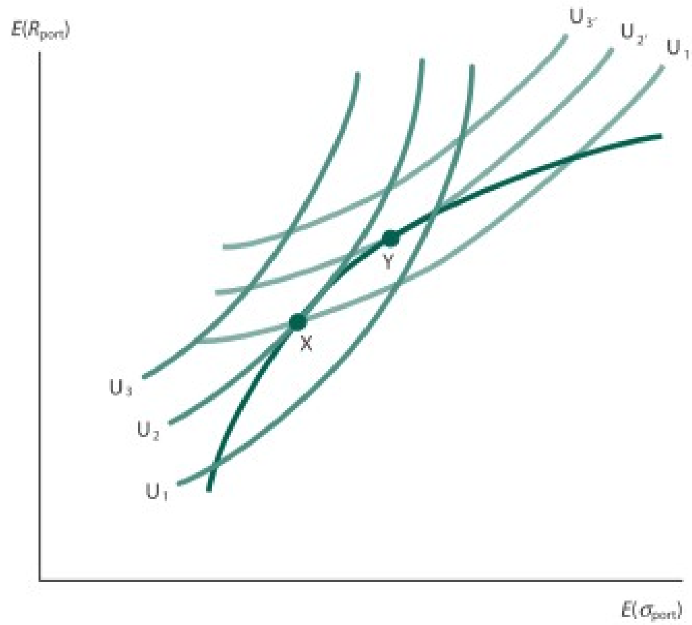


Figure 3.4: The Selection of an optimal Risky Portfolio
Source: (Reilly & Keith, 2012, p.200)

The second suggests the direct utility function maximization subject to the constraint $\sum_{i=1}^{N} \alpha_i = 1$. This method is exact only under the condition of quadratic utility function. Thus we solve the optimization problem:

$$\left.\begin{array}{l} \max_{\alpha_1,\ldots,\alpha_N} \; U(\sigma_p, E(R_p)) \\ s.c, \quad \sum_{i=1}^{N} \alpha_i = 1 \end{array}\right\} \quad \Leftrightarrow \quad \left\{\begin{array}{l} \max_{\alpha} \; \mathbf{R}'\alpha - \frac{\gamma}{2}\alpha'\Sigma\alpha \\ s.c, \quad \mathbf{e}'\alpha = 1 \end{array}\right.$$

3.3.5 The Capital Market Line

So far it has been assumed that the efficient portfolios consisted solely with risky assets. If a risk-free asset is introduced (R_f) as a possible investment vehicle, both the feasible set and efficient portfolios will change significantly. Since the risk-free asset has zero variance, so it lies on the vertical axis in as in Figure 3.5. According to the two-fund separation theorem (Tobin, 1958), the new efficient frontier can be formed by a convex combination of the risk-free asset and the market portfolio which lie on the efficient risky frontier.

Thus, the efficient set becomes a line, rather than a curve. In addition, if the investor can borrow as well as lend (lending is equivalent to buying risk-free debt securities) at the riskless rate, it is possible to move out on the line CML, and the optimal portfolio would be the one that is tangent between the indifference curve and CML line. The line CML is called the *Capital Market Line*, It has an intercept of (R_f) and a slope of $(R_M - R_f)/\sigma_M$, where R_M denotes the return for market portfolio and σ_M its volatility. Therefore, the equation for the Capital Market Line may be expressed as follows:

$$R_p = R_f + (\frac{(R_M - R_f)}{\sigma_M})\sigma_p \tag{3.9}$$

The expected rate of return on an *efficient portfolio* is equal to the riskless rate plus a risk premium that is equal to $\frac{(R_M - R_f)}{\sigma_M}$ multiplied by the portfolio's standard deviation σ_p. Thus, the CML specifies a linear relationship between an efficient portfolio's expected return and risk, with the slope of the CML being equal to the expected return on the market portfolio of risky stocks R_M minus the risk-free rate, which is called the market risk premium, all divided by the standard deviation of returns on the market portfolio σ_M.

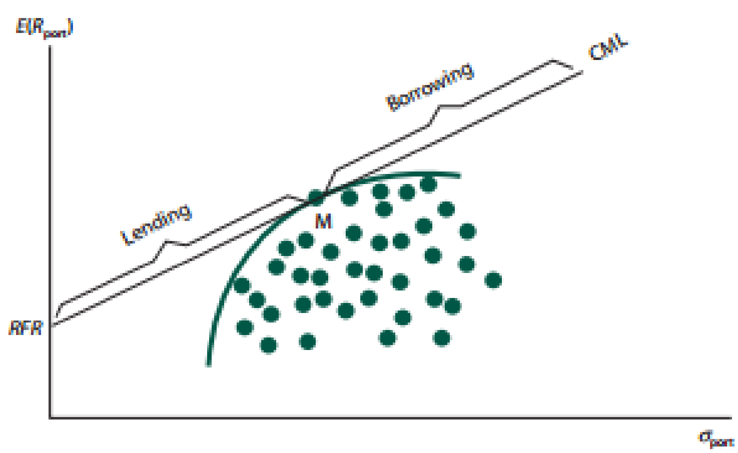


Figure 3.5: Capital line Assuming Lending or Borrowing at the Risk-Free Rate
Source: (Reilly & Keith, 2012, p.212)

Capital market theory which has represented concisely is a major step forward in how investors should think about the investment process. Unfortunately, capital market theory gives a partial and incomplete explanation for the relationship that exists between risk and return. The CML defined the risk an investor bears by the total volatility of the investment. The limitation is thus that the CML cannot provide an explanation for the risk-return trade-off for individual risky assets (Reilly & Keith, 2012).

3.4 Asset Pricing Models

3.4.1 Capital Asset Pricing Model

The capital asset pricing model (CAPM) measures the risk contribution of each risky asset to the market portfolio and can thereby assign an equilibrium return to each individual asset. The model which is an advancement of the mean-variance analysis has almost simultaneously been developed by Sharpe (1964), and Lintner (1965).

Assumptions

The CAPM is derived under the following assumptions.

- The investors are rational and price takers. They have a strictly increasing and concave utility function of wealth $U(W)$, i.e., $U^{'}(W) > 0$ and $U^{''}(W) < 0$.
- There are risky assets $R_i, i = 1, ..., N$, and a risk-free asset R_f on the market.
- Assets returns are joint normally distributed.
- All assets are marketable and perfectly divisible. The quantities of assets are fixed.
- Information is costless and simultaneously available by all investors,i.e., investors have homogenous beliefs about assets return distributions.
- Markets are frictionless.

CAPM Derivation

Consider a portfolio p which is a combination of an arbitrary risky asset H and the market portfolio M. The expected return and variance of this portfolio are given as follows:

$$E[R_p] = \alpha_H E[R_H] + (1-\alpha_H)E[R_M], \tag{3.10}$$

$$\sigma^2(R_p) = \alpha_H^2\sigma_H^2 + (1-\alpha_H)^2\sigma_M^2 + 2\alpha_H(1-\alpha_H)\sigma_{HM} \tag{3.11}$$

where α_H and $(1-\alpha_H)$ are weights of asset H and market portfolio, respectively, σ_H^2 and σ_M^2 are the variances, and σ_{HM} is the covariance of H and M.

In equilibrium asset prices adjust such that all assets are held and markets clear. The market portfolio therefore consists of all marketable assets and the weight of the assets is their market share. It follows that in equilibrium α_H is zero. Evaluating the partial derivatives of equations (3.10) and (3.11) with respect to α_H at $\alpha_H = 0$ and taking their ratio gives the risk-return trade-off, respectively, the slope of the minimum variance opportunity set of risky assets at M,

$$\frac{\partial E[R_p]/\partial\alpha_H \mid_{\alpha=0}}{\partial\sigma_{R_p}/\partial\alpha_H \mid_{\alpha=0}} = \frac{E[R_p]-E[R_M]}{(\sigma_{HM}-\sigma_M^2)/\sigma_M} \tag{3.12}$$

We know from the MV analysis that in M the CML is tangent to the minimum variance opportunity set of risky assets. Equation (3.12) is therefore equal to the slope of the CML (see equation (3.8)), i.e.,

$$\frac{E[R_H]-E[R_M]}{(\sigma_{HM}-\sigma_M^2)/\sigma_M} = \frac{E[R_M]-R_f}{\sigma_M} \tag{3.13}$$

After rearranging terms we get

$$E[R_H] = R_f + (E[R_M]-E[R_f])\frac{\sigma_{HM}}{\sigma_M^2} \tag{3.14}$$

which is the CAPM. The required expected return of any asset $E[R_i]$ is equal to the risk-free rate plus a risk premium. The risk premium is the price of risk, $E[R_M] - E[R_f]$, times the quantity of risk, σ_{iM}/σ_M^2, which is often referred to as β_i, i.e., the covariance of asset i with market portfolio scaled by the variance of market portfolio.

Critique

Three major limitations of the CAPM shall be pointed out. First, the assumptions are unrealistic (Fama & French, 2004). For example, the CAPM assumes that assets are divisible and marketable. However, human capital is not divisible. Moreover, it is quite obvious that investors do not share the same information, and markets have frictions. Most of the assumptions can be relaxed which results in variations of the original model. The most important extension is the step to multiple periods. This is done by Merton (1973), who derives an intertemporal CAPM.

Second, empirical results (see, e.g., Fama and French (1992) and Black (1993)) do not support the model. The CAPM seems to underestimate the returns of small β securities and to overestimate the return of high β securities. Furthermore, there are other factors that are able to (partially) explain the portion of security returns, which is not explained by β.

Third, Roll (1977) points out that the CAPM is a joint hypotheses of the model being valid and the market portfolio being efficient. Since the market portfolio contains all possible assets, including all types of securities, human capital, land etc., it is virtually impossible to verify that it is efficient. Hence it is impossible to conclude from a rejecting test that it is the CAPM which is rejected. It could be that the market portfolio was just not efficient. Under consideration of these limitations the main contribution of the CAPM is not necessarily the explicit pricing formula of risky assets. It is rather methodological. The CAPM clearly shows that it is the undiversifiable risk which determines asset prices.

3.4.2 Intertemporal Capital Asset Pricing Model

The static feature of CAPM implies that the amount invested into assets remains the same for a given period of time and the amounts invested into each asset could not be changed.

Where at the end of time period it was assumed that the investors consume their wealth. A more realistic setting would be to allow investors to change the amounts invested into each asset and also to withdraw a part of their investment for immediate consumption. The Intertemporal Capital Asset Pricing Model (ICAPM) as first developed by (Merton, 1973) takes these considerations into account.

The model Assumptions:

- *Market assuptions:*
  - the market is assumed to be perfect, i.e. all assets have limited liability.
  - there is no transaction costs or taxes, assets are infinitely divisible.
  - the market is always in equilibrium, hence there is no trade outside the equilibrium price, and trading takes place continuously.
- *Investor assuptions:*
  - Investors maximize a time-additive utility function of consumption, which is strictly increasing and concave, $U^{'}(W) > 0$ and $U^{''}(W) < 0$.
  - Investors only source of income are capital gains. Labor income is ignored.
  - Investors are price takers.
  - Investors can borrow and lend without any restrictions.

- *Price and return dynamics assuptions:*
  - There are N risky assets and a risk-free asset with a risk-free rate $r_f(z_t)$ on the market. The returns of the risky assets, r_t, follow a N-dimensional geometric Brownian motion,[7]

$$d\mathbf{r}_t = \mu(\mathbf{z}_t)dt + \Sigma^{\frac{1}{2}}(\mathbf{z}_t)d\mathbf{B}_t.$$

    where $d\mathbf{r}_t$ denotes the instantaneous vector of the risky security returns. $\mathbf{B}_t$ is a standard Wiener process and $d\mathbf{B}_t$ is the associated white noise. The drift $\mu(\mathbf{z}_t)$, the covariance matrix $\Sigma(\mathbf{z}_t)$, and the risk-free rate $r_f(z_t)$ are functions of the state variables $\mathbf{z}_t$.
  - All variables that can explain the prices and price changes of the assets (the state variables) follow a joint Markov process.
  - The state variables are assumed to change continuously over time, i.e. no jumps are allowed. Hence, they can be described by

$$d\mathbf{z}_t = \mu^z(\mathbf{z}_t)dt + (\Sigma^z)^{\frac{1}{2}}(\mathbf{z}_t)d\mathbf{B}_t^z,$$

    where $\mu^z(\mathbf{z}_t)$ and $(\Sigma^z)(\mathbf{z}_t)$ are functions for the drift and the covariance matrix. $d\mathbf{B}_t^z$ is a M-dimensional Wiener process. The correlation matrix between $d\mathbf{B}_t$ and $d\mathbf{B}_t^z$ is ρ^z . It is $N \times M$ dimensional. $\Sigma(\mathbf{z}_t)$ and $(\Sigma^z)(\mathbf{z}_t)$ are further assumed to be invertible and the integrability condition holds.

    To simplify notations, we denote the following $r_f(z_t)$, $\mu(\mathbf{z}_t)$, $\Sigma(\mathbf{z}_t)$, $\Sigma_{\frac{1}{2}}(\mathbf{z}_t)$, $\mu^z(\mathbf{z}_t)$, $(\Sigma)^{\frac{1}{2}}(\mathbf{z}_t)$, $\rho(\mathbf{z})$, by r_{ft}, μ_t, Σ_t, Γ_t, μ_t^z, Γ_t^z, and ρ, respectively.

[7] This implies that prices are log-normally distributed and returns normally distributed.

Optimal Portfolio and Consumption Rules

The problem of choosing an optimal portfolio and consumption policy for an investor endowed with wealth W_t who invests for T periods is formulated as follows:

$$\begin{cases} \max_{C_t,\alpha_t} E_t[\int_t^T U(C_\tau)d\tau + U(W_T)] \\ s.t. \quad dW_t = W_t(r_{f_t}dt + \alpha_t^T(\mu_\mathbf{t} - r_{f_t}\mathbf{1})dt + \alpha_t^T \Gamma_t dt\mathbf{B}_t) - C_t dt \\ 1 = \alpha_{r_{ft}} + \alpha_t^T \mathbf{1} \end{cases} \tag{3.15}$$

where E_t the expectation operator conditional on W_t and $\mathbf{z}$, under the budget constraint,

$$dW_t = W_t(r_{f_t}dt + \alpha_t^T(\mu_\mathbf{t} - r_{f_t}\mathbf{1})dt + \alpha_t^T \Gamma_t dt\mathbf{B}_t) - C_t dt \tag{3.16}$$

where α_t and $\mathbf{1}$ are $(N \times 1)$ vectors of asset weights and ones, respectively, and C_t is the consumption. The first right hand side term is the gain of the portfolio and the second term the consumption.

The solution of the optimization problem: To derive the optimal policies[8], the method of stochastic dynamic programming is used, which is based on Bellman principle of optimality that states (Bellman, 1957, p.83) "*An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.*" Thus the dynamic problem can be viewed as a method which breaks the decision problem into successive of static programming problems, if the utility function is time-additive. The principal of optimality is applied in reverse to solve the asset allocation problem. That is, at a time $T-dt$ the optimal strategy α^*_{T-dt} for the period $[T-dt, T]$ is derived. At $T-2dt$ the optimal strategy α^*_{T-2dt} is derived conditional on the solution of the remaining period,α^*_{Tdt}. Re-verse engineering from investment horizon T in portions of length d_t to the investment date t the optimal strategy α^*_t is found.

[8] For further and rigorous study see e.g.,(Stokey, Lucas, & Prescott, 1989; Ljungqvist & Sargent, 2012; Bellman, 1957)

To formally find a solution at time t , denote $J(W,\mathbf{z},t)$ as the maximal utility at t given the investment problem in (3.15),

$$J(W,\mathbf{z},t) = \max_{C_t,\alpha_t} E_t[\int_t^T U(C_\tau)d\tau + U(W_T)]. \tag{3.17}$$

This equation is known as the Bellman equation. It can be rewritten stepwise as

$$\begin{aligned}
&\max_{C_t,\alpha_t} E_t[U(C,t)dt + J(W,\mathbf{z},t+dt)] \\
\Leftrightarrow &\max_{C_t,\alpha_t} E_t[U(C,t)dt + J(W,\mathbf{z},t+dt) - J(W,\mathbf{z},t)] \\
\Leftrightarrow &\max_{C_t,\alpha_t} E_t[U(C,t)dt + \mathscr{D}dt] \\
\Leftrightarrow &\max_{C_t,\alpha_t} E_t[U(C,t)dt + \mathscr{D}] \\
\Leftrightarrow &\max_{C_t,\alpha_t} \Psi(C,\alpha_t,W,\mathbf{z},t).
\end{aligned} \tag{3.18}$$

The first step holds by a result of stochastic dynamic programming:

Theorem: *If the security prices* $\mathbf{P_t}$ *are generated by a strong diffusion process,* $U(C,t)$ *is strictly concave in C, and* $B(W,T)$ *is concave in W, then there exists a set of optimal choices,* C^* *and* α^**, satisfying t,* $J(W,\mathbf{z},T) = B(W,T)$ *, and*

$$\Psi(C_t,\alpha_t,W,\mathbf{z},t) \le \Psi(C_t^*,\alpha_t^*,W,\mathbf{z},t) = 0$$

$$for \quad t \in [0,T]$$

The theorem says the maximum obtainable utility is zero and suboptimal choices of C and α have at most zero utility. The first step therefore holds since $J(W,\mathbf{z},T)$ is zero. Equivalence two holds by recognizing that the difference between the two indirect utilities corresponds to the Dynkin operator $\mathscr{D}$, which is the expected time rate of change of the indirect utility, $J(W,\mathbf{z},T)dt$. The third equivalence is obtained by dividing both terms in equivalence two by dt.

Rewriting the objective function as $\Psi(.)$ leads to the fourth version of the optimization problem. Finally, the detailed Bellman equation for the indirect utility J can be written under application of the latter theorem as

$$\max_{C,\alpha}[\dot{J}+\{W(r_f+\alpha^T(\mu-r_f\mathbf{1}))-C\}J_W+\frac{1}{2}W^2\alpha^T\Sigma\alpha J_{WW}+(\mu^z)^T\mathbf{J_z}+ \\ \frac{1}{2}Tr(\Sigma\mathbf{J}_{zz^T})+W\alpha^T\Gamma\rho^T\Gamma^{zT}\mathbf{J}_{WV}+U(C,t)]=0 \tag{3.19}$$

subject to the boundary condition $J(W,\mathbf{z},T)=B(W,T)$.
Concerning the notation in equation (3.19), we use $\dot{J}=\dfrac{\partial J}{\partial t}$, $J_W=\frac{\partial J}{\partial W}$, $J_{WW}=\frac{\partial^2 J}{\partial W^2}$, $\mathbf{J}_z=\frac{\partial J}{\partial \mathbf{z}}$, $\mathbf{J}_{zz}=\frac{\partial^2 J}{\partial \mathbf{z}^2}$, and $\mathbf{J}_{Wz}=\frac{\partial^2 J}{\partial W\mathbf{z}}$. The operator Tr(.) takes the trace of a matrix.
First-order conditions Given the Bellman equation (3.19), the first-order conditions for the optimal C^* and α^* can be derived. They are

$$U_C(C^*)=J_W,$$

$$\begin{aligned}\alpha^* &= \frac{1}{\gamma}\Sigma^{-1}((\mu-r_f\mathbf{1})+\Gamma\rho^T\Gamma^{zT}\frac{\mathbf{J}_{Wz}}{J_W}) \\ &= \frac{1}{\gamma}(\Sigma^{-1}(\mu-r_f\mathbf{1})+\Sigma^{-1}\Gamma\rho^T\Gamma^{zT}\frac{\mathbf{J}_{Wz}}{J_W}),\end{aligned} \tag{3.20}$$

where $U_C=\frac{\partial U}{\partial C}$. $\gamma=-W\frac{J_{WW}}{J_W}$ is the relative risk aversion.
The optimal portfolio rule (3.20) consists of two terms that are both multiplied by the relative risk tolerance $1/\gamma$. These terms each of them define a portfolio. The first portfolio corresponds to the portfolio of risky assets in the mean-variance analysis.

The second portfolio hedges against shocks and shifts in the opportunity set. The intertemporal hedging portfolio is determined by $\Sigma^{-1}\Gamma\rho^T\Gamma^{zT}$ and the indirect utility function J. The term $\Sigma^{-1}\Gamma\rho^T\Gamma^{zT}$ measures the covariance of the asset returns and the predictive variable. It enables selecting portfolios with high correlation with the state variables $\mathbf{z}$. The factor $\frac{\mathbf{J}_{Wz}}{J_W}$ is the sensitivity of the marginal utility of wealth to the stochastic opportunity set.

Note that under some model specifications the mean-variance portfolio is the only portfolio held. If the investor horizon is infinitely small then the intertemporal hedge portfolio tends to be zero, as over very short time-period the investment opportunity set is not expected to change. Hence a myopic investor only invests in the mean-variance portfolio.[9]

The intertemporal hedging portfolio is going to be also zero, as a result of the asset returns being unpredictable. Moreover, Merton (1971) pointed out that investors with logarithmic utility functions would hold only the myopic portfolio, whatever their investment horizon and whether asset returns are predictable or not.

To sum up investors hold the intertemporal hedge portfolio if these three conditions are met otherwise they hold the myopic portfolio,

- the investment horizon is long-term,
- the utility function is not logarithmic, and
- asset returns are predictable.

Nonlinear PDE

In order to explicitly solve for the control variables C^* and α^* the first-order conditions are substituted back into the Bellman equation (3.19) to get:

$$\begin{aligned}
&\dot{J}+\frac{1}{\gamma}\frac{1}{W}(\mu-r_f\mathbf{1})^T\Sigma^{-1}(\mu-r_f\mathbf{1})WJ_W \\
&+\frac{1}{\gamma}(\mu-r_f\mathbf{1})^T\Gamma^{T^{-1}}\rho^T\Gamma^{zT}W\mathbf{J}_{WV} \\
&+\frac{1}{\gamma}\frac{1}{W}\frac{\mathbf{J}_{Wz}^T}{J_W}\Gamma^z\rho\rho^T\Gamma^{zT}W\mathbf{J}_{Wz} \\
&+r_fWJ_W-CJ_W+\frac{1}{2}Tr(\Gamma^z\Gamma^{zT}\mathbf{J}_{zz^T})+\mu^{zT}\mathbf{J}_z+U(C,t)=0,
\end{aligned} \tag{3.21}$$

subject to the boundary condition $J(W,\mathbf{z},T)=B(W,T)$. Equation (3.21) is a nonlinear partial differential equation (PDE) for the unknown function of indirect utility J. Nonlinear PDEs are nontrivial to solve.

