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# A Flexible Transmission Line Model For Series-line Antennas Array Design

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*Abstract-* In this paper, a transmission line model is used to design series-fed antennas arrays over a band of frequencies for satellite communications. The transmission line model is simple, precise and allowing taking into account the whole geometrical, electric and technological characteristics of the antennas arrays. To validate this last, the obtained simulation results are compared with those obtained by the moment's method (MoM). Using this transmission line approach the resonant frequency, input impedance, return loss can be determined simultaneously. Agreements between transmission line model data and the moment's methods results were achieved.

*Index Terms*- Analysis, microstrip antennas, array, transmission line model, Moment's method.

# I. INTRODUCTION

Microstrip antennas have been widely used in modern applications and became a significant matter of research in the theoretical and practical electromagnetic field. They are well-known for their good desirable physical characteristics such as their lightness, their low cost and their small overall dimensions, easy of installation and its aerodynamic profile. For all these reasons, they are suitable for many application and are used in many communication systems such as wireless area network (WLAN), mobile hand sets, radars,...etc.

Many researchers studied their basic characteristics and great efforts were also devoted to their determination (resonant frequency, the band-width, radiation... etc) by using theoretical models. These models can be classified into three groups. The transmission line model is a simple model due to its assumptions. This leads to a set of linear equations with low dimension. In the cavity model, which converts the open antenna problem into a closed one, this dimension increases. Finally, the integral equation model solves the Maxwell equations directly. The equations are hard to compute and the dimension of the set is very large. The utility of any solution, however, depends on the accuracy of the results, as well as on the simplicity of the method.

Microstrip antennas that operate as a single element usually have a relatively large half power beamwidth, low gain and low radiation efficiency. In order to improve these parameters, microstrip antennas are used in Series-fed array configuration to improve the gain and range of the radiating structure. This configuration offers a very convenient form of array fabrication because both the feed network and the radiating elements can be made photolithographically, without any need for soldering to the elements [1].

In this paper, a transmission line model is presented for the analysis of antennas array operating in 2 and 4.8 GHz for satellites communications. A rigorous method which is the moment's method was adopted to show the validity of the suggested model by comparing the results of the return losses, input phase as well as the input impedance locus. A comparison of the results showed the validity of the proposed model.



The numerically efficient procedure presented in this paper is employed to analyze two different geometries and their results are presented. A comparison of the results produced by the final model with the moment's method data showed the validity of the proposed model. This allows the analysis of very large arrays even on rather small computer.

## II. TRANSMISSION LINE MODEL ANALYSIS

In this section, an equivalent circuit model for the proposed antenna is developed. This model is capable of predicting the slot radiation conductance and the antenna input impedance near resonance. This approach provides very helpful insight as to how this antenna and its feed network operate. As mentioned before, this model is also needed to find a proper matching network for the antenna.

The antenna has a physical structure derived from a microstrip transmission line. In this model, the microstrip antenna is modelled as a length of transmission line of characteristic impedance  $Z_0$  and propagation constant  $\gamma = \alpha + j\beta$ . The fields vary along the length of the patch, witch is usually a half-wave length, and remains constant across the width. Radiation occurs mainly from the fringing fields at the open ends as shown in Fig. 1 [2-3].



Fig.1. Rectangular microstrip antenna

#### III. INPUT IMPEDANCE

The effect of radiation is accounted for by the radiation and admittance called self admittance  $Y_s$  attached to the open ends of the transmission line. The transmission line model represents the antenna by a line section which finishes by an admittance  $Y_s$  on the level of its two ends. An equivalent electric representation of this model is schematically shown in Fig. 2 [4].



Fig.2. Equivalent circuit of van de chapelle

 $Y_s$  stands for the equivalent admittance of the main slits,  $Y_m$  their mutual admittance,  $Y_c$  is the characteristic impedance of the transmission line and  $\gamma_p$  is the complex propagation constant in this line. This is a three-port model which depends on the technique used for the antenna supply.

If we consider that the antenna is fed at its extremity by a microstrip line, and if we neglect the mutual coupling between the two radiating slots, the equivalent model of the microstrip source can be represented by a transmission line section with the same characteristics, terminated at both ends by a radiation admittance  $Y_s$  of conductance *G* and susceptance *B*. A general representation is shown in Fig. 3.



Fig.3. Equivalent network of microstrip radiating element

Due to the fringing fields along the radiating edges of the antenna there is a line extension associated with the patch, which is given by the formula [5]:



$$\frac{\Delta l}{h} = 0.412 \left[ \frac{\varepsilon_{eff} + 0.3}{\varepsilon_{eff} - 0.258} \right] \left[ \frac{\frac{w}{h} + 0.264}{\frac{w}{h} + 0.813} \right]$$
(1)

The effective dielectric constant  $\varepsilon_{eff}$  due to the air dielectric boundary is given by [6]:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + 10 \frac{h}{w} \right)^2 \tag{2}$$

The resonant frequency can be estimated by using the formula [7]:

$$f_r = \frac{1}{2\sqrt{\mu_0\varepsilon_0} (L + \Delta L)\sqrt{\varepsilon_{eff}}}$$
(3)

Where :

- $\mu_0$ : Permeability of free space
- $\varepsilon_o$ : Permittivity of free space
- $\Delta L$  : Pine extension
- $\epsilon_{eff}$ : Effective dielectric constant

The antenna effective width is given by the following formula :

$$W = \frac{1}{2f_r \sqrt{\mu_0 \varepsilon_0}} \sqrt{\frac{2}{\varepsilon_r + 1}} = \frac{v_0}{2f_r} \sqrt{\frac{2}{\varepsilon_r + 1}}$$
(4)

The real length of the antenna can be given and it is given by the following formula:

$$L = \frac{1}{2f_r \sqrt{\varepsilon_{reff}} \sqrt{\mu_0 \varepsilon_0}} - \Delta L \tag{5}$$

When the antenna resonates (L~  $\lambda g/2$ ), the total admittance becomes real and is calculated using the formula [8]:

$$Y_1 = G_1 + jB_1 \tag{6}$$

The expressions of  $G_1$  and  $B_1$  are given by the relations below:

$$\begin{cases} G_1 = \frac{W}{120\lambda_0} \left[ 1 - \frac{1}{24} (k_0 h)^2 \right] \\ B_1 = \frac{W}{120\lambda_0} [1 - 0.636 \ln(k_0 h)] \end{cases}$$
(7)

Where:  $\frac{h}{\lambda_0} \langle \frac{1}{10} \rangle$ ,  $\lambda_0$  is the free space wavelength

and 
$$k_0 = \frac{2\pi}{\lambda_0}$$
.

Because the slots are identical, we have:

$$Y_2 = Y_1; G_2 = G_1; B_2 = B_1$$

The conductance of a single slot can also be obtained by using the expression field derivative from model cavity. In general, the conductance is defined by:

$$G_1 = \frac{2P_{rad}}{\left|V_0\right|^2} \tag{8}$$

By using the electric field one can calculate the radiated power:

$$P_{rad} = \frac{|V_0|}{2\pi\eta_0}^2 \times \int_{0}^{\pi} \left[ \frac{\sin\left(\frac{K_0 W}{2} \cos \theta\right)}{\cos \theta} \right]^2 \sin^3 \theta d\theta$$
(9)

The self conductance can be calculated using the following expressions:

$$G_1 = \frac{I_1}{120\pi^2}$$
(10)

Where  $I_1$  is the integral defined by:



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$$I_{1} = \int_{0}^{\pi} \left[ \frac{\sin