[9]In multi-period finance literature the mean-variance portfolio is generally called myopic portfolio

The common approach is to guess the correct function of indirect utility, which solves the PDE. An alternative approach transforms the nonlinear PDE into a set of ordinary PDEs with known solutions (Chacko & Viceira, 2005).

Critique

The ICAPM model presented concisely, here, allows to find the optimal choices for C^* and α^* for some special cases. Fortunately, the model is flexible to incorporate other price processes, expected labor income, and liabilities into the analysis. Even though the model has some nice properties still it suffers from few drawbacks, which limit its applicability such as:

- The Bellman equation can be solved either analytically, approximately or numerically in order to find the optimal control variables C^* and α^*. All three approaches are challenging, since the Bellman equation is a nonlinear PDE. Closed-form and approximate solutions are only known for some special cases.
- There are no easily applied general conditions that ensure the existence of a solution.
- A solution can only be found if the indirect utility function J is continuously differentiable.

3.5 Conclusion

This chapter provided the necessary background to deal with the subject of asset allocation in dynamic setting. The second and the third sections deals mainly with basic notions of the expected utility functions, risk aversion, and mean-variance analysis leading to asset pricing models in section 4. The capital asset pricing model (CAPM), or the one-factor capital asset pricing model which was criticized of being based on unrealistic set of assumptions. Especially, the static feature of Capital Asset Pricing Model implies that the amount invested into assets remains the same for a given period of time and the amounts invested into each asset could not be changed.

A more realistic setting would be to allow investors to change the amounts invested into each asset and also to withdraw a part of their investment for immediate consumption, due to the change in the opportunity set. The Intertemporal Capital Asset Pricing Model (ICAPM) takes these considerations in account. Hence, the ICAPM extends the CAPM to a dynamic environment.

Under ICAPM model the demand for an asset consists of two parts. The first term is the demand similar to that of the static CAPM, while the second term is an adjustment made for hedging against an unfavorable shift in the state variables. Thus, if the state variables do not vary over time, it turns out that the results of the ICAPM become identical to the standard CAPM. In general, however, the state variables will change over time.

Even the ICAPM has some nice properties, yet has some drawbacks, which limit its applicability. The most pronounced one is its difficulty to be solved analytically. The solutions are available only under mild assumptions, or solved approximately. The approach of CCV which will be used in this study is an attempt to solve the intertemporal asset allocation problem approximately.

4

Asset Return Modeling

4.1 Introduction

The question of whether stock returns are predictable or not have been an open debate among financial economists, because predictability or its lack has important implications for asset prices and as consequence portfolio allocation. The evolvement of the efficient market hypothesis (EMH) in the 1960's from the random walk theory of asset prices advanced by Samuelson (1965), shows that in an informationally efficient market price changes must be unforecastable. This theory had been supported by some statistical evidence on the random nature of equity price changes.(see e.g., Kendall (1953), and Osborne (1959)). By the early 1970s a consensus had emerged among financial economists suggesting that stock prices could be well approximated by a random walk model and that changes in stock returns were basically unpredictable.

The EMH theoretical validity did not stand long against the growing skeptical empirical literature especially after LeRoy and Porter (1981) and Shiller (1981) studies. The evidence on predictability is voluminous to cite few (see e.g., Campbell and Shiller (1988a), Fama and French (1988), Hodrick (1992), Stambaugh (1999), Campbell and Thompson (2008), and Cochrane (2008, 2011)). However, on statistical ground, Ang and Bekaert (2007), Boudoukh, Richardson, and Whitelaw (2008), and Goyal and Welch (2008) question the magnitude of return predictability in the data and argue that returns do not have significant predictability, especially when statistical measures of out-of-sample tests are employed.

Since our thesis deals mainly with international hedging demands and diversification benefits, it would be appropriate to examine the predictability of equity returns on the international stock markets. The study of Bekaert and Hodrick (1992) has detected in-sample predictability evidence in international equity and foreign exchange markets using VAR methodology for a variety of countries over the period of 1981-1989. While, the study of Rapach et al. (2005) considers both in-sample and out-of-sample tests of predictive ability of numerous macro variables. The authors employ two out-of-sample tests (The McCracken (2007) test statistic designed to test for equal predictive ability, and T. E. Clark and McCracken (2001) test statistic which is a designed to test for forecast encompassing). Rapach and Wohar (2009) investigate the intertemporal hedging demands for stocks and bonds for international investors using the methodology of CCV where the dynamic governing asset returns is described by a vector autoregressive process (VAR). In their study they include the same state variables as CCV without testing if there is an out-of-sample forecasting ability for these predictors.

In this chapter, we re-examine the predictability of stock and bond returns using the same variables as Rapach and Wohar (2009) with the aim of acquiring a better understanding of the actual nature of return predictability in international data, using Rapach et al. (2005) approach. Hence, we consider both in-sample and out of-sample tests of return predictability.

The in-sample analysis employs a predictive regression framework, with samples typically begin in 1954 and end in 2004 for three countries (Canada, the United Kingdom, and the U.S.). For our out-of-sample analysis, we reserve a period covering the bull market of the 1990s and we examine whether the predictive variables we consider contain predictive contents during this period.

The rest of the chapter is organized as follows: Section 2 presents a short review of asset returns predictability and the econometric issues related to the empirical studies; Section 3 describes the econometric methodology used in the in-sample and out-of-sample predictability tests, as well as vector autoregressive modeling approach; Section 4 describes our empirical approach and presents analysis results; Section 5 concludes.

4.2 Asset Return Predictability

4.2.1 Efficient Market Hypothesis and Return Predictability

Return modeling is a fascinated endeavor in financial economics, yet very complex and frustrating task with a long history, since we can trace back its root to the breakthrough work of Bachelier in 1900. However, and until nowadays there is still no definite consensus regarding whether asset returns are predictable or not. Thus, we may divide the literature dealing with asset return modeling into two broad categories. The first, we may denote as the *random approach* (non-predictability), whereas the second is the *predictability approach*.

Random Approach

It is well known that the first rigorous treatment of the speculative asset prices in stock markets, dated back to the remarkable Bachelier's PhD thesis "Theory of Speculation" in which he laid the mathematical foundation of Brownian motion, and deduced that " The mathematical expectation of speculator is zero." Hence, he concluded that commodity prices fluctuate randomly (Bachelier, 1964).

Although, Bachelier's contribution was way ahead of his time, his work was ignored until the rediscovery of Savage in 1955 who introduced Bachelier's ideas to financial research community (P. Bernstein, 1992).[1].

By 1965, independently Paul Samuelson (1965) and Eugene Fama (1965a, 1965b) have participated in the development of the Efficient Market Hypothesis (EMH), which would not take its final shape until Fama's seminal review (1970). The EMH asserts as Jensen (1978, p.3) states:

"A market is efficient with respect to information set Ω_t if it is impossible to make economic profits by trading on the basis of information set Ω_t.[2]"

Thus, under the EMH stock and bond returns are purely unpredictable,both at short and long horizons, since any revealed new information will reflect instantly in asset prices. For this reason, the random walks and martingales were the most suitable processes for asset return modeling, and remained the dominating models during the 60's and 70's. Moreover, the empirical studies before the inception of EMH have found a strong evidence that asset prices were random (see e.g., (Cowles, 1933; Working, 1934; Kendall, 1953; Osborne, 1959)).

Return Predictability Approach

The 1980s witnessed the raise of skepticism toward the validity of EMH, particularly after the study of LeRoy and Porter (1981) and Shiller (1981). As consequence, a vast literature started to gather evidence that numerous economic variables can predict aggregate stock returns. The most used predictor in this literature is the dividend-price ratio (e.g., (Rozeff, 1984); (J. Campbell & Shiller, 1988a); (Fama & French, 1988)). However, the best forecasting models can explain only a relative small part of asset returns, because there is a sizable unpredictable component in asset returns, in addition to the fact that competition among traders works to eliminate the forecasting ability of successful models once they

[1] Among them Paul A. Samuelson who did find later a copy of Bachelier's thesis

[2] We have changed Jensen's notation of information set from θ_t to Ω_t just to be in accordance with our use in this thesis.

are discovered (see e.g., Timmermann and Granger (2004); Timmermann (2008)). Although some brilliant attempts,ICAPM and habit formation models, to reconcile the EMH with time-varying expected returns, the predictability remains an unsettled issue due to the econometric validity of most predictive models.

Predictive Variables In what follows we limit the treatment to the most popular variables used to predict aggregate stock returns[3], since bond returns share so far the same predictors. One of the most recognized predictor is the dividend-price ratio which can help predicting bond returns (Fama & French, 1989). For the sake of simplicity we can split these predictors into two main categories: valuation ratios and macroeconomic variables.

a-Valuation ratios variables

As we have mentioned earlier, it is well-known that dividend-price ratio is the most used predictor, especially after the influential paper of J. Campbell and Shiller (1988a) who presented the evidence that dividend-yield could be used to predict stock returns. In addition to the already cited papers there are still many published papers which used dividend-price ratio as a predictor, among them ((Cochrane, 2008); (P'astor & Stambaugh, 2009)). After the relative predictability success of dividend-price ratio the literature has known a growing number of valuation ratios such as the earnings-price ratio ((J. Campbell & Shiller, 1988b), (J. Campbell & Shiller, 1998)), book-to-market ratio ((Kothari & Shanken, 1997); (Lewellen, 1999)), and dividend payout ratio (Lamont, 1998).

b- Macroeconomic variables

Theoretically, asset returns are functions of the state variables of the real economy. Hence, there is a tight relationship between macroeconomic situation and stock prices, particularly, when the real economy displays significant business-cycle fluctuations (Cochrane, 2011). Most often stock prices are considered a leading indicator of economic condition.

[3]Most predictability literature is based on U.S. stock markets

As consequence, the predictability research has extended to include macroeconomic variables such as, nominal interest rates Ang and Bekaert (2007), inflation ((Nelson, 1976); (J. Campbell & Vuolteenaho, 2004)), interest rate spreads ((J. Campbell, 1987);(Fama & French, 1989)), labor income ((Santos & Veronesi, 2006)), aggregate output (Rangvid, 2006), and expected business conditions (S. Campbell & Diebold, 2009).[4]

4.2.2 Econometric Issues

Studies on return predictability typically employ predictive regression method which is an ordinary least squares (OLS) regression of returns on lagged predictors.

$$r_{t+1} = \alpha + \beta x_t + \varepsilon_{t+1} \tag{4.1}$$

Where r_{t+1} is the excess stock return from the end of period t to the end of period $t+1$, x_t is a predictive variable at the end of time t used to predict the excess return (such as the dividend-price ratio), and ε_{t+1} is a zero-mean disturbance term.

The predictive regression approach to return forecasting didn't go beyond critics. One of the well-known biases in the context of predictive regressions which complicate in-sample tests of return predictability is Stambaugh bias (Keim & Stambaugh, 1986; Stambaugh, 1999). This bias arises when the predictor is both highly persistent and correlated with return disturbance. Potentially and more importantly, testing the null hypothesis of no predictability creates additional size distortions when using the conventional t-statistic approach.

In addition to Stambaugh bias, the use of predictive regressions implicitly assumes that variables have homogenous effects across the return distribution, which is an over-stringent assumption, and often produces a misleading and insufficient picture of variable effects on returns (Zhu, 2013).

[4] For a thorough review of predictive variables, see Rapach and Zhou (2013)

Although the advances [5] to produce a robust in-sample inference, but out-of-sample validation was a real challenge for most return predictability models (see e.g. Brennan and Xia (2005); Butler, Grullon, and Weston (2005); Goyal and Welch (2008)). The influential study of Goyal and Welch (2008) shows that out-of-sample equity premium forecasts based on the bivariate predictive regression fail to outperform the simple historical average. However, Cochrane (2008) argues that this is not an evidence against the predictability, but evidence of the difficulty to exploit predictability with trading strategies. As result, of the econometric shortcomings of the predictive regressions which prove of being influential and in favor of EMH; the orthodox advocates of EMH gain some ground (Malkiel, 2011). Fortunately, some studies assert that the unreliability of return predictability, however, can result from the econometric methods themselves; as pointed out by Lamoureux and Zhou (1996).

In attempt to extend the predictability inquiries from the mean to the full distribution, the study of Pedersen (2010) and Zhu (2013) employ a quantile regression framework and find significant out-of-sample predictability of the entire stock return distribution. In the same line of improving the out-of-sample return predictability, but in different direction Pesaran and Timmermann (1995) have demonstrated the importance and relevance of model uncertainty and parameter instability for stock return forecasting. In such approach the forecaster knows neither the “best” model specification nor its corresponding parameter. More recently, return forecasting literature has seen studies which provide strategies that produce significant statistical and economical out-of-sample gains. In what follows, we choose to cite the strategies as Rapach and Zhou (2013) presented them.

[5] For example, many studies have addressed the problem of predictive regressions with overlapping returns and develop econometric procedures for making more reliable inferences include (Hodrick, 1992),(Goetzmann & Jorion, 1993),(Valkanov, 2003), (Boudoukh, Richardson, & Whitelaw, 2008), and (Hjalmarrson, 2011)

- economically motivated model restrictions (e.g., (J. Campbell & Thompson, 2008); (Ferreira & Santa-Clara, 2011));
- forecast combination (e.g., (Rapach, Strauss, & Zhou, 2010));
- diffusion indices (e.g., (Ludvigson & Ng, 2007); (Kelly & Pruitt, 2015) ; (Neely, Rapach, Tu, & Zhou, 2014));
- regime shifts (e.g., (Guidolin & Timmermann, 2007); (Henkel, Martin, & Nadari, 2011) ; (Dangl & Halling, 2012)).

4.3 Asset Return Modeling and Testing

4.3.1 Bivariate Testing

Setup

Following much of the extant literature [6], we analyze asset return predictability using a predictive regression framework. The predictive regression model takes the form,

$$y_{t+1}^{k} = \alpha + \beta . z_t + \gamma . y_t + u_{t+1}^{k}, \tag{4.2}$$

where y_t is the real return to hold an asset from period $t-1$ to period t, $y_{t+1}^{k} = y_{t+1} + ... + y_{t+k}$ is the real return to hold an asset from period t to $t+k$, z is a variable which may potentially predict future real returns, and u_{t+1}^{k} is a disturbance term.

Under the null hypothesis, i.e. ($\beta = 0$), the variable z_t has no predictive power for future returns, whereas under the alternative hypothesis, z_t does have predictive power for future returns ($\beta \neq 0$) [7]. Note that we include a lagged return term in Eq. (4.2) as a control variable when testing the predictive ability of z_t , as in, e.g., Lettau and Ludvigson (2001).

[6] This subsection is based extensively on the article of Rapach et al. (2005)

[7] Inoue and Kilian (2005) recommend using a one-sided alternative hypothesis if theory makes strong predictions about the sign of β in Eq.(4.2), as this increases the power of in-sample tests.

Suppose we have observations for y_t, and z_t, for $t = 1, \ldots, T$. This leaves us with $T-k$ usable observations with which to estimate the in-sample predictive regression model. The predictive ability of z_t is typically assessed by examining the t-statistic corresponding to $\hat{\beta}$, the OLS estimate of β in Eq.(4.2), as well as the goodness-of-fit measure R^2.

Out-of-sample Scheme

Evaluating models' ability to forecast is one approach to determine its usefulness. Recently, out-of-sample prediction ability plays an important role in determining the appropriateness of a model (see e.g. Lettau and Ludvigson (2001); Rapach et al. (2005)). In our bivariate tests of return predictability we follow much of Rapach et al. (2005) out-of-sample procedure.

Our out-of-sample tests are based on the following recursive scheme. First, we divide the total sample of T observations into in-sample and out-of-sample portions, where the in-sample portion spans the first H observations for y_t and z_t, and the out-of-sample portion spans the last P observations for the two variables.

The first out-of-sample forecast for the "unrestricted" predictive regression model, Eq.(4.2), is generated in the following manner. Estimate the unrestricted predictive regression model via OLS using data available through period H; denote the OLS estimates of α, β and γ in Eq.(4.2) using data available through period H as $\hat{\beta}_{1,H}, \hat{\alpha}_{1,H}$, and $\hat{\gamma}_{1,H}$.

Using the OLS parameter estimates from (4.2) z_H and y_H, construct a forecast for y^k_{H+1} based on the unrestricted predictive regression model using

$$\hat{y}^k_{1,H+1} = \hat{\alpha}_{1,H} + \hat{\beta}_{1,H} \cdot z_H + \hat{\gamma}_{1,H} \cdot y_R \tag{4.3}$$

Denote the forecast error $\hat{u}^k_{1,H+1} = y^k_{H+1} - \hat{y}^k_{1,H+1}$. The initial forecast for the "restricted" predictive model is generated in a similar manner, except we set $\beta = 0$ in Eq. (4.2) . That is, we estimate the restricted regression model, Eq. (4.2) with $\beta = 0$, via OLS using data available through period H in order to form the forecast

$$\hat{y}^{k}_{0,H+1} = \hat{\alpha}_{0,H} + \hat{\gamma}_{0,H} \cdot y_H, \tag{4.4}$$

where $\hat{\alpha}_{0,H}$ and $\hat{\gamma}_{0,H}$ are the OLS estimates of α and β in Eq.(4.2) with $\beta = 0$ using data available through period H.

Denote the forecast error corresponding to the restricted model as

$$\hat{u}^{k}_{0,H+1} = y^{k}_{H+1} - \hat{y}^{k}_{0,H+1} \tag{4.5}$$

In order to generate a second set of forecasts, we update the above procedure one period by using data available through period $H+1$. That is, we estimate the unrestricted and restricted predictive regression models using data available through period $H+1$, and we use these parameter estimates and the observations for z_{H+1} and y_{H+1} in order to form unrestricted and restricted model forecasts for y^{k}_{H+2} and their respective forecast errors, $\hat{u}^{k}_{1,H+2}$ and $\hat{u}^{k}_{0,H+2}$. We repeat this process through the end of the available sample, leaving us with two sets of $T-H-k+1$ recursive forecast errors, one each for the unrestricted and restricted regression models ($\{\hat{u}^{k}_{1,t+1}\}_{t=H}^{T-k}$ and $\{\hat{u}^{k}_{0,t+1}\}_{t=H}^{T-k}$).

The next step is to compare the out-of-sample forecasts from the unrestricted and restricted predictive regression models. If the unrestricted model forecasts are superior to the restricted model forecasts, then the variable z_t improves the out-of-sample forecasts of y^{k}_{t+1} relative to the first-order autoregressive (AR) benchmark model which excludes z_t.

In order to compare the predictive ability of two nested regression models, as in our case, a metric is needed to rank and compare forecasts. The Theil's U ratio of the unrestricted forecast root-mean-squared error (RMSE) to the restricted model forecast RMSE. If Theil's U is less than one, that means the unrestricted model forecasts are superior to the restricted model. To test the statistical significance of the superiority of one model over the other we use the McCracken (2007) MSE-F and (T. E. Clark & McCracken, 2001) ENC-NEW statistic.

MSE-F test of equal forecast accuracy

The MSE-F statistic is a variant of the (Diebold & Mariano, 1995) and (West, 1996) statistic designed to test for equal predictive ability. The test used to evaluate the null hypothesis that the unrestricted model forecast mean-squared error (MSE) is equal to the restricted model forecast MSE against the one-sided (upper-tail) alternative hypothesis that the unrestricted model forecast MSE is less than the restricted model forecast MSE. The MSE-F statistic is based on the loss differential,

$$\hat{d}_{t+1}^{k} = (\hat{u}_{0,t+1}^{k})^2 - (\hat{u}_{1,t+1}^{k})^2$$

Letting

$$\bar{d} = (T - H - k + 1)^{-1} \sum_{t=H}^{T-k} \hat{d}_{t+1}^{k} = M\hat{S}E_0 - M\hat{S}E_1,$$

where

$$M\hat{S}E_i = (T - H - k + 1)^{-1} \sum_{t=H}^{T-k} (\hat{u}_{i,t+1}^{k})^2,\ i = 0, 1$$

The McCracken (2007) MSE-F is given by

$$MSE - F = (T - H - k + 1).\bar{d}/M\hat{S}E_1 \tag{4.6}$$

A significant MSE-F statistic indicates that the unrestricted model forecasts are statistically superior to those of the restricted model.

Encompassing test ENC-NEW

Most of times, there are several competing forecasts which come from different models. The concept of forecast combination is based on blending together all the competing forecasts into a single forecast, usually through a linear combination (weighted average)(Harvey, Leybourne, & Newbold, 1998). Forecast encompassing is based on optimally constructed composite forecasts (Rapach et al., 2005). Fang (2003, p.87) states that:

"Forecast encompassing tests are used to determine whether one of a pair of forecasts contains all the useful information for prediction. If this is not the case and rather both models contain some information, there is potential to form a combined forecast that blends the useful information of the two (or more) forecasts."
The ENC-NEW test statistic is a variant of Harvey et al. (1998) statistic designed to test for forecast encompassing.

In our setup, if forecasts from the restricted regression model embody the unrestricted model forecasts, this implies that the included variable in the unrestricted model has no additional information for predicting returns relative to the restricted model which excludes the variable; on the other hand, if the restricted model forecasts do not embody the unrestricted model forecasts, then the variable does contain useful information for predicting returns beyond the information already contained in a model that excludes the variable. The T. E. Clark and McCracken (2001) ENC-NEW statistic takes the form,

$$ENC-NEW = (T-H-k+1).\bar{c}/M\hat{S}E_1 \tag{4.7}$$

where $\hat{c}^k_{t+1} = \hat{u}^k_{0,t+1}(\hat{u}^k_{0,t+1} - \hat{u}^k_{1,t+1})$ and $\bar{c} = (T-H-k+1)^{-1}\sum_{t=H}^{T-k}\hat{c}^k_{t+1}$; Under the null hypothesis, the weight attached to the unrestricted model forecast in the optimal composite forecast is zero, and the restricted model forecasts encompass the unrestricted model forecasts. Under the one-sided (upper-tail) alternative hypothesis, the weight attached to the unrestricted model forecast in the optimal composite forecast is greater than zero, so that the restricted model forecasts do not encompass the unrestricted model forecasts.

Bootstrap Algorithm

T. Clark and McCracken (2005) recommend to base inference on bootstrap procedure when comparing the forecast power from nested models with $k > 1$ (as the case here). For this reason, carry out our test inferences on a bootstrap procedure similar to that used by Kilian (1999), and T. Clark and McCracken (2005).

Under the null hypothesis of no return predictability the bootstrap data-generating process is obtained by fitting the restricted regression model

$$y_t = a_0 + a_1.y_{t-1} + \varepsilon_{1,t} \tag{4.8}$$

$$z_t = b_0 + b_1.z_{t-1} + ... + b_p.z_{t-p} + \varepsilon_{2,t} \tag{4.9}$$

where the disturbance vector $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$ is independently and identically distributed with covariance matrix Σ. We first estimate Eqs.(4.8) and (4.9) via OLS, with the lag order(p) in Eq.(4.9) selected using the Akaike Information Criterion (AIC)[8], and compute the OLS residuals $\{\hat{\varepsilon}_t = (\hat{\varepsilon}_{1,t}, \hat{\varepsilon}_{2,t})'\}_{t=1}^{T-p}$ In order to generate a series of disturbances for our pseudo-sample, we draw with replacement $T+100$ times from the OLS residuals $\{\hat{\varepsilon}_t\}_{t=1}^{T-p}$ giving us a pseudo-series of disturbance terms $\{\hat{\varepsilon}_t^*\}_{t=1}^{T+100}$ Note that we draw from the OLS residuals in tandem, thus preserving the contemporaneous correlation between the disturbances in the original sample. Denote the OLS estimates of a_0 and a_1 in Eq. (4.8) by $\hat{a}_0$ and $\hat{a}_1$, and the OLS estimate of $(b_0, b_1, ..., b_p)$ in Eq.(4.9) by $(\hat{b}_0, \hat{b}_1, ..., \hat{b}_p)$. Using $\{\hat{\varepsilon}_t^*\}_{t=1}^{T+100}$, $(\hat{a}_0, \hat{a}_1, \hat{b}_0, \hat{b}_1, ..., \hat{b}_p)$ in Eqs. (4.8) and (4.9), and setting the initial observations for y_{t-1} and $z_{t-1}, ..., z_{t-p}$ equal to zero in Eqs. (4.9) and (4.9), we can build up a pseudo-sample of $T+100$ observations for y_t and z_t, $\{y_t^*, z_t^*\}_{t=1}^{T+100}$. We drop the first 100 transient startup observations in order to randomize the initial y_{t-1} and $(z_{t-1}, ..., z_{t-p})$ observations, leaving us with pseudo-sample of T observations, matching the original sample. For the pseudo-sample, we calculate the t-statistic corresponding to b in the in-sample predictive regression model given in Eq. (4.2), and the two out-of-sample statistics given in Eqs. (2) and (3). We repeat this process 1000 times, giving us an empirical distribution for the in-sample t-statistic and each of the out-of-sample statistics.

[8] We set a maximum lag order of 12

For each statistic, the p-value is the proportion of the bootstrapped statistics that are greater than the statistic computed using the original sample. As both of the out-of-sample tests are one-sided (upper-tail), an out-of-sample statistic is significant at, say, the 10% level if the p-value is less than or equal to 0.10. As the in-sample t-test is two-sided, the in-sample t-statistic is significant at the 10% level if the p-value is less than or equal to 0.05 or greater than or equal to 0.95 .

4.3.2 Vector Autoregressive Model

Model Presentation

Vector Autoregressive models (VAR) are a natural generalization of the univariate autoregressive models, and suitable tool for forecasting. Moreover, as economic or financial theory are rarely able to identify which variables are endogenous or exogenous the use of a VAR model circumvents this identification issue as all variables treated endogenously. Their setup is that a system of linear equations, where each equation expresses the evolution of a variable as a linear function of the previous lags of every variable in the system (Luetkepohl, 2011). The VAR model can be represented on matrix form as follows

$$\mathbf{y}_t = \Phi_0 + \Phi_1 \mathbf{y}_{t-1} + \Phi_2 \mathbf{y}_{t-2} + ... + \Phi_p \mathbf{y}_{t-p} + \mathbf{v}_t \tag{4.10}$$

where $\mathbf{y}_t$ is an m-dimensional vector of realizations at time t, Φ_0 is an m-dimensional vector of intercepts, Φ_p is a $m \times m$ matrix of slope coefficients at lag p, and $\mathbf{v}_t$ is an $m \times 1$ vector of white noise process with the additional assumption that $\mathbf{v}_t$ is i.i.d $N(0, \Sigma)$. Theoretically, the structure of VAR models can include numerous lags making them very flexible, however, increasing the lag number results in augmenting parameters' number to be estimated which quickly becomes an issue as the degrees of freedom will be used up.

VAR models can be estimated with standard methods, such as ordinary least squares (OLS) and maximum likelihood (ML) methods, or Bayesian estimation. The parameters can be estimated efficiently by OLS for each equation separately. For a normally distributed process the OLS estimates are asymptotically equivalent to the corresponding ML estimates (Luetkepohl,2011).

Companion VAR Form

Any stationary VAR(P)system can be rewritten as a VAR(1). Suppose $\mathbf{y}_t$ follows a VAR(p) process like equation (4.10), by subtracting the mean and stacking p of $\mathbf{y}_t$ into a large column vector denoted $\mathbf{z}_t$, a VAR(p) can be transformed into a VAR(1) by constructing the companion form (see Canova (2007, p.119-121)). The companion form is given by

$$\mathbf{z}_t = \Upsilon_t \mathbf{z}_{t-1} + \xi_t \tag{4.11}$$

where

$$\mathbf{z}_t = \begin{bmatrix} \mathbf{y}_t - \mu \\ \mathbf{y}_{t-1} - \mu \\ \vdots \\ \mathbf{y}_{t-p+1} - \mu \end{bmatrix}$$

$$\Upsilon_t = \begin{bmatrix} \Phi_1 & \Phi_2 & \dots & \Phi_{p-1} & \Phi_p \\ \mathbf{I}_k & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_k & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{I}_k & \mathbf{0} \end{bmatrix}$$

and

$$\xi_t = \begin{bmatrix} \varepsilon_t \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}$$

The companion form allows the statistical properties of any VAR(p) to be directly computed using only the results of a VAR(1) noting that

$$E[\xi_t \xi_t^{'}] = \begin{bmatrix} \Sigma & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix}$$

Stationarity Test

In order to verify the validity of any VAR it is required to address the stationarity issue. As such, the variables contained in the VAR system should be stationary as autocorrelated errors could lead to inefficient coefficient estimation, and an invalid inference which lead to poor forecasting power of the specified model (Granger & Newbold, 1974). To test whether a system is stationary in its weakest form, it is required that the mean and variance are constant over time. Expressed mathematically

$$E(\mathbf{z}_t) = \mu \quad \forall t \tag{4.12}$$

$$E((\mathbf{z}_t - \mu)(\mathbf{z}_{t-h} - \mu)^{'}) = \Sigma_z(h) \quad \forall t,\, h = 0, 1, 2, ... \tag{4.13}$$

where the second condition enforces that the auto-covariance depends only on the time

period between the two vectors, h, and not the specific point in time t (Lutkepohl, 2005,

p. 24). Hence, for the stationarity assumption to hold the time series cannot exhibit trends or shifts in means as well as auto-covariance. It is quite common to meet non-stationary financial and macroeconomic time series. If a time series contains a unit root, it is possible to first-difference the data and thereby remedy the problem of non-stationarity. If this first-differenced time series is stationary it is said to be integrated of the order one, $I(1)$. Though the data is now stationary the slope coefficients resulting from future estimations will now no longer be in levels and thus interpretation becomes more difficult. For this reason some econometricians claim that classical methods can obtain consistent estimates of VAR coefficients even with existence of unite roots, see e.g. (Stock, Sims, & Watson, 1990).

There are several tests and methods that can be employed to check if a time series has a unit root, i.e. non-stationary time series against a stationary time series, see e.g. Stock (1994). In what follows we focus on the augmented Dickey-Fuller test (Dickey & Fuller, 1979).

Augmented Dickey-Fuller test Testing for a unit root with the Augmented Dickey-Fuller test is done using the following regression

$$\triangle z_{i,t} = \Psi z_{i,t-1} + u_t \tag{4.14}$$

where i denotes the variable. Under the null hypothesis z_t contains a unit root, meaning that $\Psi = 0$ in the above equation. This is tested against the alternative one-sided hypothesis that $\Psi < 0$ and thus stationary of the time series. The resulting test statistic $\frac{\hat{\Psi}}{\widehat{SE(\hat{\Psi})}} i$ is then evaluated against the Dickey-Fuller critical values rather than the usual t-values as the null hypothesis is of non-stationarity (see (Hamilton, 1994)). Each variable will be tested for stationarity and if we fail to reject the H_0 that means we need to apply first-difference and teste it again. If we are now able to reject the null hypothesis for the first-difference we deduce that the variable is integrated of order one, or difference-stationary.

Note that many of the state variables applied in asset allocation literature are known from economic theory to be non-explosive we will rely on the augmented Dickey-Fuller results when specifying our model.

4.4 Empirical Methodology

4.4.1 VAR Setup for Asset Returns

A vector autoregressive (VAR) dynamics is considered for U.S. asset returns in e.g. Barberis (2000), and Campbell and Viceira (1999). Moreover, Bekaert and Hodrick (1992), Rapach et al. (2005) and Rapach and Wohar (2009) used VAR to model return dynamics in international setting.

We follow CCV VAR model when investing domestically and a two-country VARs extension of (Rapach & Wohar, 2009) when investing internationally. Thus we assume that the dynamics of state variables are well characterized by a VAR(1). The use of a VAR (1) specification is in principle not restrictive, since any vector autoregression can be written as a VAR (1) as discussed in subsubsection (4.3.2). The manager has access to N risky assets. Let $R_{1,t}$ denote the ex post real short rate at time t and $r_{1,t} = \log(R_{1,t})$ the log (or continuously compounded) real return on this asset that is used as a benchmark to compute excess returns. The log excess return vector is defined as:

$$\mathbf{x}_{t+1} \equiv \begin{bmatrix} r_{2,t+1} - r_{1,t+1} \\ r_{3,t+1} - r_{1,t+1} \\ \vdots \\ r_{n,t+1} - r_{1,t+1} \end{bmatrix} \tag{4.15}$$

In addition to the N-log excess returns of the risky assets, and the log return on the benchmark asset a vector of instruments $\mathbf{s_{t+1}}$ which includes a selection of predictor variables. The m-vector of state variables is given by

$$\mathbf{z}_{t+1} = \begin{bmatrix} r_{1,t+1} \\ \mathbf{x_{t+1}} \\ \mathbf{s_{t+1}} \end{bmatrix} \tag{4.16}$$

Since we retain the same VAR(1) system as CCV, the data-generating process for the state vector $\mathbf{z_{t+1}}$ is given by

$$\mathbf{z_{t+1}} = \Phi_0 + \Phi_1 \mathbf{z_t} + \mathbf{v_t}, \tag{4.17}$$

where Φ_0 is the vector of intercepts, Φ_1 is the matrix of slope coefficients and $\mathbf{z_t}$ are the innovations in the model assumed to be *iid* normal distributed:

$$\mathbf{v}_{t+1} \sim N(0, \Sigma_v) \tag{4.18}$$

where

$$\Sigma_v = \begin{bmatrix} \sigma_1^2 & \sigma_{1\mathbf{x}}^{'} & \sigma_{1\mathbf{s}}^{'} \\ \sigma_{1\mathbf{x}} & \Sigma_{xx} & \Sigma_{xs}^{'} \\ \sigma_{1\mathbf{s}} & \Sigma_{xs} & \Sigma_{xs} \end{bmatrix} \tag{4.19}$$

where σ_1^2 is the benchmark asset return variance, $\sigma_{1\mathbf{x}}$ is the vector of covariances between innovations to the benchmark asset return and innovations to the excess returns on the remaining assets, $\sigma_{1\mathbf{s}}$ is the vector of covariances between innovations to the benchmark asset return and innovations to the instruments, Σ_{xx} is the variancecovariance matrix for the innovations to the excess returns, Σ_{xs} is the matrix of covariances between innovations to the excess returns and innovations to the instruments, Σ_{ss} is the variancecovariance matrix for the innovations to the instruments.

The innovations can be cross-sectionally correlated, but are assumed homoskedastic and independently distributed over time. The assumption of homoskedastic variance is restrictive, because it rules out the possibility that the state variables predict changes in the risk of the assets the state variables can "only" affect portfolio choice through the ability to predict changes in expected return. Prior Research (Chacko & Viceira, 1999) explores the role of changing risk for asset allocation using a continuous-time version of Campbell and Viceira (1999) framework and concludes that changes in the risk of equities do not have large effects on intertemporal hedging demand. This result is not sufficient to ignore the role of changing risk for the overall portfolio choice for long-term investors, but other results show the role of predictability in assets returns plays a more important role for asset allocation than changing risk ((Aït-Sahalia & Brandt, 2001)). On this background, the homoskedasticity assumption is presumed not to be too restrictive, to obtain useable results for the purpose of this study.

It is quite feasible to use the CCV approach to solve a multi-period portfolio choice problem with five risky assets and six instruments. This allows us to extend the empirical application in CCV and analyze a multi-period portfolio choice problem for a SWF in the U.S. which, in addition to domestic bills, stocks, and bonds, has access to stocks and bonds from a foreign country (Canada, or the U.K.), and where the manager considers six instruments (the domestic and foreign nominal bill yields, dividend yields, and term spreads) that potentially contribute to return predictability (see (Rapach & Wohar, 2009)). The same exercise is used but instead of the U.S. perspective we consider a SWF in Canada which has access to U.S. or U.K. stock markets. Thus, the return dynamics are characterized by a VAR(1) process that includes the five returns and six instruments.

4.4.2 Data Description

This subsection will provide an overview and brief description of the data set used in the following empirical implementation. Our data set consists of monthly data for three countries: the United States, Canada, and the United Kingdom. The full sample begins in 1954:6 and ends in 2004:5 for all countries, thus giving 600 observations. Whereas the out-sample data used to test asset returns predictability consist of (197) observations. The data used in this study is mainly provided by David Rapach public available dataset,[9] though we still explain the transformations made to them. In addition to Rapach's data set we used inflation rates for Canada and the United Kingdom, as well as exchange rates from British pound to Canadian dollars which are from two websites.[10]

In line with J. Campbell et al. (2003); Rapach and Wohar (2009); Engsted and Pedersen (2012) we have three asset return variables, namely the 3-month Treasury bills, stock and bond returns, in addition to three instrumental variables in order to check for asset predictability. The returns used are total return indexes (includes dividends) and are continuously compounded returns.

In what follows, a description is given to the variables we consider in our study along with the changes that have been made to the raw data:

- The log real return on a 3-month Treasury bill is calculated as:

$$rtbr = \log(1 + R_{tb}) - \log(1 + cpi)$$

  which is the difference in the logs of the total return index for bills for the given and previous months minus the difference in the logs of the consumer price index for the given and previous months,(rtbr);

[9] The data source is from Global Financial Data, which can be downloaded from David Rapach website. http://sites.slu.edu/rapachde/home/research

[10] www.statbureau.org. Form which inflation rates retrieved. Whereas, exchange rates from, https://www.quandl.com/data/FRED/EXUSUK-U-S-U-K-Foreign-Exchange-Rate

- The log excess stock (bond) return is calculated as:

$$xsr = \log(1 + R_{stock}) - rtbr$$

$$xbr = \log(1 + R_{bond}) - rtbr$$

  which is the difference in the logs of the total return index for stocks (10-year government bonds) for the given and previous months minus the difference in the logs of the total return index for bills for the given and previous months,(xsr and xbr, respectively);

- The nominal bill yield is the deviation in the nominal 3-month Treasury bill yield from 12-month backward-looking moving average,(bills);

- The term spread, is the difference between the 10-year government bond yield and the 3-month Treasury bill,(spread);

- The Log dividend yield (dy).

Descriptive statistics

Table 4.1: Summary statistics, 1954:06-2004:05.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Country	Variables	rtbr	xbr	xsr	bill	spread	dy
Canada							
	Mean	2.37	0.99	3.12	0.006	1.27	1.13
	Std. Dev.	1.40	7.53	15.81	1.36	1.5	0.35
	ρ_1	0.21	0.10	0.09	0.92	0.95	0.99
	Jarque-Bera	63.28	1187.01	382.44	52.79	110.65	87.35
U.K.							
	Mean	1.71	0.83	4.63	0.023	0.81	1.49
	Std. Dev.	2.04	4.93	18.78	1.38	2.11	0.28
	ρ_1	0.30	0.32	0.11	0.90	0.97	0.98
	Jarque-Bera	1046.80	339.67	1815.54	25.76	46.63	17.75
U.S.							
	Mean	1.37	1.01	5.4	-0.002	1.40	1.14
	Std. Dev.	0.99	5.76	14.77	1.05	1.18	0.38
	ρ_1	0.38	0.14	0.03	0.88	0.94	0.99
	Jarque-Bera	20.74	851.31	165.39	89.99	5.23	93.48

Notes: rtbr = log real 3-month Treasury bill return; xsr=log excess stock return; xbr= log excess bond return; bill= nominal 3-month Treasury bill yield; dy =log dividend yield; spreadt=10-year government bond yield - 3-month Treasury bill yield. ρ_1 = first order autocorrelation.

Table 4.1 reports summary statistics for the three risky asset returns and three instruments for each of the three countries. The mean and standard deviation for the returns are presented in annualized percentage. The mean excess stock return for the U.S., and U.K. are close with 5.4% and 4.63%, respectively. While Canada exhibits the lower mean excess stock returns with 3.12%. The mean excess bond returns range from 0.83% to 1.01%. The mean excess bond return is lower than the mean excess stock return for each country. The standard deviation of excess stock returns is approximately 24 times larger than the standard deviation of excess bond returns. As a matter of fact, the standard deviation of the real bill return is significantly lower than of excess bond return for each country.

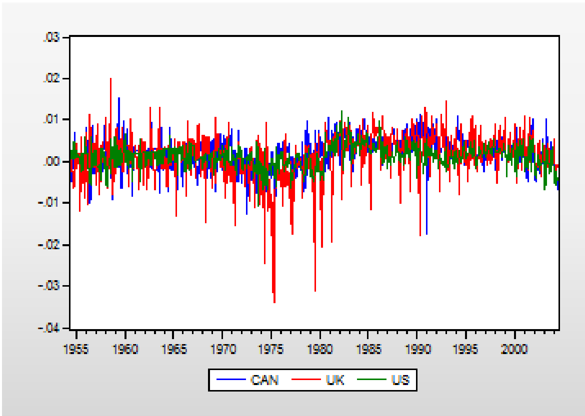
Given the results of normality test (Jarque-Bera) It is worth noting that the normality assumption is violated for all variables. Even that, we follow much of the conventional practice of neglecting the normality issue if samples are large, which is our case. Moreover, it is not a necessary condition to have normal distribution in order to proceed in our setup.

Since the state variables are well-defined it would be appropriate to present the historical development of these time series over the full sample (1954-2004). First, we present time series graphically to get a preliminary overview of the main features and the behavior of the first two moments and the series' stationarity.

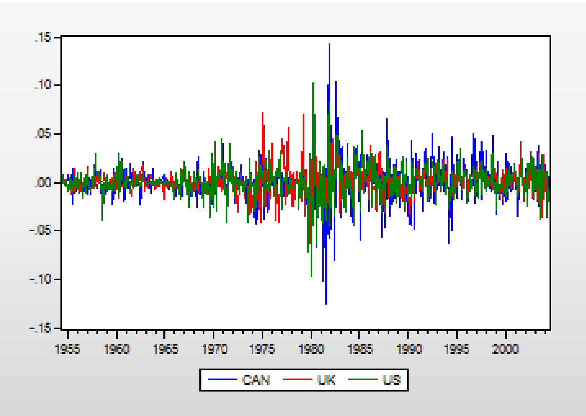
Figure 4.1 shows the main historical market movements; such as the 1973 oil crisis, and the bursting of the tech bubble in the early 2000s. For the Log real returns on Treasury bills we observe almost the same co-movement between the three country. However, the U.K. series seems more volatile. The long-term bond markets demonstrate much more stable development with the one major deflection being the inflation-driven movements around 1980. Excess stock returns show remarkable stability for most sample periods, except for major international crises. Generally, excess returns seem to be stable over our full sample, which is less obvious to the instrumental variables, especially with the dividend yield variable.

Stationarity:

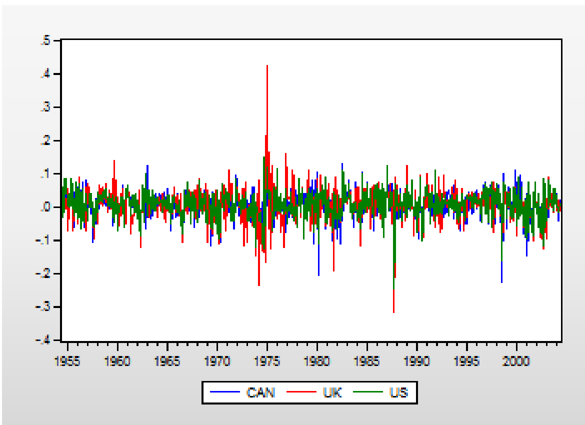
As previously stated in section (4.3) stationarity of applied time series is important in our study as we are going to use VAR model. To test whether the time series have a unit root, or not, we employ the Augmented Dickey-Fuller test which under the null-hypothesis the series have a unit root. The number of lags in the ADF-test are determined by Akaikes Information Criterion. The results reported in Table 4.2 confirm our visual inspection of time series plots that all variables are stationary, but dividend yield variables in Canada and the U.S. are not.



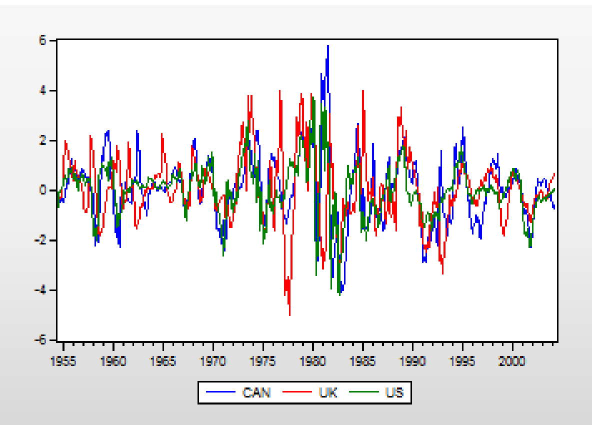
(a) log real return on Treasury bill



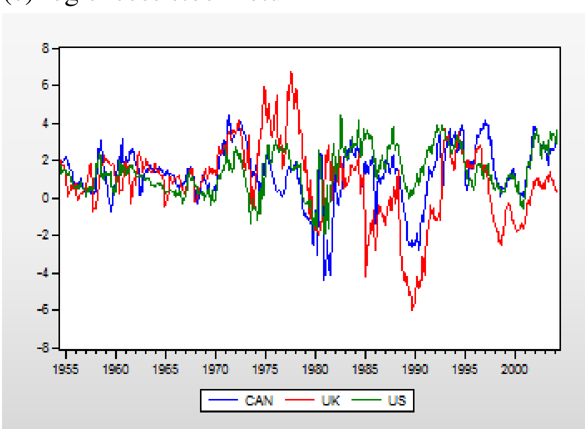
(b) log excess bond return



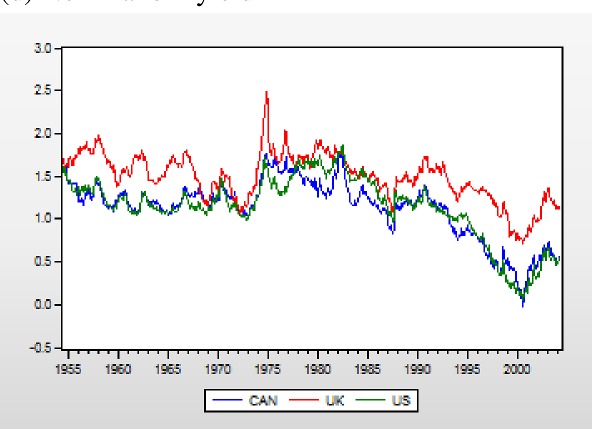
(c) log excess stock return



(d) Nominal bill yield



(e) Term spread



(f) Dividend yield

Figure 4.1: Time series plots: Asset returns and predictors

Table 4.2: Augmented Dickey-Fuller tests for unit roots in level and the first difference:

	Canada		U.K.		U.S.	
Variables	ADF	p-value	ADF	p-value	ADF	p-value
rtbr	-2.14**	(0.03)	-2.34**	(0.02)	-2.00**	(0.04)
xbr	-22.14*	(0.00)	-16.51*	(0.00)	-21.19*	(0.00)
xsr	-22.23*	(0.00)	-21.87*	(0.00)	-23.53*	(0.00)
bill	-7.79*	(0.00)	-6.91*	(0.00)	-7.19*	(0.00)
spread	-3.18*	(0.00)	-3.02*	(0.00)	-2.48**	(0.02)
dy	-1.32	(0.17)	-3.40***	(0.05)	-1.32	(0.17)
$\triangle$ dy	-22.36*	(0.00)	-	-	-24.40*	(0.00)

One-sided (lower tail) test of H0: Nonstationary vs. H1: Stationary
*,**, and *** denote significance level of 1%, 5%, and 10%, respectively.

To verify whether dividend yield time series are integrated, we difference the series and we run ADF-test again. From the panel we conclude that the dividend yield are indeed integrated of order one. Statistically, it is preferably to treat the non-stationarity through differencing the time series. However, there are instances were differencing is unnecessary as stated by Stock et al. (1990, p.136):

"This work shows that the common practice of attempting to transform models to stationary form by difference or cointegration operators whenever it appears likely that the data are integrated is in many cases unnecessary."

In addition, differencing variables would potentially lose their predictive power in comparison to their levels. Especially, we are dealing with a ratio. Another reason for not performing difference is that, most economic variables can not be explosive by construction. In our case it is well-known that both dividends and prices will not take extreme values for long time.

Based on these results we will proceed in the empirical section assuming all variables are stationary.

4.4.3 Result Analysis

Bivariate Testing Results

Tables (6.5, and 6.6) in Appendix B report in-sample regression results for the Equation (4.2), and the out-sample tests. The out-sample portion begins in 1988:01 until the end of our sample for each country.

In what follows we briefly discuss the results for each country. From Table 6.5 which reports excess stock return predictability results, we see that term spread exhibits the strongest predictive ability in both in-sample and out-sample in Canada. The in-sample t-statistic and both MSE-F and ENC-new out-sample tests are significant at each considered horizon. Nominal T-bills yield exhibits strong in-sample predictive ability as well, however, out-sample results are less impressive as only the ENC-new statistic is significant at 24-month horizon. For dividend yield variable the test statistics show no predictive ability in this sample.

Turning to the results for the U.K., there is evidence of in-sample and out-sample predictive ability for dividend yield at almost all horizons. There is some out-sample predictive ability evidence for term spread at 3, and 12 months, while in-sample results show only a little evidence at 12-month horizon. The nominal T-bills yield variable in turn shows no predictive power at all. In contrast to the U.K. the U.S. results demonstrate no evidence of predictability for the dividend yields. Whereas, both the nominal T-bill yield and term spread variables have only in-sample predictability power in most horizons, for nominal T-bill yield at 1, 3, and 12-month horizons, while term spread at 3, 12, and 24-month horizons.

The excess bond return predictability results reported in table 6.6 indicate that in Canada case the term spread exhibits the strongest in-sample and out-sample predictive ability among other variables. We see that term spread has significant in-sample predictive ability of 1, 3. While the out-sample results show that there is predictive ability at 1, 3, and 12 months horizons.

The interesting thing about nominal T-bill yield variable is the strong out-sample predictability evidence, while the in-sample predictability is limited to 1-month horizon. The studies of Clark and McCracken (2001,2005) find that there are some instances where the out-sample tests (MSE-F and ENC-new) can have more power compared to the in-sample tests. In the case of Dividend yield, the tests show no evidence of predictability.

The U.k. results show relatively less predictability power compared to Canada. The term spread is the only variable which has significant evidence in out-samples tests at 1, and 3 months horizons. As for the dividend yield, it is significant only in the 1-month in-sample test, while the nominal T-bills yield shows no predictive ability at all. For the U.S. only the term spread demonstrates in-sample and out-sample predictive ability at 1, 3, and 12 months horizons.

VAR estimation Results

Tables 4.3, 4.4, and 4.5 report the estimation results for the VAR systems in Canada, the U.K., and the U.S., respectively. The top section of each panel reports coefficient estimates (with t-statistics in parentheses) and the R^2 statistic for each equation in the system. The bottom section of each panel shows the covariance structure of the innovations in the VAR system. The entries below the main diagonal are correlation statistics, and the entries on the main diagonal are standard deviations multiplied by 100.

The first row of each panel corresponds to the real bill rate equation. In all countries lagged real bill rate has positive coefficient and significant t-statistics. While yield spread has negative and significant t-statistic only in Canada and the U.k. For the rest variables we see that dividend yield helps predict real bill rates only in Canada, whereas lagged nominal short-term interest rate is statistically significant only in the U.k. The fit of the equation in the U.K. and U.S. seem close with an R^2 of 15.9% and 15.8%, respectively. While in Canada the R^2 is 9.8%.

In fact predicting excess stock returns is well-known to be difficult, especially in short time horizons. That is the reason why in all countries the second row corresponding to the equation for the excess stock return has the lowest R^2 which ranges from 2.8% to 4.3%. In Canada no variable is significant to predict excess stock return, while in the U.K. two variables are significant, the real T-bill and dividend yield. In the U.S. two variables help predicting excess stock returns as well, the dividend yield and nominal bills. These results are similar to some extinct with Ang and Bekaert (2007) as they show that the dividend yields and short-term interest rates are robust predictors for the stock returns in the U.S., U.K.

Turning to excess bond returns the model shows significant predictability in each country compared to stocks. The results related to the U.S. and Canada bear remarkable similarity for both of them show four significant lagged variables (real bill rates, excess stock returns, excess bond returns, and yield spread) have the ability to predict future excess bond returns, with explanatory regression R^2 power of 7.1% and 8.9%, respectively. The positive yield spread coefficient implies that the expected future bond return is higher when the yield spread is high. On the other hand, the Uk differs slightly as the VAR model doesn't show significant predictability power of any instrumental variable. However, lagged excess bond and stock returns as well as real T-bill returns have significant predictive power. Since our main interest is to model asset returns, we skip the interpretation of the remaining state variables estimation results.

The bottom section panel of each VAR-system results illustrates the correlation between innovations in the VAR system, which can be interpreted as the correlation between the unexpected returns and shocks in predictor variables. In all countries the unexpected log excess stock returns exhibit high negative correlation with shocks to the log dividend yield ratio (-90.8%, -78.9%, -96.4%, for Canada, U.K., and U.S., respectively). These results are in line with previous empirical results of Rapach and Wohar (2009), and CCV. The unexpected log excess bond returns show moderate negative correlation with shocks to the nominal bill rate (Canada - 44.9%, U.K. - 46.7%, and - 65.7% in U.S.), but weak correlation with the yield spread (Canada - 16.6%, U.K. 3.7%, and 4.9% in U.S.).

Table 4.3: VAR estimation results, Canada, 1954:06-2004:05

1 dependent variable	2 rtbr-t	3 xsr-t	4 xbr-t	5 bills-t	6 dy-t	7 spread-t	8 R^2
VAR slope coefficient estimates and goodness-of-fit measures							
rtbr-t+1	0.147	-0.002	0.006	0.000	-0.001	-0.001	0.098
	(3.099)*	(-0.794)	(0.783)	(-1.417)	(-2.568)*	(-4.911)*	
xsr-t+1	-0.113	0.064	0.168	-0.001	0.010	0.003	0.028
	(-0.233)	(1.338)	(1.531)	(-0.405)	(1.407)	(1.517)	
xbr-t+1	0.829	-0.101	0.136	0.000	0.003	0.003	0.089
	(3.357)*	(-3.702)*	(1.975)*	(-0.126)	(1.357)	(2.787)*	
bills-t+1	-12.686	1.245	-6.158	0.945	-0.016	0.070	0.863
	(-2.313)*	(2.282)*	(-3.662)*	(5.812)*	(-0.383)	(3.447)*	
dy-t+1	-0.204	-0.039	-0.201	0.003	0.988	-0.001	0.982
	(-0.398)	(-0.775)	(-1.747)**	(1.493)	(129.621)*	(-0.711)	
spread-t+1	1.660	0.297	4.180	-0.052	-0.042	0.920	0.913
	(0.371)	(0.626)	(3.166)*	(-2.563)*	(-1.221)	(53.394)*	
Cross-correlations of VAR residuals							
	rtbr	xsr	xbr	bills	dy	spread	
rtbr	1.000						
xsr	0.024	1.000					
xbr	0.020	0.271	1.000				
bills	0.043	-0.202	-0.449	1.000			
dy	-0.072	-0.908	-0.309	0.217	1.000		
spread	-0.052	0.051	-0.166	-0.778	-0.049	1.000	

Notes: rtbr= log real 3-month Treasury bill return; xsr = log excess stock return; xbr= log excess bond return; bills= 3-month Treasury bill yield; dy = log dividend yield; spread= 10-year government bond yield 3-month Treasury bill yield. t-statistics are given in parentheses; Significance levels:t=1.645 (10%), t=1.960 (5%), where ** and * indicate the significance at 10% and 5%, respectively.

Table 4.4: VAR estimation results, UK, 1954:06-2004:05

1 dependent variable	2 rtbr-t	3 xsr-t	4 xbr-t	5 bills-t	6 dy-t	7 spread-t	8 R^2
VAR slope coefficient estimates and goodness-of-fit measures							
rtbr-t+1	0.210	0.000	-0.001	-0.001	-0.001	-0.001	0.159
	(3.569)*	(-0.014)	(-0.026)	(-3.277)*	(-1.469)	(-6.057)*	
xsr-t+1	0.901	0.092	0.226	-0.001	0.027	0.001	0.043
	(1.942)**	(1.612)	(1.149)	(-0.387)	(2.420)*	(0.721)	
xbr-t+1	0.282	0.042	0.271	0.000	0.004	0.000	0.140
	(2.567)*	(2.439)*	(5.748)*	(0.291)	(1.620)	(1.500)	
bills-t+1	2.752	-0.093	-7.130	0.913	-0.249	0.054	0.829
	(0.546)	(-0.203)	(-3.783)*	(42.422)*	(-3.422)*	(3.877)*	
dy-t+1	-0.385	-0.448	-0.122	0.002	0.984	0.000	0.978
	(-1.272)	(-7.437)*	(-0.898)	(1.479)	(125.385)*	(0.479)	
spread-t+1	-7.930	-1.013	1.828	-0.031	0.148	0.950	0.937
	(-1.557)	(-2.140)*	(1.130)	(-1.652)**	(2.255)*	(72.063)*	
Cross-correlations of VAR residuals							
	rtbr	xsr	xbr	bills	dy	spread	
rtbr	1.000						
xsr	-0.124	1.000					
xbr	-0.053	0.278	1.000				
bills	0.101	-0.264	-0.467	1.000			
dy	0.082	-0.789	-0.292	0.225	1.000		
spread	-0.098	0.131	0.037	-0.840	-0.085	1.000	

Notes: rtbr= log real 3-month Treasury bill return; xsr = log excess stock return; xbr= log excess bond return; bills= 3-month Treasury bill yield; dy = log dividend yield; spread= 10-year government bond yield 3-month Treasury bill yield. t-statistics are given in parentheses; Significance levels:t=1.645 (10%), t=1.960 (5%), where ** and * indicate the significance at 10% and 5%, respectively.

Table 4.5: VAR estimation results, U.S., 1954:06-2004:05

1 dependent variable	2 rtbr-t	3 xsr-t	4 xbr-t	5 bills-t	6 dy-t	7 spread-t	8 R^2
VAR slope coefficient estimates and goodness-of-fit measures							
rtbr-t+1	0.376	0.003	0.008	0.000	0.000	0.000	0.158
	(7.660)*	(1.176)	(1.087)	(-0.646)	(0.426)	(0.538)	
xsr-t+1	0.862	-0.007	0.250	-0.004	0.009	0.001	0.040
	(1.430)	(-0.131)	(2.049)*	(-1.711)**	(1.646)**	(0.713)	
xbr-t+1	0.684	-0.066	0.166	0.001	0.000	0.002	0.071
	(2.477)*	(-3.490)*	(2.583)*	(0.870)	(0.125)	(2.651)*	
bills-t+1	-10.930	1.650	-6.854	0.880	-0.017	0.047	0.792
	(-1.162)	(2.662)*	(-2.871)*	(27.116)*	(-0.280)	(2.443)*	
dy-t+1	-0.907	0.033	-0.276	0.005	0.993	-0.001	0.987
	(-1.455)	(0.600)	(-2.228)*	(2.056)*	(178.480)*	(-0.325)	
spread-t+1	-1.168	-0.356	3.217	-0.011	0.001	0.932	0.886
	(-0.153)	(-0.721)	(1.786)**	(-0.393)	(0.010)	(51.014)*	
Cross-correlations of VAR residuals							
	rtbr	xsr	xbr	bills	dy	spread	
rtbr	1.000						
xsr	0.055	1.000					
xbr	-0.012	0.131	1.000				
bills	0.060	-0.046	-0.657	1.000			
dy	-0.083	-0.964	-0.126	0.035	1.000		
spread	-0.096	-0.074	0.049	-0.747	0.085	1.000	

Notes: rtbr= log real 3-month Treasury bill return; xsr = log excess stock return; xbr= log excess bond return; bills= 3-month Treasury bill yield; dy = log dividend yield; spread= 10-year government bond yield 3-month Treasury bill yield. t-statistics are given in parentheses; Significance levels:t=1.645 (10%), t=1.960 (5%), where ** and * indicate the significance at 10% and 5%, respectively.

4.5 Conclusion

In this chapter, we examine the predictability of asset returns using variables from previous empirical research that have substantial effects on optimal portfolio. Namely, dividend yields, interest rates, and term spreads (yield spreads). Three countries are taken into consideration, Canada, the U.K. and the U.S. In the first part we consider both in-sample and out-of sample tests of predictability. For our out-of-sample analysis, we follow the same approach as Rapach et al. (2005) which is based on Clark and McCracken (2001) and McCracken (2007) tests. While, the second part we model the asset return using a vector autoregressive model.

Our out-of sample period covers the period from 1988:01 to 2004:05 a period for which it is notoriously difficult to predict asset returns. When we analyze each of the variables in turn for each country, term spread stand out in terms of predictive ability -both in-sample and out-of-sample- and to lesser extinct nominal T-bills yield. Turning to the results for the U.K., there is strong evidence of in-sample and out-sample predictive ability for dividend yield to predict stocks and less impressive evidence using term spread. In contrast, the term spread has strong evidence to predict bonds while dividend yield has limited in-sample power. The U.S. results demonstrate that only the term spread has in-sample and out-sample bond predictive ability. While both the nominal T-bill yield and term spread variables have only in-sample stock predictability power.

Given literature empirical results and our in-sample and out-sample tests findings, we kept the same variables to test them in multivariate setup. Since our VAR estimation results are in line to some extinct with prior models in asset allocation literature. We are more inclined to go through the process of asset allocation using this model specification. The results from VAR estimation - Φ_0, Φ_1, and Σ_v - will serve as input to our portfolio allocation implementation in the next chapter.

5

Dynamic Portfolio Choice

5.1 Introduction

One of the most critical decisions for long term investors in general and SWFs in particular is the selection of asset classes and their strategic weights in order to maximize utility function assuming that SWF seeks to maximize financial returns, which is the stated goal by Alberta and Alaska SWFs. In a highly concurrent financial environment institutional investors exploit and seize every single opportunity whether it is return predictability, investor profile characteristics or even econometric and optimization models. Besides the critical question about how much SWF should allocate to stocks, bonds, and T-bills, SWFs face another serious decision problem which may affect and undermine the very existence of SWF, the optimal consumption (withdrawal) must be treated equally as asset allocation.

Since the seminal work of Merton (1969, 1971) and Samuelson (1969), financial economists have realized the important implications of return predictability on dynamic portfolio choice problems. More specifically, return predictability can give rise to intertemporal hedging demands for assets. Solutions to multi-period portfolio choice problems can be either closed-form analytical solutions under very restrictive and usually unrealistic assumptions.[1]. For example, Kim and Omberg (1996), Wachter (2002), Chacko and Viceira (2005), and Liu (2007), or numerical approximate solutions typically rely on complex numerical methods.

Most applied numerical methods in dynamic portfolio choice literature can be classified into the following five broad areas, see (Garlappi & Skoulakis, 2010):

(1) dynamic stochastic programming (see e.g., Consigli and Dempster (1998)); (2) numerical solution of partial differential equations (e.g., Brennan et al. (1997)); (3) state-space discretization (e.g., Balduzzi and Lynch (1999)) ;Lynch and Balduzzi (2000), Barberis (2000), Brandt (1999)); (4) Malliavin calculus and Monte Carlo methods (e.g., Detemple and Rindisbacher (2003); (5) analytical approximations, such as log-linearization of the budget constraint (e.g., J. Campbell and Viceira (1999); J. Campbell et al. (2003) or perturbation methods (e.g., Das and Sundaram (2002)).

Due to the well-known problem of "curse of dimensionality" which may rise when the number of the state variables is large and/or when the vector of primitive shocks is high-dimensional, the numerical methods have a limited application in large-scale problems. Particularly, the curse of dimensionality issue is more pronounced in cases where methods rely on discretization of the state space in which expectations are computed via either quadrature or binomial approximations, and numerical solution of partial differential equations. Generally, methods that rely on Malliavin calculus or analytical approximations are less affected by the curse of dimensionality (Garlappi & Skoulakis, 2010).

[1] Typically requires particular assumptions about preferences, market completeness, and absence of frictions

The approach of J. Campbell and Viceira (1999) and CCV blends together a relatively simple numerical procedure and approximate analytical method. As result it has the potential of being applied with a relatively large number of assets and return predictors (Rapach & Wohar, 2009). CCV use their approach to analyze optimal dynamic asset allocation across U.S. bills, stocks, and bonds when return predictability is described by a first-order vector autoregressive (VAR) process. CCV consider an investor who maximizes the expected utility of lifetime consumption over an infinite horizon, where the utility function is of the Epstein-Zin-Weil form (Epstein and Zin (1989, 1991); Weil (1989)). The empirical results in CCV, as well as the other studies cited above, indicate that return predictability can generate important intertemporal hedging demands. However, the literature focuses almost exclusively on domestic investments in U.S. assets even optimal allocation models for oil-based SWFs consider only the case of domestic stock markets(e.g, (Scherer, 2009b), (Gintschel & Scherer, 2008)).In order to study intertemporal hedging demands in an international setting, Rapach and Wohar (2009) extended CCV approach.

In this chapter, we apply the CCV approach to strategic asset allocation to analyze international hedging demands and diversification effects on dynamic asset allocation across different asset classes and countries, where the returns dynamics are characterized by a VAR(1) process, the empirical findings of the previous chapter serve as an input to multi-period choice problem. For a set of plausible values for the parameters relating to intertemporal references, the CCV approach is applied in order to estimate the mean total, myopic, and intertemporal hedging demands for domestic bills, stocks, and bonds in Canada and U.S. We also present estimates of the intertemporal hedging demands for domestic stocks and bonds for each month over the sample in each country. In addition to examining the implied domestic intertemporal hedging demands, we consider the case of a multi-period portfolio choice problem for Canadian and American SWF which can invest in foreign stocks and bonds.

The rest of the chapter is organized as follows: Section 2 describes CCV framework because it is the backbone to our model; Section 3 presents the model with empirical results; Section 4 concludes.

5.2 CCV Framework [2]

5.2.1 Preferences and Utility Function Choice

Although the conventional power-utility model has many attractive features, yet the poor empirical performance of the representative agent optimizing models (see e.g., Hansen and Singleton (1983) and Mehra and Prescott (1985)) has raised a legitimate concern of the convenience of power-utility specification. Particularly, the highly restrictive feature of constraining the coefficient of relative risk aversion with the elasticity of intertemporal substitution (EIS) as one to be reciprocal to the other , $\gamma = 1/\psi$, which seems not clear. As these two concepts are not tightly linked in reality as J. Campbell and Viceira (2002, p.31) noted:

"Risk aversion describes the consumer's reluctance to substitute consumption across states of the world and is meaningful even in an atemporal setting, whereas the elasticity of intertemporal substitution describes the consumer's willingness to substitute consumption over time and is meaningful even in a deterministic setting."

Many papers have attempted to address this issue, but it was not possible to reach an acceptable solution until Epstein and Zin (1989, 1991) and Weil (1989) who used the theoretical framework of Kreps and Porteus (1978) to develop a general and more flexible version of the standard power utility model. The recursive Epstein-Zin utility keeps the desirable feature of the scale-independence of power utility, but separates the two parameters of risk aversion from the elasticity of intertemporal substitution. The distinction between these two concepts plays an important role in the asset allocation for long term investors as RRA is a key factor in defining portfolio composition, whereas the EIS has a quite small role. On the other hand, the EIS has a great importance in consumption decisions.

[2] The model presentation is based on CCV and J. Campbell and Viceira (2002, Chapter 2,3, and 4)

The Epstein-Zin utility function is defined recursively by:

$$U_t = \{(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}}\}^{\frac{\theta}{1-\gamma}} \tag{5.1}$$

where $\theta = (1-\gamma)/(1-1/\psi)$. The Epstein-Zin utility function nest the CRRA/power utility function as a special case, when $\gamma = 1/\psi$, and when $\gamma = \psi$, equation (5.1) becomes the log utility function. Hence, $\theta = 1$ in CRRA/power utility case and $\theta = 0$ in the log utility case.

Considering the fact that the Epstein-Zin function resolves the biggest drawback of the CRRA/utility function. It can be characterized as the best quantification of rational intertemporal investor preferences, that is why it is often used when working with long-term portfolio problem, and it will be our choice in this thesis as well. The non-linear recursion result in equation (5.1) appears very complicated, but Epstein and Zin shows, using dynamic programming arguments that, equation (5.1) under the condition that equation (3.6) holds yields the following Euler Equation:

$$1 = E\{[\delta(\frac{C_{t+1}}{C_t})^{-\frac{1}{\psi}}]^{\theta}[\frac{1}{1+R_{p,t+1}}]^{1-\theta}(1+R_{i,t+1})\} \tag{5.2}$$

Where $R_{i,t+1}$ is the simple return on any available asset, including the risk free asset and the portfolio itself. Equation (5.2) can be simplified if we set $R_{i,t+1} = R_{p,t+1}$. Assuming that portfolio returns and consumption growth are jointly lognormal; one can derive this expression for expected consumption growth:

$$E_t(\Delta c_{t+1}) = \psi \log \delta + \psi E_t r_{p,t+1} + \frac{\theta}{2\psi} Var(\Delta c_{t+1} - \psi r_{p,t+1}) \tag{5.3}$$

Equation (5.3) describes which factors affect expected consumption growth in the Epstein-Zin framework taking the before mentioned assumptions into account. One should notice that EIS plays a major role in determining expected consumption growth, while RRA only has an indirect effect through theta. The other factors that directly influence expected consumption growth are; time preference, expected portfolio return and the uncertainty regarding future return and consumption growth. The uncertainty is expressed in the variance term. A preference for consumption in the future (a large time discount factor) and high-expected portfolio returns equals higher expected consumption growth. This makes sense because an investor with preference for future consumption will postpone consumption and has a higher future consumption rate. In addition, higher returns increases aggregate wealth, which therefore makes an increase in consumption possible. Further, a higher expected return creates incentive to save for future consumption. The uncertainty term increases expected consumption growth for $\theta > 0$, meaning that a larger uncertainty regarding consumption growth and portfolio returns increases precautionary savings, and lowers current consumption. Ceteris paribus, the investor needs to save more to have a higher consumption in the future, which results in a lower consumption in the present. For the case of $\theta < 0$ it is vice versa higher uncertainty leads to lower precautionary savings. Both the time discount factor and the portfolio return effects are increased by increasing EIS, whereas the uncertainty effect is decreased with increasing EIS value. Alas, the bigger the willingness of the investor to tilt consumption to the future the larger the effect of time preferences and portfolio return on expected consumption growth. On the other hand, the effect of uncertainty is lowered by this willingness.

If we assume one single risky asset, a risk free asset and jointly lognormal asset returns and consumption, the risk premium on the risky asset in the Epstein-Zin framework is:

$$E_t r_{t+1} - r_{f,t+1} + \frac{\sigma_t^2}{2} = \theta \frac{Cov_t(r_{t+1} \Delta c_{t+1})}{\psi} + (1-\theta) Cov_t(r_{t+1}, r_{p,t+1}) \qquad (5.4)$$

The simple excess return on a risky asset is a weighted average of its covariance with consumption growth (divided by EIS), and portfolio return (market return in a multi asset case), where the weighting is a function of $\theta = (1-\gamma)/(1-1/\psi)$. The relationship depicted by equation (5.4) is an asset pricing equilibrium model that explains the risk premium on a risky asset. The higher the assets covariance with consumption growth and the portfolio return the higher its required excess return/risk premium.

It can be seen that the investor in the Epstein-Zin setup must be compensated for intertemporal risk related to future consumption and myopic risk relating to the portfolio/market return. Both equation (5.3) and (5.4) is of great importance when solving the intertemporal portfolio problem.

5.2.2 Solution Methodology

Log-linearization

The solution builds on the presented theoretical framework. Therefore, the solution should satisfy the loglinear Euler equations for consumption and asset pricing implied by the Epstein-Zin model.

The simple return on the portfolio from t to $t+1$ is given as:

$$R_{p,t+1} = \sum_{i=1}^{n} \alpha_{i,t}(R_{i,t+1} - R_{1,t+1}) + R_{1,t+1} \tag{5.5}$$

Since it is more convenient to work with log returns, following Campbell and Viceira (1999) and CCV, the return on the portfolio in (5.5) can be presented in log return approximately as follows:

$$r_{p,t+1} = r_{1,t+1} + \alpha'_t \mathbf{x}_t + \frac{1}{2}\alpha'_t({\sigma_{\mathbf{x}}}^2 - \Sigma_{\mathbf{xx}}\alpha_t) \tag{5.6}$$

where ${\sigma_{\mathbf{x}}}^2 \equiv diag(\Sigma_{\mathbf{xx}})$ is the vector consisting of the diagonal elements of $\Sigma_{\mathbf{xx}}$ which is the variances of the excess returns. This approximation holds exactly in continuous time and is highly accurate for short time intervals. (The derivations of this approximation is provided in (J. Campbell & Viceira, 2002))

The intertemporal budget constraint in (3.6) is nonlinear. To linearize it, we log-linearize around the unconditional mean of the log consumption-wealth ratio to get,

$$\triangle w_{t+1} \backsimeq r_{p,t+1} + k + (1 - \frac{1}{\rho})(c_t - w_t) \tag{5.7}$$

where $\triangle$ is the difference operator, $\rho \equiv 1 - \exp(E(c_t - w_t))$ and $k \equiv \log(\rho) + (1-\rho)\log(1-\rho)/\rho$. When consumption is chosen optimally, ρ depends on the optimal level of c_t relative to w_t. This approximation is exact when the intertemporal substitution $\psi = 1$, in which case $c_t - w_t$ is constant and $\rho = \delta$, where δ is the time discount factor.

If we apply a second-order Taylor expansion to the Euler equation (5.2) around the conditional means of $\triangle c_{t+1}, r_{p,t+1}, r_{i,t+1}$ to obtain

$$0 = \theta \log \delta - \frac{\theta}{\psi} E_t \Delta c_{t+1} - (1-\theta) E_t r_{p,t+1} + E_t r_{i,t+1} + \frac{1}{2} Var_t[-\frac{\theta}{\psi} \Delta c_{t+1} - (1-\theta) r_{p,t+1} + r_{i,t+1}] \tag{5.8}$$

The loglinearized Euler equation is exact if consumption and asset returns are jointly log-normally distributed, which is the case when the elasticity of intertemporal substitution $\psi = 1$. It can be usefully transformend as follows. Setting $i = 1$ in (5.8), subtracting from the general form of (5.8), and noting that $\Delta c_{t+1} = \Delta(c_{t+1} - w_{t+1}) + \Delta w_{t+1}$, we obtain, for asset $i = 2, ..., n$,

$$\begin{aligned} E_t(r_{i,t+1} - r_{1,t+1}) + \frac{1}{2} Var_t[r_{i,t+1} - r_{1,t+1}] = {} & \frac{\theta}{\psi}(\sigma_{i,c-w,t} - \sigma_{1,c-w,t}) \\ & + \gamma(\sigma_{i,p,t} - \sigma_{1,p,t}) - (\sigma_{i,1,t} - \sigma_{1,1,t}) \end{aligned} \tag{5.9}$$

where $\sigma_{i,c-w,t} = Cov_t(r_{i,t+1}, c_{t+1} - w_{t+1})$, $\sigma_{1,c-w,t} = Cov_t(r_{1,t+1}, c_{t+1} - w_{t+1})$, $\sigma_{i,p,t} = Cov_t(r_{i,t+1}, r_{p,t+1})$, $\sigma_{1,p,t} = Cov_t(r_{1,t+1}, r_{p,t+1})$, $\sigma_{i,1,t} = Cov_t(r_{i,t+1}, r_{1,t+1})$, and $\sigma_{1,1,t} = Var_t(r_{1,t+1})$. The left hand side of this equation is the risk premium on asset i over asset 1. The equation relates asset i's risk premium to its excess covariance with consumption growth, its excess covariance with the portfolio return, and the covariance of its excess return with the return on asset 1.

Since consumption growth and portfolio return are endogenous so this is a first-order condition describing the optimal solution rather the solution itself.

Solving the Model

In order to solve dynamic programming problem, mainly three methods are used: Value function iteration, policy function iteration, or guess and verify method (Ljungqvist & Sargent, 2012, p.106)[3]. CCV applied guess and verify method which depends on guessing the right solution to Bellman equation. This method relies on luck to find the solutions, that is the reason why it is not used very often. CCV guess the optimal portfolio and consumption rules take the form of,

$$\alpha_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{z}_t \tag{5.10}$$

$$c_t - w_t = b_0 + \mathbf{B}_\mathbf{1}^{'} \mathbf{z}_t + \mathbf{z}^{'}{}_t \mathbf{B}_2 \mathbf{z}_t \tag{5.11}$$

Thus, the optimal portfolio rule is linear in the VAR state vector but the optimal consumption rule is quadratic. $\mathbf{A}_0, \mathbf{A}_1, b_0, \mathbf{B}_1$, and $\mathbf{B}_2$ are constant coefficient matrices to be determined, with dimensions $(n-1) \times 1, (n-1) \times m, 1 \times 1, m \times 1$,and $m \times m$, respectively.As previously stated n denotes the number of assets and m is the number of variables in the state vector. It should be noted that only $m + (m^2 - m)/2$ elements of the matrix $\mathbf{B}_2$ are determined, because the matrix is symmetric around the main diagonal.

To verify this guess and solve for the parameters of the solution, we write the conditional moments that appear in (5.9) as functions of the VAR parameters and the unknown parameters of (5.10) and (5.11). We then solve for the parameters that satisfy (5.9). Since the vector of excess returns is written as $\mathbf{x}_t$, the conditional expectation on the left hand side of (5.9) is

$$E_t(\mathbf{x}_{t+1}) + \frac{1}{2} Var_t(\mathbf{x}_{t+1}) = \mathbf{H}_x \Phi_0 + \mathbf{H}_x \Phi_1 \mathbf{z}_t + \frac{1}{2} \sigma_x^2 \tag{5.12}$$

where $\mathbf{H}_x$ is a selection matrix that selects the vector of excess returns from the full state vector. The conditional covariances of the right hand side of (5.9) can be written as linear

[3]For further and rigorous study see e.g.,(Stokey et al., 1989)

functions of the state variables (J. Campbell & Viceira, 2002).

$$\sigma_{c-w,t} - \sigma_{1,c-w,t}\mathbf{1} = \Lambda_0 + \Lambda_1 \mathbf{z}_t \tag{5.13}$$

$$\sigma_{p,t} - \sigma_{1,p,t}\mathbf{1} = \Sigma_{xx}\alpha_t + \sigma_{1x} \tag{5.14}$$

$$\sigma_{1,t} - \sigma_{1,1,t}\mathbf{1} = \sigma_{1x} \tag{5.15}$$

where **1** is a vector of ones.

Optimal portfolio choice

Solving the Euler equation (5.9) for the portfolio rule we have

$$\alpha_t = \frac{1}{\gamma}\Sigma_{xx}^{-1}[E_t(\mathbf{x}_{t+1}) + \frac{1}{2}Var_t(\mathbf{x}_{t+1}) + (1-\gamma)\sigma_{1x}] + \frac{1}{\gamma}\Sigma_{xx}^{-1}[-\frac{\theta}{\psi}(\sigma_{c-w,t} - \sigma_{1,c-w,t}\mathbf{1})] \tag{5.16}$$

Where $E_t(\mathbf{x}_{t+1}) + Var_t(\mathbf{x}_{t+1})/2$ and $\sigma_{c-w,t} - \sigma_{1,c-w,t}\mathbf{1}$ are the linear functions of $\mathbf{z}_t$ given in equation (5.12) and (5.13) , respectively. This asset allocation solution is a multiple-asset generalization of Campbell and Viceira (1999). The equation (5.16) describes the optimal portfolio choice as the sum of the two components: the myopic demand and the intertemporal hedging demand.

The first term on the right hand side of equation (5.16) is the myopic asset demand, which is determined by excess return scaled by the inverse asset covariance matrix adjusted by $(1-\gamma)$ times the reciprocal of relative risk aversion. Investors with $\gamma \neq 1$ adjust their allocation slightly by a term $(1-\gamma)\sigma_{1x}$ due to the fact that the benchmark asset is not risk free. An investor with $\gamma = 1$ clearly does not adjust for this fact. In short, what drive the myopic demand for assets are the same factors that determine the Markowitz portfolio.

The second term is the intertemporal hedging demand. Since, in this model the investment opportunity set is assumed time-varying in this model, because the expected returns depend on the state variables. Merton (1969, 1971, 1973) shows that a rational risk averse investor will hedge against adverse changes in investment opportunities. When investment opportunities are constant over time, hedging demand becomes zero for any level of risk aversion.

Substituting the equations (5.12) and (5.13) in (5.16) and rearranging the terms yields the initial guess for the portfolio rules, see CCV.

$$\alpha_t \equiv \mathbf{A}_0 + \mathbf{A}_1 \mathbf{z}_t \tag{5.17}$$

where

$$\mathbf{A}_0 = (\frac{1}{\gamma})\Sigma_{xx}^{-1}(\mathbf{H}_x\Phi_0 + \frac{1}{2}\sigma_x^2 + (1-\gamma)\sigma_{1x}) + (1-\frac{1}{\gamma})\Sigma_{xx}^{-1}(\frac{-\Lambda_0}{1-\psi}) \tag{5.18}$$

$$\mathbf{A}_1 = (\frac{1}{\gamma})\Sigma_{xx}^{-1}\mathbf{H}_x\Phi_1 + (1-\frac{1}{\gamma})\Sigma_{xx}^{-1}(\frac{-\Lambda_1}{1-\psi}) \tag{5.19}$$

Equation (5.17) which verify the initial guess of the optimal portfolio rule expresses $\mathbf{A}_0$, and $\mathbf{A}_1$ matrices as function of the underlying parameters which describe the dynamics of the state variables and investor preferences. Moreover, $\mathbf{A}_0$, and $\mathbf{A}_1$ depend as well on the consumption-wealth ratio equation, $\mathbf{B}_1$ and $\mathbf{B}_2$ through Λ_0 and Λ_1. The term $1-1/\gamma$ in equations (5.18) and (5.19) reflects the intertemporal hedging on the optimal portfolio choice. Intertemporal hedging demand considerations affect both the optimal portfolio allocation to risky assets through $\mathbf{A}_0$ and $\mathbf{A}_1$, and the sensitivity of the optimal portfolio allocation to changes in state variables through $\mathbf{A}_1$

5.2.3 Measuring Value Function Loss

In order to measure economic benefits of an asset allocation strategy we should apply a measure to compare competing strategies, to do so we discuss the welfare losses associated with suboptimal or restricted asset allocations. Epstein and Zin (1989, 1991) consider the value function, which is defined as the utility function per unit of wealth evaluated in an optimal consumption-wealth ratio. They show that the value function can be expressed as a power function of the optimal consumption-wealth ratio:

$$V_t = U_t/W_t = (1-\delta)^{-\psi/(1-\psi)}(C_t/W_t)^{1/1-\psi} \tag{5.20}$$

The value function can be used to assess the welfare loss due to suboptimal or restricted asset allocation, provided that consumption is chosen optimally given the suboptimal portfolio rule (Campbell and Viceira, 1999, 2001; CCV). The Expected Value Loss (EVL) is obtained as the relative difference between the expected value of V_t corresponding to the suboptimal or restricted allocation which we denote as $IE(V_t^{restr})$ and the expected value of V_t corresponding to the optimal asset allocation (denoted $IE(V_t^{opt})$):

$$EVL = 1 - \frac{IE(V_t^{restr})}{IE(V_t^{opt})} \tag{5.21}$$

The EVL value ranges from 0 to 1 and measures the welfare loss due to suboptimal asset allocation. An EVL of 0 indicates that there is no welfare loss relative to the optimal asset allocation. However, an EVL of 1 corresponds with the largest welfare loss possible. For example, when applied to the demand for foreign assets, the associated EVL measures the welfare loss per unit of wealth due to ignoring international diversification. It is possible to apply the same approach to obtain the EVL corresponding to ignoring hedging demand.[4] We therefore focus on the EVL to evaluate the economic importance of foreign asset demands.

[4]Portfolio weights (and their statistical significance) are not necessarily informative about welfare effects (Spierdijk & Zaghum, 2014).

5.3 Emperical Application

5.3.1 Data description

In this subsection we briefly describe the subsample we are going to use in the allocation exercise. Our data set consists of 328 observations from the same monthly data considered in chapter 4 for three countries: the United States, Canada, and the United Kingdom. The sample begins in 1977:1 and ends in 2004:5 for all countries. The reason to choose the year 1977 is related to the date of inception of the Canadian SWF (Alberta Heritage Fund) and the U.S. SWF (Alaska Permanent Fund), both of them created in 1976.[5]

In what follows a summary description is given to the variables through simple statistics: The Table 5.1 reports summary statistics: mean and standard deviation for the

Table 5.1: Summary statistics:1977:01-2004:05

Variables	rtb	xsr	xbr	bill	dy	spread
Canada						
Mean	3.57	2.89	2.09	-0.13	1.00	1.14
Std. Dev.	1.29	16.84	9.41	1.59	0.40	1.80
Sharp ratio	2.76	0.17	0.22			
U.k.						
Mean	3.21	5.46	2.13	-0.16	1.41	0.14
Std. Dev.	1.84	16.93	5.50	1.54	0.29	2.40
Sharp ratio	1.74	0.32	0.39			
U.S.						
Mean	1.92	5.62	1.81	-0.08	1.06	1.76
Std. Dev.	1.04	15.40	6.83	1.20	0.48	1.30
Sharp ratio	1.86	0.36	0.27			

three risky asset returns and three instruments for each of the three countries. The mean and standard deviation for the returns are expressed in annualized percentage. In addition to the reported two moments, the Sharpe ratio is presented as well, which is computed using the annualized mean and standard deviation[6].

[5]For more details see,`http://www.swfinstitute.org/sovereign-wealth-fund-rankings/`

[6]The Sharpe ratio is the average return earned in excess of the risk-free rate per unit of volatility

The mean excess stock return for the U.S., and U.K. is between 5% and 6%. While Canada exhibits the lowest mean excess stock returns (2.89%). The mean excess bond returns range from 1.81% in the U.S. to 2.13% in the U.K., where Canada is 2.09 %. The mean excess bond return is lower than the mean excess stock return for each country. The difference between stock and bond excess returns is more pronounced in the case of U.S., and U.K. Whereas, the difference in Canada seems not important (0.8%).

The standard deviation of excess stock returns is about 2-3 times larger than the standard deviation of excess bond returns for all countries. The real bill return standard deviation is considerably below that of excess bond return for each country.

The U.S., and U.K. have the highest Sharpe ratios for excess stock returns (0.36, 0.32, respectively), while Canada has the smallest (0.17). Turning to the Sharpe ratios for excess bond returns, the ratios seem close in Canada and the U.S. ranging from 0.22 to 0.27. For the U.K., the Sharpe ratio for excess bond returns is considerably higher at 0.39. Note that in the U.S. the Sharpe ratio for excess stock returns is approximately 1.34 times higher than for excess bond returns. By contrast, in Canada, and the U.K., the Sharpe ratio for excess stock returns is actually less than the Sharpe ratio for excess bond returns. These ratios lead us to expect a higher demand on average for stocks in the U.S., while in Canada, and the U.K. we would expect a higher demand on average for bonds.

5.3.2 Result Analysis

In our application we apply the CCV approach to estimate Eqs. (7) and (8) for a infinitely lived investor (SWF) in Canada, and the U.S. We assume an annual discount factor $\delta = 0.92$; hence the monthly basis equals to $0.92^{1/12}$. The VAR parameter estimates reported in chapter 4 serve as an input to the multi-period asset allocation exercises. We consider three plausible values for $\gamma = 3, 7$, and 10. These γ values are in line with those considered in other studies.[7] We report estimates of the mean asset demands for various assets over the sample period (From 1977:01 to 2004:05) for each γ value using the CCV numerical procedure.

Domestic asset demands for SWF in Canada and U.S.

Table 5.2 reports the mean total, myopic, and intertemporal hedging demands (in percentages) for domestic bills, stocks, and bonds in Canada, and U.S. The results are generated using the CCV approach for different γ values of 3,7, and 10.

The total mean demands across the three assets sum to 100; the mean myopic demands across assets also sum to 100, while the mean hedging demands sum to 0. For the U.S., there are large mean total and intertemporal hedging demands for stocks for each reported γ value. As expected, the mean total demand for stocks decreases as γ increases. Furthermore, the mean hedging demand for stocks also decreases as γ increases. The mean hedging demands for bonds are negative and fairly large in magnitude, contributing to the smaller total demands for bonds vis-a-vis stocks. The mean total demand for bills is negative for each reported γ value, so that the investor typically shorts bills. These results for the U.S. mean hedging demands for stocks are close but not similar to the mean hedging demand for stocks reported in (Rapach & Wohar, 2009). For example, the U.S. mean hedging demands for stocks equals to 80.07% using monthly data for 1952:04-2004:05, and $\gamma = 7$, and $\psi = 1$;

[7] For example, CCV include tabulated results for $\gamma = 5$; Balduzzi and Lynch (1999) consider $\gamma = 6$; Barberis (2000) considers $\gamma = 5, 10$; Lynch (2001) considers $\gamma = 4$.

the mean hedging demands for bonds in the U.S. in Table 5.2 are negative and seems very close to the results (-24.72,and -19.42) reported in (Rapach & Wohar, 2009) for γ values 7 and 10, respectively. The most remarkable result for the U.S. in Table 5.2, (Rapach & Wohar, 2009), and CCV as well is the considerable mean total and intertemporal hedging demands for domestic stocks.

While there are a number of factors which may affect the intertemporal hedging demands especially in this kind of multivariate setup as emphasized by CCV, still there are two reasonable explanations which may be given to clarify the sizable intertemporal hedging demand for domestic stocks in the U.S.

- (i) the positive coefficient on the lagged dividend yield in the excess stock return equation of the VAR and,
- (ii) the strong negative correlation between innovations to excess stock returns and the dividend yield.

To see how these factors generate a strong intertemporal hedging demand for stocks, consider a negative innovation to excess stock returns next period. Due to the large Sharpe ratio for stocks in the U.S., investors are usually long in stocks, so that the negative innovation to excess stock returns represents a worsening of the investor's investment opportunities next period. However, a negative innovation to excess stock returns next period tends to be accompanied by a positive innovation to the dividend yield next period, and according to the positive coefficient on the lagged dividend yield in the excess stock return equation of the VAR, the higher dividend yield next period leads to higher expected stock returns two periods from now. Thus, by looking beyond one-period-ahead as an investor with $\gamma > 1$ will do and taking into account return predictability, as well as the negative correlation between innovations to stock returns and the dividend yield, stocks become a good hedge against themselves, in that they hedge exposure to future adverse return shocks.

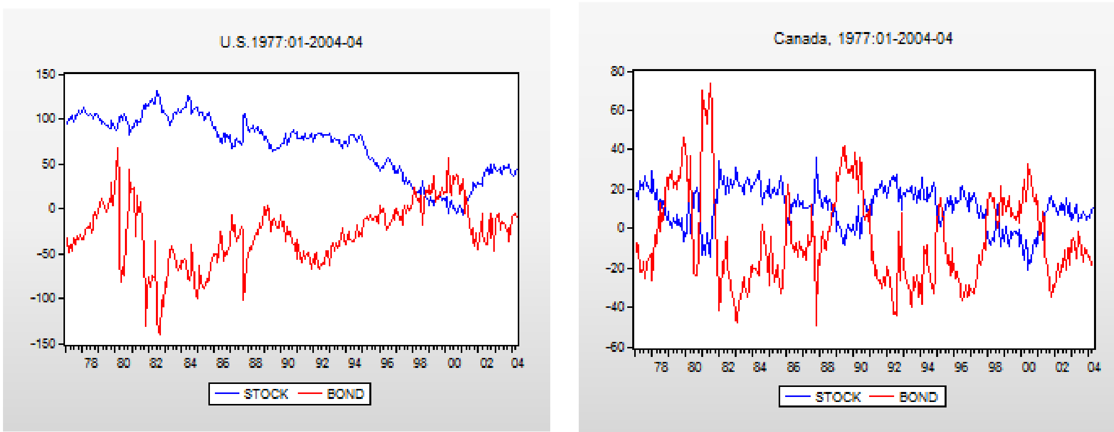


Figure 5.1: Historical intertemporal hedging demands for domestic stocks and bonds for investors in the U.S. and Canada when $\gamma = 7$ and $\psi = 1$

In order to further our understanding of the intertemporal hedging demands for domestic stocks and bonds in the U.S., Panel (a) of Fig. 5.1 represents the estimated hedging demands for stocks and bonds for each month over the sample in the U.S. when $\gamma = 7$. Generally, the hedging demand for stocks appears considerably less volatile than the hedging demand for bonds. The hedging demand for stocks is typically well above the hedging demand for bonds in most sample period, with the exception that the hedging demand for bonds does move above the hedging demand for stocks during the late 1990s and 2000, which is the period that witnessed the bursting of the dot com bubble but stock hedging demands rebound after 2001.

Turning to Canada's results in Table 5.2 the mean total demands for domestic stocks are moderately large, however, they are considerably smaller than the mean total demands for domestic stocks in the U.S. The mean hedging demands for stocks in Canada are also much smaller than the corresponding demands in the U.S. The mean total demands for bonds in Canada are very close to those in U.S. especially if we consider the case of $\gamma = 7$, and 10, but they differ remarkably when $\gamma = 3$. The mean hedging demands for bonds in Canada are negative and getting closer to zero more than in the U.S., particularly when γ getting bigger. From Panel (b) of Fig. 5.1, we see that the hedging demand for stocks is above the hedging demand for bonds when $\gamma = 7$, though we see drops in the average hedging demands for stocks especially in times of perturbations (Oil crisis, Asian financial crisis, and the dot com bubble bursting). In addition, the Panel (b) shows that the hedging demand for bonds is much more volatile than the hedging demand for stocks in Canada. Since the Sharpe ratios for domestic stocks in Canada are considerably smaller than the Sharpe ratio for stocks in the U.S., an investor in Canada is likely to hold fewer stocks than an investor in the U.S.

The results reported in Table 5.2 assume different values for γ with fixed value of EIS, however, empirical evidence suggests that the EIS varies across economic agents as well. Therefore we compute mean asset demands for the two countries assuming different ψ values of 0.3 and 1.5 with $\gamma = 7$ in Table 5.3.In order to make comparison feasible with the results in Table 5.2, we again report the mean asset demands for $\psi = 1$ in Table 5.3. The mean total and hedging demands for stocks change little as we vary ψ for Canada, e.g. hedging demands for stocks changes from 12.72 to 15.68 as ψ increases from 1.0 to 1.5. However, there are sizable increases in the mean total and hedging demands for stocks from 72.84 to 134.13 as ψ increases for the U.S. from 1.0 to 1.5. Intuitively, as the EIS increases, agents become more willing to trade future for current consumption, and they hold more stocks, which have a relatively high expected return. As they hold more stocks (for a given degree of relative risk aversion), the hedging demand for stocks increases, as stocks are a good hedge against themselves in the U.S.

It is also interesting to note that the patterns in Table 5.3 generally agree with theoretical results derived by Bhamra and Uppal (2006). They show that the EIS affects the magnitude, but not the sign, of the intertemporal hedging demand for the risky asset. In line with their theoretical results, we see from Table 5.3 that changes in the value of ψ only affect the magnitude, and not the sign, of the mean hedging demands for stocks.

International hedging demands

In the following asset allocation exercises we use the CCV approach to analyze a multi-period portfolio choice problem for SWF in Canada which, in addition to domestic bills, stocks, and bonds, has access to stocks and bonds from a foreign country, namely U.S. and the U.K. We take, in turn, the case of SWF in U.S. which can invest in domestic assets, in addition to foreign assets (Canada, U.k.). In order to keep the VAR parameter space to a reasonable size, we take just two countries in the same VAR system as we have seen in chapter 4.

To refresh the reader and facilitate the comprehension of our findings, we reintroduce the components of the state vector in the case of international investing. The log real return on a 3-month Treasury bill, and the log excess returns on Canadian (U.S.) stocks and bonds and foreign stocks and bonds (the four log excess returns are computed according to domestic log real return on a 3-month Treasury bill). The instrument set includes the Canadian (U.S.) nominal bill yield, dividend yield, and term spread, as well as their foreign counterparts. Assuming $\delta = 0.92$ on an annual basis, and $\psi = 1$. Following the same setup as in domestic settings: γ takes values of 3, 7, and 10. The results are reported in Table 6.7 in Appendix B.

A number of results stand out in Table 6.7. First, an investor in the U.S. continues to have substantial mean total and intertemporal hedging demands for domestic stocks, regardless of the foreign country. Thus, it is still optimal for an investor in the U.S. with access to foreign stocks and bonds from Canada, and the U.K. to have sizable mean total and hedging demands for domestic stocks. It is interesting to note that, the mean total demands for the foreign U.K. assets are significantly higher than the Canadian stocks and bonds. However, the stock hedging demands are very close.

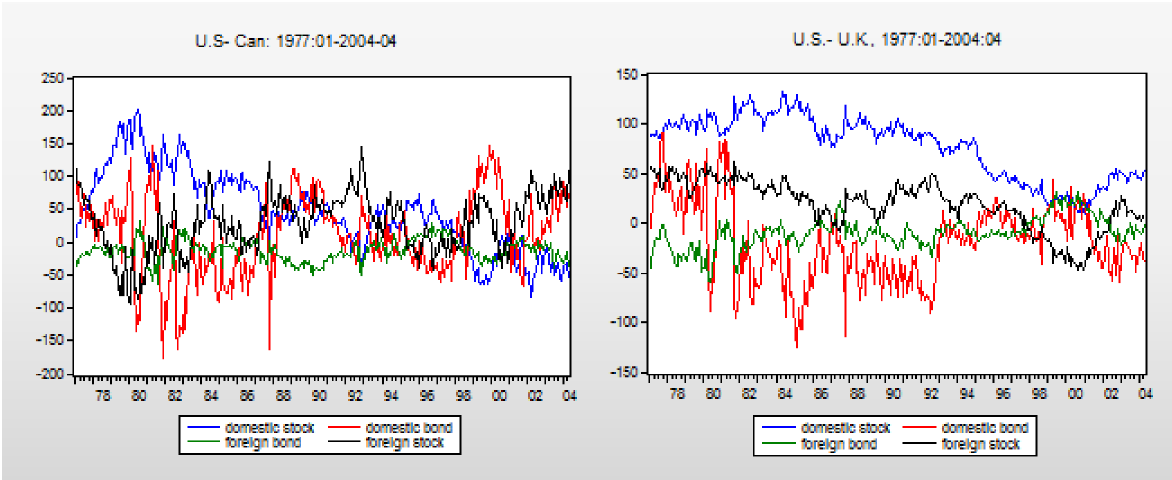


Figure 5.2: Historical intertemporal hedging demand for domestic stocks, domestic bonds, foreign stocks, and foreign bonds for an investor in the U.S. which can also invest in foreign stocks and bonds when $\gamma = 7$ and $\psi = 1$

Fig. 5.2 presents the intertemporal hedging demands for domestic and foreign stocks and bonds for each month over the sample when $\gamma = 7$. The profiles of the hedging demands for domestic stocks and bonds in each panel of Fig. 5.2 are similar to those for the U.S. in Panel A of Fig. 5.1 we see that the hedging demands for foreign stocks and bonds are typically small in magnitude throughout the sample for the U.K. However, when Canada stocks are considered we see foreign hedging demands are higher than domestic stocks especially between 1990 and 1992, and after 1997. The sizable intertemporal hedging demands for domestic stocks for U.S. investor who also has access to foreign stocks and bonds can again be largely explained by the relatively high Sharpe ratio for U.S. stocks. Table 6.8 reports mean asset demands for ψ values of 0.3 and 1.5 and $\gamma = 7$ for a SWF in the U.S. which can also invest in foreign stocks and bonds. (The mean asset demands for $\psi = 1$ from Table 6.7 are included in Table 6.8 to facilitate comparison of results.) The pattern of results is similar to that in Table 5.3 for an investor in the U.S.: the mean hedging demand for domestic stocks increases as ψ increases, and the sign of the hedging demand is unchanged for different values of ψ. The same intuition for the former results also applies: investors become more willing to trade future for current consumption.

Our final empirical exercise is another extension that analyzes asset demands for an investor in Canada which has access to domestic bills, stocks, and bonds, as well as stocks and bonds from foreign countries (U.K., U.S.). The output of international VAR(1) systems serve as an input in this multi-period asset allocation exercise.

Table 6.7 which reports the mean asset demands for γ values of 3, 7, and 10, indicates that the mean intertemporal hedging demands for domestic stocks in the case of U.S. being the foreign stock market are fairly small in magnitude and close to zero with negative sign in almost all γ cases. While the total and mean hedging demands for foreign stocks are very extensive. The total mean and intertemporal hedging demands for U.S. bonds are larger in magnitude, but they are negative in sign which means short selling bonds. When an investor in Canada can invest in U.K. assets, in addition to domestic assets, the total and the mean intertemporal hedging demands for domestic stocks are very large in magnitude, while other assets (domestic bonds, and foreign stocks and bonds) all are substantial however show negative sign which means they are shorted. Overall, the results in Table 6.7 indicate that access to U.S. stocks and bonds for a Canadian investor generates sizable intertemporal hedging demands for U.S. stocks.

Fig. 5.3 presents the intertemporal hedging demands for domestic and foreign stocks and bonds for each month over the sample in each country when $\gamma = 7$, and the figure reinforces the conclusions from Table 6.7. The U.S. stocks represent the largest intertemporal hedging demand among international assets over most of the sample. What accounts for the sizable intertemporal hedging demand for U.S. stock that emerges when investor in Canada has access to U.S. stocks and bonds?

In the previous discussion about an investor in U.S. which invests domestically, we have discussed how the high Sharpe ratio for U.S. stocks, together with the relationship between U.S. excess stock returns and the dividend yield, create sizable intertemporal hedging demands for domestic stocks for investors in the U.S. When investors outside the U.S. have access to U.S. stocks, the relatively high (local currency) Sharpe ratio for U.S. stocks makes U.S. stocks relatively attractive to these investors.

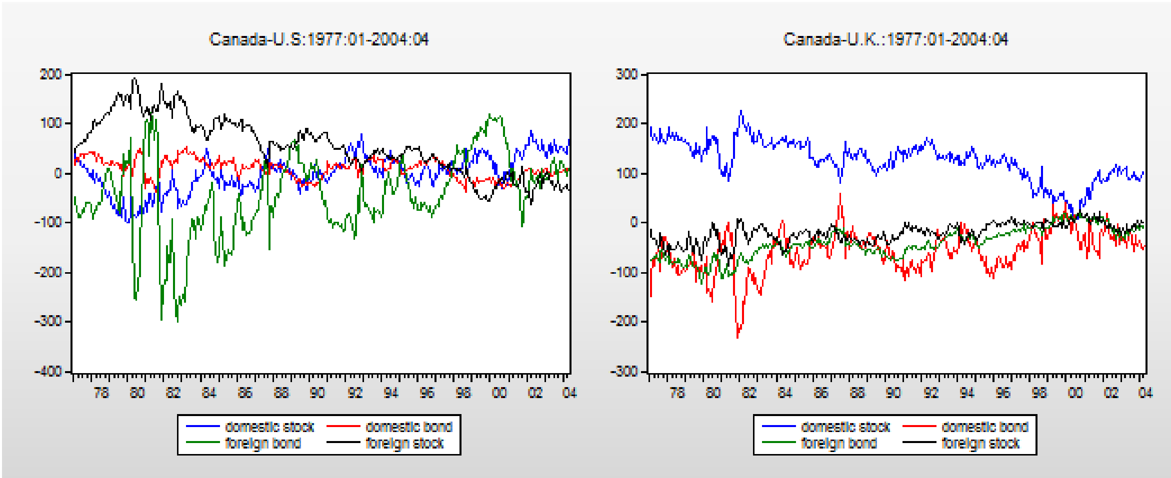


Figure 5.3: Historical intertemporal hedging demand for domestic stocks, domestic bonds, foreign stocks, and foreign bonds for an investor in Canada which can also invest in foreign stocks and bonds when $\gamma = 7$ and $\psi = 1$

International diversification gain

One way to quantify the gains from international diversification is by calculating for each SWF (Canada, U.S.) the optimal asset allocation, conditional on the restriction that it cannot invest in foreign assets, which means invests domestically only. The resulting welfare implications are based on the associated EVLs discussed in subsection (5.2.3). For $\gamma = 7$, we observe that the welfare losses due to no international diversification for a SWF in the U.S.: Canada, 0.87; U.K., 0.79.

We apply the same method to quantify the gains from international diversification in the case of a SWF in Canada which has access to the U.S. and U.K. stocks and bonds in addition to domestic assets. The investor welfare losses due to no international diversification are: U.S., 0.88. U.K. 0.98.

We observe that the welfare losses due to no international diversification are substantial and more important in Canada than U.S. This implies that the SWF in Canada has more to gain through international diversification than the SWF in U.S. These results are in line with the empirical findings (See e.g., (Driessen & Laeven, 2007)).

Table 5.2: Mean demands for domestic assets for SWF in Canada and the U.S.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Canada			U.S.		
CRRA	$\gamma = 3$	$\gamma = 7$	$\gamma = 10$	$\gamma = 3$	$\gamma = 7$	$\gamma = 10$
stocks						
Total demand	76.42	35.40	24.54	186.31	114.47	90.09
Myopic demand	53.14	22.68	15.82	97.62	41.63	29.04
Hedging demand	23.28	12.72	8.72	88.69	72.84	61.05
bonds						
Total demand	35.45	13.87	9.53	56.07	16.98	10.04
Myopic demand	49.25	20.95	14.58	93.30	40.17	28.22
Hedging demand	-13.81	-7.08	-5.05	-37.22	-23.19	-18.18
bills						
Total demand	-11.87	50.73	65.92	-142.38	-31.45	-0.13
Myopic demand	-2.39	56.37	69.60	-90.92	18.20	42.75
Hedging demand	-9.48	-5.64	-3.67	-51.46	-49.65	-42.88

This reinforces the idea that U.S. stock markets are very attractive to investors outside the U.S. given the fact that U.S. stocks offer them well-diversified access to assets with high expected return (and Sharpe ratio) that also serves as a good hedge against itself (Rapach & Wohar, 2009).

Table 5.3: Mean asset demands for SWF in the U.S. and Canada which can invest domestically assuming different values for the elasticity of intertemporal substitution ψ and a coefficient of relative risk aversion $\gamma = 7$

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Canada			U.S.		
CRRA	$\psi = 0.3$	$\psi = 1.0$	$\psi = 1.5$	$\psi = 0.3$	$\psi = 1.0$	$\psi = 1.5$
stocks						
Total demand	32.74	35.40	38.36	89.40	114.47	175.77
Myopic demand	22.68	22.68	22.68	41.63	41.63	41.63
Hedging demand	10.07	12.72	15.68	47.77	72.84	134.13
bonds						
Total demand	12.77	13.87	15.13	10.71	16.98	33.12
Myopic demand	20.95	20.95	20.95	40.17	40.17	40.17
Hedging demand	-8.18	-7.08	-5.82	-29.46	-23.19	-7.05
bills						
Total demand	54.49	50.73	46.51	-0.11	-31.45	-108.88
Myopic demand	56.37	56.37	56.37	18.20	18.20	18.20
Hedging demand	-1.89	-5.64	-9.86	-18.31	-49.65	-127.08

5.4 Conclusion

In this chapter, we have investigated international diversification and the intertemporal hedging demands for stocks and bonds for SWFs with Epstein-Zin-Weil preferences and infinite horizons in Canada, and the U.S. When we analyze allocations across domestic bills, stocks, and bonds for investors in each country, our findings stress the sizable mean intertemporal hedging demands for domestic stocks in the U.S. Interestingly even when an SWF in the U.S. also has access to foreign stocks and bonds, still it continues to have sizable mean hedging demands for U.S. stocks, and the only foreign asset for which it exhibits relatively large intertemporal hedging demands is U.K. stocks.

The demand for domestic assets for SWF in Canada shows that the mean hedging demands are considerably smaller than corresponding demands in the U.S. When the SWF has access to U.S. stocks, it displays substantial intertemporal hedging demands for U.S. stocks. The excess stock return-dividend yield relationship in the U.S. helps to account for the strong intertemporal hedging demands for U.S. stocks. In contrast to U.S. assets, the U.K. assets are shorted. Overall, our results indicate that U.S. stocks provide an attractive intertemporal hedging instruments for international investors with Epstein-Zin-Weil preferences and infinite horizons.

Our findings stress the importance of international diversification, which are substantial and more important in Canada than U.S. This implies that the SWF in Canada have more to gain through international diversification than the SWF in U.S. This reinforces the idea that U.S. stock markets are very attractive to investors outside the U.S. given the fact that U.S. stocks offer them well-diversified access to assets with high expected return (and Sharpe ratio) that also serves as a good hedge against itself.

6

Conclusion

The decision of asset allocation is considered the most important factor driving the financial performance of any portfolio, in general and for SWF in particular. Since the SWFs are pools assets owned by government their performance does not affect special type of investors, but the well-being of the whole citizens. For this reason, the well- performance is more pronounced in the case of commodity based SWFs, because these funds transfer non-renewal wealth to financial wealth.

In this thesis, the focus is mainly concentrated on the analysis of the optimal asset allocation policy for oil-based SWFs for many reasons: a) the asset under management for commodity funds is estimated at about 60% of total sovereign wealth fund assets, oil-based SWFs are the dominating force in commodity funds. b) The peculiarity of commodity funds, in general, and oil-based SWFs in particular as they transfer non-renewable wealth into renewable wealth.

In this thesis our aim is twofold. First, we examine the international predictability of stocks and bonds for three countries, namely Canada, the United States, and the United

Kingdoms using in-sample and out-sample tests applying the same approach as Rapach et al. (2005), which is based on Clark and McCracken (2001) and McCracken (2007) tests. In our study we have exploited the same variables as CCV, (Rapach & Wohar, 2009), and (Engsted & Pedersen, 2012). Thus, we use nominal bill yield, dividend yield, and term spread as predictors. For the sake of comparability and estimation benefits we apply long time sample. The sample begins in 1954:06 and ends in 2004:05 for all countries.

Second, in order to analyze the dynamic asset allocation domestically and internationally and evaluate the effects of intertemporal hedging demands as well as international diversification effects, we extend the CCV approach to analyze dynamic asset allocation across domestic bills, stocks, and bonds for an SWF in Canada, and the U.S., where the returns dynamics are characterized by a VAR(1) process. For a set of plausible values for the parameters relating to intertemporal preferences, we use the CCV approach to estimate the mean total, myopic, and intertemporal hedging demands for domestic bills, stocks, and bonds in each country. In addition we consider a multi-period portfolio choice problem for SWF in the U.S. which has access to stocks and bonds from a foreign country (Canada, U.K.). The same exercise goes with SWF in Canada which can invest in a foreign country (U.K., U.S.). The sample begins in 1977:01 and ends in 2004:05 for all countries.

The out-of sample test findings, term spread stand out in terms of predictive ability -both in-sample and out-of-sample- and to lesser extinct nominal T-bills yield. Turning to the results for the U.K., there is strong evidence of in-sample and out-sample predictive ability for dividend yield to predict stocks and less impressive evidence using term spread. In contrast, the term spread has strong evidence to predict bonds while dividend yield has limited in-sample power. The U.S. results demonstrate that only the term spread has in-sample and out-sample bond predictive ability. While both the nominal T-bill yield and term spread variables have only in-sample stock predictability power.

Given literature empirical results and our in-sample and out-sample tests findings, we kept the same variables to test them in multivariate setup. Since our VAR estimation results are in line to some extinct with prior models in asset allocation literature. We are more inclined to go through the process of asset allocation using this model specification. The results from VAR estimation - Φ_0, Φ_1, and Σ_v - will serve as input to our portfolio allocation implementation.

In chapter 5, we have investigated international diversification and the intertemporal hedging demands for stocks and bonds for SWFs with Epstein-Zin-Weil preferences and infinite horizons in Canada, and the U.S. When we analyze allocations across domestic bills, stocks, and bonds for investors in each country, our findings stress the sizable mean intertemporal hedging demands for domestic stocks in the U.S. Interestingly even when an SWF in the U.S. also has access to foreign stocks and bonds, still it continues to have sizable mean hedging demands for U.S. stocks, and the only foreign asset for which it exhibits relatively large intertemporal hedging demands is U.K. stocks.

The demand for domestic assets for SWF in Canada shows that the mean hedging demands are considerably smaller than corresponding demands in the U.S. When the SWF has access to U.S. stocks, it displays substantial intertemporal hedging demands for U.S. stocks. The excess stock return-dividend yield relationship in the U.S. helps to account for the strong intertemporal hedging demands for U.S. stocks. In contrast to U.S. assets, the U.K. assets are shorted. Overall, our results indicate that U.S. stocks provide especially attractive intertemporal hedging instruments for international investors with Epstein-Zin-Weil preferences and infinite horizons.

Our findings stress the importance of international diversification, which are substantial and more important in Canada than U.S. This implies that the SWF in Canada have more to gain through international diversification than the SWF in U.S. This reinforces the idea that U.S. stock markets are very attractive to investors outside the U.S. given the fact that U.S. stocks offer them well-diversified access to assets with high expected return (and Sharpe ratio) that also serves as a good hedge against itself.

Appendix A: Useful Background

Asset Returns

a- One period simple return

The asset returns can be measured in terms of absolute price change, relative price change, and logarithmic return. In order to define these. Let P_t denote the price of an asset at time t . The absolute price change over the period $t-1$ to t is given by $P_t - P_{t-1}$, the relative price change by

$$R_t = (P_t - P_{t-1})/P_{t-1}$$

The gross return on security is given by

$$1 + R_t = \frac{P_t + d_t}{P_{t-1}}$$

where d_t is the dividend. In what follows we assume that dividends are included in P_t. Hence the above equation becomes:

$$1 + R_t = P_t/P_{t-1}$$

and the log price change by

$$r_t = \Delta \ln(P_t) = \ln(1 + R_t)$$

Log-returns are called continuously compounded returns, whereas R_t known as discretely compounded return. In daily or higher frequency where there are small relative price changes the log-price change and the relative price change are almost identical $r_t \approx R_t$.

$$r_t^e = r_t - r_t^f$$

where r_t^e is the excess return and r_t^f is typically the return of a riskless short-term asset, like three months government bond.

b- Multi-period gross return

Single-period returns can be used to compute multi-period returns. Denote the return over the most recent h periods by $R_t(h)$ then

$$R_t(h) = \frac{P_t - P_{t-h}}{P_{t-h}}$$

and

$$r_t(h) = \ln(P_t/P_{t-h}) = r_t + r_{t-1} + ... + r_{t-h+1}$$

where r_{t-i}, $i = 0; 1; 2; ...; h-1$ are the single-period returns.

c- Portfolio returns

The return of a portfolio composed of N assets with α_i weights
$\alpha_{i,t-1}$,$(\Sigma_{i=1}^{N} \alpha_{i,t-1} = 1, \alpha_{i,t-1} \geq 0)$ we have

$$R_{p,t} = \Sigma_{i=1}^{N} \alpha_{i,t-1} R_{i,t},$$

where $R_{p,t}$ is the portfolio return and $R_{i,t}$ is the return of asset i .
In terms of log-returns

$$r_{p,t} = \ln(\Sigma_{i=1}^{N} \alpha_{i,t-1} e^{r_{i,t}}),$$

where r_{pt} is the continuously compounded return of the portfolio. Often $r_{p,t}$ is approximated by $\Sigma_{i=1}^{N} \alpha_{i,t-1} r_{i,t}$.

Ito process

If we let P_t denote the price of an asset at time t and h the number of time units. The expected return μ and the variance σ^2 are given respectively by:

$$\mu = \frac{1}{h} E[\frac{P_{t+h} - P_t}{P_t}]$$

,

$$\sigma^2 = \frac{1}{h} E[(\frac{P_{t+h} - P_t}{P_t} - h\mu)^2].$$

Assuming that μ, and σ^2 exist and are finite and that $\lim_{h\to 0}\sigma^2 > 0$, which means that for small time intervals the uncertainty can not be eliminated or dominate the analysis.
Define y_t to be iid $N(0,1)$ distributed, then the return dynamics can be written as:

$$\frac{P_{t+h}-P_t}{P_t} = h\mu + \sqrt{h}\sigma y_t.$$

By taking the limit with respect to h we get the stochastic differential equation form of the return dynamics:

$$\frac{dP}{P} = \lim_{h\to 0}\frac{P_{t+h}-P_t}{P_t} = \mu dt + \sqrt{dt}\sigma y_t$$

Define dz to be a Wiener process:

$$dz = y_t\sqrt{dt}$$

Then insert it into the differential equation of return dynamics results that return follow an Ito process:

$$\frac{dP}{P} = \mu dt + \sigma dz$$

Appendix B: Predictability and Asset Allocation Results

Table 6.1: VAR estimation results, Canada-U.S.,1954:06-2004:05

1 dependent variable	2 rtbr-t	3 xsr-t	4 xbr-t	5 fxsr-t	6 fxbr-t	7 bills-t	8 dy-t	9 spread-t	10 fbills-t	11 fdy-t	12 fspread-t	13 R^2
VAR slope coefficient estimates and goodness-of-fit measures												
rtbr-t+1	0.053	-0.014	0.009	0.015	-0.008	0.000	-0.006	-0.001	0.000	0.004	0.001	0.176
	(1.125)	(-2.709)*	(1.084)	(2.656)*	(-0.758)	(-1.676)**	(-4.668)*	(-6.672)*	(1.859)**	(3.603)*	(5.405)*	
xsr-t+1	-0.189	-0.002	-0.049	0.073	0.302	-0.001	0.008	0.003	-0.001	0.002	-0.001	0.046
	(-0.396)	(-0.020)	(-0.369)	(1.027)	(2.014)*	(-0.215)	(0.494)	(1.588)	(-0.268)	(0.103)	(-0.251)	
xbr-t+1	0.773	-0.143	0.055	0.055	0.102	0.000	0.015	0.002	0.001	-0.012	0.001	0.112
	(3.153)*	(-3.677)*	(0.761)	(1.582)	(1.501)	(-0.245)	(1.611)	(2.168)*	(0.379)	(-1.299)	(1.282)	
fxsr-t+1	-0.223	0.006	-0.013	-0.030	0.208	0.003	-0.007	0.003	-0.008	0.017	-0.001	0.033
	(-0.438)	(0.079)	(-0.095)	(-0.386)	(1.465)	(0.999)	(-0.426)	(1.343)	(-1.894)**	(1.079)	(-0.336)	
fxbr-t+1	0.246	-0.128	0.161	0.024	-0.016	0.003	0.013	0.001	-0.001	-0.012	0.001	0.089
	(1.100)	(-3.383)*	(2.327)*	(0.741)	(-0.238)	(2.222)*	(1.524)	(0.662)	(-0.852)	(-1.461)	(1.184)	
bills-t+1	-10.859	1.344	-2.008	-0.223	-4.114	0.815	-0.085	0.038	0.225	0.058	0.015	0.881
	(-2.056)*	(1.824)**	(-1.232)	(-0.279)	(-2.498)*	(23.004)*	(-0.440)	(1.827)**	(5.035)*	(0.307)	(0.601)	
dy-t+1	-0.059	0.023	0.024	-0.076	-0.303	0.003	0.972	0.000	0.000	0.016	-0.002	0.982
	(-0.116)	(0.272)	(0.176)	(-0.989)	(-1.997)*	(1.087)	(54.079)*	(-0.231)	(-0.123)	(1.004)	(-0.869)	
spread-t+1	-0.027	1.005	1.340	-0.823	2.621	0.071	-0.109	0.949	-0.211	0.076	-0.015	0.923
	(-0.006)	(1.655)**	(0.848)	(-1.154)	(1.695)**	(2.055)*	(-0.633)	(54.005)*	(-4.659)*	(0.450)	(-0.693)	
fbills-t+1	-15.583	3.541	-3.794	-0.794	-0.542	-0.018	-0.290	0.014	0.902	0.253	0.038	0.804
	(-3.157)*	(3.243)*	(-2.275)*	(-0.956)	(-0.315)	(-0.492)	(-1.421)	(0.593)	(19.179)*	(1.257)	(1.489)	
fdy-t+1	0.252	0.020	-0.046	0.031	-0.159	-0.001	0.026	-0.002	0.007	0.967	0.001	0.987
	(0.523)	(0.274)	(-0.357)	(0.411)	(-1.233)	(-0.395)	(1.684)**	(-1.142)	(1.941)**	(66.531)*	(0.516)	
fspread-t+1	10.046	-1.261	1.022	0.376	1.066	-0.017	0.208	-0.012	0.005	-0.193	0.940	0.888
	(2.339)*	(-1.717)**	(0.807)	(0.548)	(0.741)	(-0.526)	(1.307)	(-0.605)	(0.127)	(-1.234)	(41.421)*	
Cross-correlations of VAR residuals												
	rtbr-t	xsr-t	xbr-t	fxsr-t	fxbr-t	bills-t	dy-t	spread-t	fbills-t	fdy-t	fspread-t	
rtbr-t	1.000											
xsr-t	0.026	1.000										
xbr-t	0.016	0.260	1.000									
fxsr-t	0.066	0.773	0.278	1.000								
fxbr-t	0.053	0.239	0.654	0.409	1.000							
bills-t	0.038	-0.185	-0.453	-0.135	-0.383	1.000						
dy-t	-0.072	-0.908	-0.293	-0.748	-0.301	0.196	1.000					
spread-t	-0.048	0.032	-0.200	-0.053	-0.044	-0.752	-0.026	1.000				
fbills-t	-0.058	-0.065	-0.435	-0.091	-0.541	0.399	0.119	-0.114	1.000			
fdy-t	-0.079	-0.748	-0.229	-0.940	-0.263	0.056	0.709	0.106	0.058	1.000		
fspread-t	0.022	-0.013	-0.057	-0.053	0.026	-0.242	-0.021	0.312	-0.745	0.075	1.000	

Notes: rtbr= log real 3-month Treasury bill return; xsr = log excess stock return; xbr= log excess bond return; bills= 3-month Treasury bill yield; dy = log dividend yield; spread= 10-year government bond yield 3-month Treasury bill yield. t-statistics are given in parentheses; Significance levels:t=1.645 (10%), t=1.960 (5%), where ** and * indicate the significance at 10% and 5%, respectively.

Table 6.2: VAR estimation results, U.S-Canada, 1954:06-2004:05

1 dependent variable	2 rtbr-t	3 xsr-t	4 xbr-t	5 fxsr-t	6 fxbr-t	7 bills-t	8 dy-t	9 spread-t	10 fbills-t	11 fdy-t	12 fspread-t	13 R^2
VAR slope coefficient estimates and goodness-of-fit measures												
rtbr-t+1	0.314	0.015	0.005	-0.013	0.002	0.000	0.003	0.000	0.000	-0.004	0.000	0.205
	(6.280)*	(3.703)*	(0.501)	(-3.510)*	(0.395)	(-0.468)	(3.587)*	(0.825)	(-1.208)	(-4.182)*	(-2.124)*	
xsr-t+1	1.031	-0.025	0.307	0.025	-0.044	-0.005	0.015	0.000	0.001	-0.004	0.003	0.045
	(1.719)**	(-0.337)	(2.057)*	(0.369)	(-0.435)	(-1.418)	(1.087)	(-0.202)	(0.422)	(-0.257)	(1.494)	
xbr-t+1	0.590	0.036	0.049	-0.130	0.112	0.001	-0.010	0.002	0.000	0.011	0.000	0.136
	(2.101)*	(1.338)	(0.744)	(-4.040)*	(2.352)*	(0.741)	(-1.438)	(3.233)*	(0.387)	(1.551)	(-0.747)	
fxsr-t+1	1.100	0.086	0.602	0.030	-0.127	-0.001	-0.010	0.000	0.002	0.022	0.003	0.051
	(1.469)	(0.995)	(2.598)*	(0.349)	(-1.061)	(-0.305)	(-0.548)	(-0.093)	(0.525)	(1.103)	(1.591)	
fxbr-t+1	1.354	0.079	0.461	-0.126	-0.086	0.001	-0.019	0.003	0.001	0.023	0.001	0.134
	(3.201)*	(1.745)**	(4.623)*	(-2.809)*	(-1.317)	(0.513)	(-1.878)**	(2.444)*	(0.643)	(2.124)*	(0.897)	
bills-t+1	-6.976	-1.406	-4.717	3.628	-1.806	0.873	0.219	0.021	-0.002	-0.227	0.031	0.805
	(-0.737)	(-1.556)	(-1.817)	(3.282)*	(-1.298)	(17.520)*	(1.069)	(0.827)	(-0.056)	(-1.085)	(1.394)	
dy-t+1	-0.929	0.025	-0.316	0.006	0.025	0.007	0.972	0.002	-0.001	0.020	-0.003	0.987
	(-1.478)	(0.332)	(-2.062)*	(0.088)	(0.246)	(1.815)**	(65.990)*	(0.742)	(-0.438)	(1.257)	(-1.640)	
spread-t+1	-3.418	0.738	2.733	-1.311	0.551	0.019	-0.142	0.951	-0.026	0.130	-0.026	0.887
	(-0.453)	(1.030)	(1.216)	(-1.749)	(0.453)	(0.446)	(-0.896)	(42.029)*	(-0.800)	(0.815)	(-1.331)	
fbills-t+1	-7.692	-0.225	-4.742	1.059	-2.416	0.220	0.046	0.010	0.812	-0.059	0.043	0.880
	(-1.038)	(-0.263)	(-1.945)*	(1.444)	(-1.713)**	(4.753)*	(0.249)	(0.409)	(22.082)*	(-0.311)	(2.370)*	
fdy-t+1	-1.531	-0.084	-0.548	-0.016	0.082	-0.002	0.023	-0.002	0.003	0.964	-0.001	0.983
	(-2.079)*	(-1.028)	(-2.738)*	(-0.198)	(0.745)	(-0.458)	(1.401)	(-0.741)	(0.977)	(52.875)*	(-0.692)	
fspread-t+1	-8.145	-1.037	0.248	0.961	2.553	-0.225	0.114	-0.020	0.076	-0.155	0.950	0.924
	(-1.257)	(-1.298)	(0.108)	(1.519)	(1.842)**	(-4.667)*	(0.689)	(-0.873)	(2.123)*	(-0.916)	(57.011)*	
Cross-correlations of VAR residuals												
	rtbr-t	xsr-t	xbr-t	fxsr-t	fxbr-t	bills-t	dy-t	spread-t	fbills-t	fdy-t	fspread-t	
rtbr-t	1.000											
xsr-t	0.063	1.000										
xbr-t	-0.032	0.145	1.000									
fxsr-t	-0.016	0.758	0.104	1.000								
fxbr-t	0.002	0.290	0.597	0.441	1.000							
bills-t	0.091	-0.058	-0.636	-0.076	-0.401	1.000						
dy-t	-0.083	-0.965	-0.140	-0.738	-0.286	0.045	1.000					
spread-t	-0.114	-0.069	0.021	-0.005	-0.021	-0.748	0.080	1.000				
fbills-t	0.046	-0.065	-0.297	-0.221	-0.475	0.401	0.049	-0.247	1.000			
fdy-t	-0.019	-0.730	-0.129	-0.899	-0.360	0.102	0.706	-0.016	0.191	1.000		
fspread-t	-0.077	-0.089	-0.150	0.066	-0.090	-0.128	0.099	0.318	-0.759	-0.041	1.000	

Notes: rtbr= log real 3-month Treasury bill return; xsr = log excess stock return; xbr= log excess bond return; bills= 3-month Treasury bill yield; dy = log dividend yield; spread= 10-year government bond yield 3-month Treasury bill yield. t-statistics are given in parentheses; Significance levels:t=1.645 (10%), t=1.960 (5%), where ** and * indicate the significance at 10% and 5%, respectively.

Table 6.3: VAR estimation results, U.S.-UK, 1954:06-2004:05, 1954:06-2004:05

1 dependent variable	2 rtbr-t	3 xsr-t	4 xbr-t	5 fxsr-t	6 fxbr-t	7 bills-t	8 dy-t	9 spread-t	10 fbills-t	11 fdy-t	12 fspread-t	13 R^2
VAR slope coefficient estimates and goodness-of-fit measures												
rtbr-t+1	0.313	0.006	0.010	-0.003	-0.003	0.000	0.002	0.000	0.000	-0.002	0.000	0.201
	(6.229)*	(1.856)**	(1.270)	(-1.085)	(-0.607)	(-0.118)	(3.361)*	(0.398)	(-3.131)*	(-2.975)*	(-4.240)*	
xsr-t+1	0.917	-0.045	0.277	0.050	-0.065	-0.003	-0.001	0.002	0.000	0.015	0.000	0.045
	(1.492)	(-0.806)	(2.282)*	(1.119)	(-0.974)	(-1.375)	(-0.074)	(1.098)	(0.323)	(1.234)	(-0.418)	
xbr-t+1	0.710	-0.057	0.173	-0.016	-0.035	0.001	-0.002	0.002	0.000	0.005	0.000	0.087
	(2.401)*	(-2.698)*	(2.640)*	(-0.905)	(-1.163)	(0.876)	(-0.679)	(3.330)*	(0.718)	(1.231)	(-1.027)	
fxsr-t+1	0.944	0.068	0.051	0.017	0.112	-0.002	-0.021	0.005	0.003	0.050	0.000	0.047
	(1.071)	(0.806)	(0.307)	(0.214)	(1.070)	(-0.785)	(-1.361)	(1.646)	(1.325)	(1.935)**	(0.027)	
fxbr-t+1	-0.196	-0.052	0.202	0.040	0.129	0.000	-0.006	0.003	0.003	0.010	-0.001	0.074
	(-0.440)	(-1.381)	(2.109)*	(1.124)	(2.037)*	(-0.141)	(-0.940)	(1.770)**	(1.763)**	(1.086)	(-0.623)	
bills-t+1	-10.230	1.541	-7.529	0.204	1.765	0.875	0.040	0.037	0.006	-0.117	0.011	0.795
	(-1.028)	(2.382)*	(-3.109)*	(0.391)	(2.228)*	(24.246)*	(0.379)	(1.873)**	(0.288)	(-0.842)	(0.912)	
dy-t+1	-0.800	0.072	-0.312	-0.055	0.085	0.004	0.996	-0.001	0.000	-0.006	0.001	0.987
	(-1.241)	(1.250)	(-2.546)*	(-1.197)	(1.210)	(1.783)**	(106.872)*	(-0.511)	(0.187)	(-0.465)	(0.591)	
spread-t+1	-2.991	-0.450	3.893	0.113	-1.049	-0.001	-0.016	0.928	-0.033	0.043	-0.004	0.887
	(-0.402)	(-0.873)	(2.138)*	(0.307)	(-1.698)	(-0.048)	(-0.185)	(50.334)*	(-1.936)**	(0.347)	(-0.457)	
fbills-t+1	17.864	1.365	0.349	-0.965	-2.213	0.094	0.216	0.010	0.907	-0.467	0.044	0.837
	(1.828)**	(2.258)*	(0.174)	(-1.781)**	(-1.912)**	(2.672)*	(1.752)**	(0.316)	(35.020)*	(-2.949)*	(3.239)*	
fdy-t+1	-1.022	-0.033	-0.072	-0.432	0.268	0.002	0.028	-0.001	0.002	0.952	0.000	0.978
	(-1.721)**	(-0.531)	(-0.666)	(-6.413)*	(3.535)*	(1.047)	(2.846)*	(-0.632)	(1.401)	(64.615)*	(-0.292)	
fspread-t+1	-21.562	-0.337	-2.571	-0.643	2.207	-0.064	-0.108	-0.015	-0.018	0.249	0.959	0.939
	(-2.439)*	(-0.563)	(-1.597)	(-1.109)	(2.093)*	(-1.983)**	(-0.949)	(-0.545)	(-0.784)	(1.655)**	(75.306)*	
Cross-correlations of VAR residuals												
	rtbr-t	xsr-t	xbr-t	fxsr-t	fxbr-t	bills-t	dy-t	spread-t	fbills-t	fdy-t	fspread-t	
rtbr-t	1.000											
xsr-t	0.063	1.000										
xbr-t	-0.018	0.129	1.000									
fxsr-t	0.107	0.489	0.073	1.000								
fxbr-t	0.007	0.012	0.228	0.476	1.000							
bills-t	0.072	-0.043	-0.654	-0.082	-0.206	1.000						
dy-t	-0.086	-0.965	-0.126	-0.467	-0.006	0.032	1.000					
spread-t	-0.110	-0.076	0.049	0.028	0.075	-0.751	0.088	1.000				
fbills-t	-0.045	-0.026	-0.062	-0.298	-0.308	0.053	0.025	-0.025	1.000			
fdy-t	-0.106	-0.452	0.020	-0.695	-0.080	-0.021	0.451	0.022	0.220	1.000		
fspread-t	-0.012	-0.021	-0.037	0.136	0.051	0.003	0.014	0.048	-0.833	-0.071	1.000	

Notes: rtbr= log real 3-month Treasury bill return; xsr = log excess stock return; xbr= log excess bond return; bills= 3-month Treasury bill yield; dy = log dividend yield; spread= 10-year government bond yield 3-month Treasury bill yield. t-statistics are given in parentheses; Significance levels:t=1.645 (10%), t=1.960 (5%), where ** and * indicate the significance at 10% and 5%, respectively.

Table 6.4: VAR estimation results, Canada-U.k., 1954:06-2004:05

1 dependent variable	2 rtbr-t	3 xsr-t	4 xbr-t	5 fxsr-t	6 fxbr-t	7 bills-t	8 dy-t	9 spread-t	10 fbills-t	11 fdy-t	12 fspread-t	13 R^2
VAR slope coefficient estimates and goodness-of-fit measures												
rtbr-t+1	0.114	-0.001	0.009	0.000	0.000	0.000	0.001	0.000	0.000	-0.001	-0.001	0.129
	(2.463)*	(-0.325)	(1.189)	(-0.065)	(-0.215)	(1.308)	(1.419)	(-1.160)	(-3.129)*	(-1.125)	(-4.241)*	
xsr-t+1	-0.176	0.049	0.173	-0.008	-0.012	0.000	0.014	0.004	0.000	-0.002	-0.001	0.030
	(-0.368)	(0.860)	(1.539)	(-0.133)	(-0.632)	(-0.016)	(1.185)	(1.416)	(0.072)	(-0.166)	(-0.663)	
xbr-t+1	0.770	-0.126	0.141	-0.011	-0.018	0.000	0.005	0.004	0.001	0.000	-0.001	0.103
	(3.127)*	(-4.257)*	(1.998)*	(-0.323)	(-2.046)*	(0.355)	(1.090)	(2.678)*	(0.516)	(0.034)	(-1.163)	
fxsr-t+1	-0.033	0.166	-0.169	0.206	0.059	0.004	0.013	0.002	-0.002	-0.025	-0.001	0.178
	(-0.116)	(4.283)*	(-2.028)*	(4.520)*	(3.902)*	(2.610)*	(1.734)**	(1.345)	(-1.258)	(-2.689)*	(-1.068)	
fxbr-t+1	0.061	-0.120	0.594	0.219	0.058	0.012	0.043	0.002	-0.001	-0.112	-0.001	0.058
	(0.042)	(-0.827)	(2.192)*	(1.018)	(0.912)	(2.004)*	(1.184)	(0.313)	(-0.120)	(-2.123)*	(-0.331)	
bills-t+1	-15.800	1.777	-5.550	0.855	0.254	0.995	0.118	0.127	-0.060	0.084	-0.056	0.867
	(-2.659)*	(2.797)*	(-3.348)*	(1.295)	(1.229)	(36.799)*	(1.260)	(4.297)*	(-2.190)*	(0.624)	(-2.965)*	
dy-t+1	-0.050	-0.026	-0.209	0.037	0.013	0.001	0.976	-0.004	0.002	0.010	0.002	0.982
	(-0.099)	(-0.437)	(-1.808)**	(0.629)	(0.704)	(0.221)	(79.993)*	(-1.282)	(1.122)	(0.734)	(1.304)	
spread-t+1	4.773	0.218	3.619	-0.732	0.049	-0.094	-0.168	0.856	0.023	-0.094	0.059	0.916
	(0.983)	(0.367)	(2.768)*	(-1.287)	(0.274)	(-3.488)*	(-2.096)*	(35.443)*	(0.896)	(-0.808)	(3.410)*	
fbills-t+1	13.185	1.546	2.736	3.130	0.391	0.073	0.218	-0.004	0.886	-0.434	0.047	0.838
	(2.094)*	(2.496)*	(1.879)**	(3.757)*	(1.859)**	(2.132)*	(1.682)**	(-0.105)	(33.939)*	(-3.155)*	(2.165)*	
fdy-t+1	0.067	-0.006	0.066	0.051	0.181	0.003	0.030	0.001	0.001	0.958	-0.001	0.978
	(0.142)	(-0.104)	(0.697)	(0.857)	(6.839)*	(1.539)	(2.656)*	(0.272)	(0.743)	(68.005)*	(-0.390)	
fspread-t+1	-12.113	-0.444	-3.285	-1.157	0.217	-0.029	-0.028	0.026	-0.018	0.231	0.943	0.939
	(-1.948)**	(-0.769)	(-2.728)*	(-1.520)	(0.965)	(-0.934)	(-0.233)	(0.829)	(-0.748)	(1.731)**	(48.735)*	
Cross-correlations of VAR residuals												
	rtbr-t	xsr-t	xbr-t	fxsr-t	fxbr-t	bills-t	dy-t	spread-t	fbills-t	fdy-t	fspread-t	
rtbr-t	1.000											
xsr-t	0.020	1.000										
xbr-t	0.012	0.268	1.000									
fxsr-t	-0.002	-0.089	-0.238	1.000								
fxbr-t	-0.067	-0.518	-0.125	0.273	1.000							
bills-t	0.019	-0.206	-0.458	0.097	0.069	1.000						
dy-t	-0.060	-0.910	-0.307	0.111	0.488	0.227	1.000					
spread-t	-0.028	0.057	-0.159	0.036	0.014	-0.778	-0.056	1.000				
fbills-t	0.025	-0.049	-0.109	0.431	0.231	0.106	0.069	-0.050	1.000			
fdy-t	-0.027	-0.440	-0.092	0.288	0.785	0.059	0.415	0.003	0.209	1.000		
fspread-t	-0.012	-0.007	-0.020	-0.004	-0.101	-0.046	-0.006	0.077	-0.836	-0.062	1.000	

Notes: rtbr= log real 3-month Treasury bill return; xsr = log excess stock return; xbr= log excess bond return; bills= 3-month Treasury bill yield; dy = log dividend yield; spread= 10-year government bond yield 3-month Treasury bill yield. t-statistics are given in parentheses; Significance levels:t=1.645 (10%), t=1.960 (5%), where ** and * indicate the significance at 10% and 5%, respectively.

Table 6.5: In-sample and out-of-sample stock predictability test results, 1988:01-2004:05 out-of-sample period, for Canada,U.K., and U.S.

Variables	bills				spread				dy			
Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Canada												
Slope coefficient	-0.004	-0.010	-0.048	-0.043	0.004	0.012	0.045	0.048	0.002	0.007	0.033	0.047
t-statistic	-2.099	-1.593	-2.220	-2.902	2.046	2.064	2.024	2.397	1.172	1.060	1.527	1.494
	(0.987)	(0.920)	(0.971)	(0.990)	(0.017)	(0.036)	(0.045)	(0.031)	(0.263)	(0.364)	(0.279)	(0.338)
R-squared	0.016	0.019	0.075	0.049	0.015	0.023	0.066	0.059	0.011	0.011	0.034	0.053
Theil's U	1.001	1.006	1.030	0.987	0.995	0.988	0.950	0.937	1.014	1.042	1.177	1.246
MSE-F	-0.512	-2.376	-10.535	4.508	1.940	4.715	19.801	24.210	-5.274	-15.482	-51.431	-61.592
	(0.606)	(0.733)	(0.897)	(0.126)	(0.028)	(0.034)	(0.023)	(0.037)	(0.972)	(0.981)	(0.974)	(0.959)
ENC-new	0.483	0.366	2.418	6.032	2.031	5.225	20.761	16.617	-1.054	-3.004	-6.038	-10.890
	(0.147)	(0.291)	(0.202)	(0.085)	(0.014)	(0.015)	(0.012)	(0.054)	(0.933)	(0.943)	(0.843)	(0.870)
U.K.												
Slope coefficient	-0.002	-0.008	-0.036	-0.038	0.003	0.009	0.032	0.036	0.007	0.022	0.090	0.162
t-statistic	-1.063	-1.274	-1.278	-1.381	1.167	1.202	1.930	1.149	3.338	2.560	4.535	4.973
	(0.870)	(0.879)	(0.881)	(0.901)	(0.115)	(0.128)	(0.049)	(0.148)	(0.000)	(0.032)	(0.003)	(0.006)
R-squared	0.013	0.009	0.032	0.021	0.014	0.011	0.027	0.020	0.029	0.053	0.197	0.364
Theil's U	1.002	1.006	1.040	1.012	0.999	0.994	0.949	1.013	1.001	1.009	1.049	0.959
MSE-F	-0.710	-2.420	-13.829	-3.933	0.319	2.313	20.435	-4.414	-0.338	-3.619	-16.890	14.999
	(0.673)	(0.775)	(0.947)	(0.681)	(0.176)	(0.100)	(0.029)	(0.492)	(0.458)	(0.788)	(0.847)	(0.115)
ENC-new	-0.080	-0.171	-0.912	1.329	0.533	2.748	15.029	-1.112	6.064	16.474	64.085	125.458
	(0.434)	(0.447)	(0.593)	(0.248)	(0.144)	(0.074)	(0.043)	(0.514)	(0.000)	(0.001)	(0.000)	(0.000)
U.S.												
Slope coefficient	-0.006	-0.014	-0.041	-0.028	0.004	0.011	0.039	0.048	0.003	0.009	0.037	0.066
t-statistic	-3.577	-3.033	-2.197	-1.516	2.485	2.250	2.316	2.520	1.679	1.649	1.641	1.568
	(0.999)	(0.994)	(0.966)	(0.905)	(0.008)	(0.023)	(0.041)	(0.046)	(0.203)	(0.267)	(0.346)	(0.386)
R-squared	0.022	0.036	0.068	0.020	0.011	0.023	0.062	0.053	0.005	0.014	0.054	0.096
Theil's U	1.014	1.044	1.124	1.028	1.009	1.026	1.011	0.972	1.012	1.038	1.177	1.228
MSE-F	-5.241	-15.932	-38.506	-9.315	-3.457	-9.708	-3.952	10.153	-4.687	-14.035	-51.377	-58.368
	(0.994)	(0.996)	(0.999)	(0.834)	(0.977)	(0.979)	(0.580)	(0.135)	(0.940)	(0.949)	(0.962)	(0.915)
ENC-new	-0.652	-3.983	-12.121	-3.026	0.220	-0.143	6.964	7.699	-0.890	-2.569	-13.197	-21.440
	(0.929)	(0.994)	(1.000)	(0.836)	(0.248)	(0.421)	(0.117)	(0.164)	(0.882)	(0.887)	(0.954)	(0.965)

Note: p-values given in parentheses; bold entries indicate significance at the 10% level; 0.00 indicates < 0.005.

Table 6.6: In-sample and out-of-sample bond predictability test results, 1988:01-2004:05 out-of-sample period, for Canada,U.K., and U.S.

Variables	bills				spread				dy			
Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Canada												
Slope coefficient	-0.002	-0.006	-0.014	-0.019	0.003	0.008	0.017	0.016	0.000	-0.001	-0.008	-0.028
t-statistic	-1.958	-1.553	-1.672	-1.413	2.994	2.218	1.736	0.824	0.145	-0.400	-0.814	-1.586
	(0.970)	(0.923)	(0.933)	(0.924)	(0.001)	(0.023)	(0.099)	(0.274)	(0.529)	(0.729)	(0.805)	(0.906)
R-squared	0.016	0.023	0.034	0.026	0.024	0.043	0.049	0.019	0.010	0.005	0.016	0.051
Theil's U	0.999	0.997	0.975	0.971	0.994	0.976	0.951	1.000	1.023	1.041	1.072	0.997
MSE-F	0.285	1.226	9.712	10.355	2.527	9.731	19.348	0.052	-8.730	-15.042	-24.072	1.013
	(0.174)	(0.137)	(0.064)	(0.026)	(0.019)	(0.004)	(0.033)	(0.301)	(0.999)	(0.991)	(0.906)	(0.278)
ENC-new	0.796	2.639	6.614	5.897	3.516	10.779	12.567	0.569	-2.416	-5.095	-8.590	17.024
	(0.085)	(0.061)	(0.082)	(0.060)	(0.003)	(0.003)	(0.050)	(0.360)	(0.999)	(0.995)	(0.937)	(0.121)
U.K.												
Slope coefficient	0.000	-0.001	-0.006	-0.010	0.001	0.003	0.007	0.001	0.001	0.004	0.012	0.014
t-statistic	-0.639	-0.447	-0.815	-1.431	1.462	1.107	1.083	0.066	1.926	1.671	1.590	1.041
	(0.741)	(0.688)	(0.800)	(0.903)	(0.068)	(0.153)	(0.183)	(0.460)	(0.037)	(0.102)	(0.146)	(0.274)
R-squared	0.102	0.015	0.014	0.017	0.105	0.022	0.018	0.001	0.107	0.032	0.047	0.032
Theil's U	0.999	0.998	0.987	0.979	0.998	0.993	1.010	1.049	1.017	1.086	1.267	1.333
MSE-F	0.249	0.585	4.720	7.549	0.871	2.648	-3.457	-15.866	-6.675	-29.646	-69.694	-75.671
	(0.173)	(0.215)	(0.108)	(0.069)	(0.080)	(0.089)	(0.500)	(0.785)	(0.997)	(1.000)	(1.000)	(0.991)
ENC-new	0.146	0.439	3.069	4.674	1.315	4.307	2.301	-7.317	0.106	-1.424	-8.050	-14.585
	(0.266)	(0.277)	(0.150)	(0.100)	(0.039)	(0.037)	(0.248)	(0.865)	(0.300)	(0.856)	(0.939)	(0.953)
U.S.												
Slope coefficient	0.000	-0.002	-0.010	-0.009	0.002	0.005	0.019	0.019	0.000	0.000	-0.005	-0.013
t-statistic	-0.221	-0.468	-1.553	-0.904	2.425	2.270	2.965	1.550	0.250	-0.137	0.510	-0.762
	(0.605)	(0.697)	(0.922)	(0.835)	(0.004)	(0.023)	(0.021)	(0.114)	(0.457)	(0.600)	(0.695)	(0.757)
R-squared	0.020	0.003	0.029	0.010	0.029	0.031	0.097	0.044	0.020	0.000	0.010	0.020
Theil's U	1.001	1.004	0.995	0.991	1.003	1.008	0.978	1.018	1.019	1.041	1.067	1.066
MSE-F	-0.293	-1.558	1.978	3.297	-1.161	-3.093	8.415	-6.213	-7.230	-14.935	-22.385	-20.697
	(0.432)	(0.667)	(0.186)	(0.183)	(0.831)	(0.819)	(0.104)	(0.549)	(0.996)	(0.988)	(0.887)	(0.730)
ENC-new	-0.138	-0.436	3.171	2.211	1.636	5.163	20.306	5.396	-1.853	-4.952	-8.986	-8.773
	(0.560)	(0.620)	(0.153)	(0.219)	(0.019)	(0.023)	(0.027)	(0.186)	(0.990)	(0.992)	(0.943)	(0.811)

Note: p-values given in parentheses; bold entries indicate significance at the 10% level; 0.00 indicates < 0.005.

Table 6.7: Mean asset demands for SWF in Canada and the U.S. which can also invest in foreign stocks and bonds.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	Domestic stocks			Domestic bonds			Foreign stocks			Foreign bonds			Domestic bills		
CRRA	Total demand	Myopic demand	Hedging demand	Total demand	Myopic demand	Hedging demand	Total demand	Myopic demand	Hedging demand	Total demand	Myopic demand	Hedging demand	Total demand	Myopic demand	Hedging demand
Canada invests in foreign country:															
U.S., 1977:01-2004:04															
$\gamma = 3$	-6.49	-6.02	-0.47	200.53	181.46	19.08	167.58	86.94	80.64	-319.93	-227.88	-92.06	58.31	65.50	-7.19
$\gamma = 7$	-1.56	-2.34	0.78	92.98	78.03	14.95	96.66	36.83	59.83	-150.61	-98.10	-52.51	62.53	85.58	-23.05
$\gamma = 10$	0.32	-1.51	1.84	65.27	54.76	10.51	71.68	25.55	46.13	-102.23	-68.90	-33.33	64.95	90.10	-25.15
U.K., 1977:01-2004:04															
$\gamma = 3$	260.11	64.83	195.28	-263.43	-150.04	-113.38	-632.04	-600.64	-31.40	-53.70	15.46	-69.16	789.06	770.39	18.66
$\gamma = 7$	167.13	27.91	139.23	-125.44	-64.45	-60.99	-285.43	-257.58	-27.85	-43.23	6.77	-50.00	386.96	387.35	-0.39
$\gamma = 10$	132.46	19.60	112.86	-88.09	-45.19	-42.90	-203.71	-180.39	-23.33	-35.35	4.82	-40.17	294.70	301.16	-6.46
U.S. invests in foreign country:															
Canada, 1977:01-2004:04															
$\gamma = 3$	177.83	115.37	62.46	81.61	85.03	-3.41	10.83	-21.78	32.62	3.62	12.31	-8.69	-173.90	-90.92	-82.98
$\gamma = 7$	104.50	48.75	55.75	39.18	37.30	1.87	14.46	-8.76	23.22	-0.44	4.79	-5.23	-57.69	17.92	-75.61
$\gamma = 10$	81.05	33.76	47.29	31.55	26.57	4.98	13.14	-5.82	18.96	-2.02	3.09	-5.11	-23.71	42.41	-66.12
U.K., 1977:01-2004:04															
$\gamma = 3$	168.89	84.17	84.72	51.29	71.10	-19.81	55.43	21.69	33.75	34.50	54.57	-20.06	-210.11	-131.53	-78.59
$\gamma = 7$	112.31	36.07	76.23	22.26	30.62	-8.35	34.73	8.96	25.77	7.82	23.65	-15.83	-77.11	0.71	-77.82
$\gamma = 10$	91.97	25.25	66.73	16.06	21.51	-5.45	26.27	6.09	20.18	4.16	16.69	-12.53	-38.47	30.46	-68.93

Table 6.8: Mean asset demands for SWF in Canada and the U.S. which can also invest in foreign stocks and bonds, assuming different values for ψ the elasticity of intertemporal substitution and a coefficient of relative risk aversion γ equal to 7.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	Domestic stocks			Domestic bonds			Foreign stocks			Foreign bonds			Domestic bills		
CRRA	Total demand	Myopic demand	Hedging demand	Total demand	Myopic demand	Hedging demand	Total demand	Myopic demand	Hedging demand	Total demand	Myopic demand	Hedging demand	Total demand	Myopic demand	Hedging demand
Canada invests in foreign country:															
U.S., 1977:01-2004:04															
$\psi = 0.3$	-10.28	-2.34	-7.94	76.16	78.03	-1.87	64.37	36.83	27.54	-134.49	-98.10	-36.39	104.24	85.58	18.66
$\psi = 1.0$	-1.56	-2.34	0.78	92.98	78.03	14.95	96.66	36.83	59.83	-150.61	-98.10	-52.51	62.53	85.58	-23.05
$\psi = 1.5$	7.54	-2.34	9.88	112.23	78.03	34.20	128.44	36.83	91.61	-170.99	-98.10	-72.90	22.79	85.58	-62.79
U.K., 1977:01-2004:04															
$\psi = 0.3$	71.18	27.91	43.28	-146.34	-64.45	-81.89	-250.99	-257.58	6.59	-12.30	6.77	-19.08	438.44	387.35	51.10
$\psi = 1.0$	167.13	27.91	139.23	-125.44	-64.45	-60.99	-285.43	-257.58	-27.85	-43.23	6.77	-50.00	386.96	387.35	-0.39
$\psi = 1.5$	164.94	27.91	137.04	-125.88	-64.45	-61.43	-284.73	-257.58	-27.15	-42.67	6.77	-49.44	388.34	387.35	0.99
U.S. invests in foreign country:															
Canada, 1977:01-2004:04															
$\psi = 0.3$	68.51	48.75	19.76	17.52	37.30	-19.79	-1.66	-8.76	7.10	-1.90	4.79	-6.68	17.53	17.92	-0.39
$\psi = 1.0$	104.50	48.75	55.75	39.18	37.30	1.87	14.46	-8.76	23.22	-0.44	4.79	-5.23	-57.69	17.92	-75.61
$\psi = 1.5$	144.55	48.75	95.80	56.17	37.30	18.87	32.26	-8.76	41.02	4.28	4.79	-0.51	-137.26	17.92	-155.18
U.K., 1977:01-2004:04															
$\psi = 0.3$	65.25	36.07	29.18	5.81	30.62	-24.81	22.73	8.96	13.77	16.31	23.65	-7.34	-10.09	0.71	-10.80
$\psi = 1.0$	112.31	36.07	76.23	22.26	30.62	-8.35	34.73	8.96	25.77	7.82	23.65	-15.83	-77.11	0.71	-77.82
$\psi = 1.5$	150.55	36.07	114.48	35.44	30.62	4.82	43.82	8.96	34.86	1.14	23.65	-22.50	-130.96	0.71	-131.67

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DYNAMIC ASSET ALLOCATION FOR OIL-BASED SOVEREIGN WEALTH FUNDS: HEDGING DEMANDS AND INTERNATIONAL DIVERSIFICATION EFFECTS

The goal of this study is finding the dynamic asset allocation strategy for oil-based sovereign wealth funds. We have investigated the intertemporal hedging demands for assets for SWF in the U.S., and Canada, which can invest domestically and internationally. Using an Epstein-Zin-Weil utility function, where the dynamics governing asset returns are described by a vector autoregressive process. Our findings stress the importance of the mean intertemporal hedging demands for domestic stocks in the U.S. and to smaller extinct in Canada. A SWF in the U.S. displays small mean intertemporal hedging demands for foreign assets, while SWF in Canada exhibits sizable mean hedging demands for U.S. stocks. The international diversification seems more beneficial in Canada than U.S.
Keywords: Dynamic asset allocation, sovereign wealth funds, hedging demands, international diversification.

ALLOCATION DYNAMIQUES D'ACTIFS DES FONDS SOUVERAINS D’ORIGINE PERTROLIERE : LES EFFETS DE LA DEMANDE DE LA COUVERTURE ET LA DIVERSIFICATION INTERNATIONALE.

L'objectif de cette étude est de trouver la stratégie de l’allocation dynamique d'actifs des fonds souverains. Nous avons étudié la demande de couverture intertemporelle pour les fonds souverains aux États-Unis et au Canada. Ces derniers peuvent ainsi investir à l'échelle nationale et internationale en utilisant une fonction d'utilité de type Epstein-Zin-Weil. La dynamique régissant les rendements est modélisée par un processus vectoriel autorégressif (VAR). Les résultats montrent l'importance de la demande de couverture intertemporelle pour les actions domestique aux États-Unis et avec un degré moins au Canada. Un fond souverain aux États-Unis a montré une importance mineure de couverture intertemporelle pour les actions internationales, alors qu’au Canada la demande aux actions américaines a été élevée.
Ainsi, la diversification internationale semble plus Bénéfique au Canada qu’aux États-Unis.

Mots-clés : Allocation dynamique d'actifs, fonds souverains, demande de couverture intertemporelle, diversification internationale.

التوزيع الديناميكي لأصول الصناديق السيادية ذات الأصل النفطي: الطلب التحوطي وأثرالتنويع الدولي.

تهدف هذه الدراسة إيجاد إستراتيجية للتوزيع الديناميكي الأمثل لأصول صناديق الثروة السيادية ذات الأصل النفطي. ومن أجل ذلك قمنا بدراسة الطلب التحوطي لصناديق الثروة السيادية في كل من الولايات المتحدة و كندا على الأسهم والسندات محليا ودوليا وهذا باستخدام دالة منفعة من نوع Epstein-Zin-Wei حيث تمت نمذجة عوائد الأصول بنموذج متجة الانحدار الذاتي VAR. النتائج التي توصلنا إليها تؤكد على أهمية متوسط الطلب التحوطي للأسهم المحلية في الولايات المتحدة وبنسبة أقل للأسهم المحلية في كندا. أما في حالة صندوق سيادي في الولايات المتحدة ولديه إمكانية الاستثمار في الأسهم والسندات الأجنبية يلاحظ أن متوسط الطلب التحوطي على الأصول الأجنبية يعتبر ضئيلانسبيا. أما في حالة كندا يلاحظ أن الطلب التحوطي لأسهم الولايات المتحدة يعتبر كبير ومهم. أما فيما يخص التنويع الدولي، فهي معتبرة وأكثر أهمية في حالة الصندوق السيادي الكندي منه في الولايات المتحدة.

الكلمات المفتاحية: التوزيع الديناميكي للأصول، صناديق الثروة السيادية، الطلب التحوطي، التنويع الدولي.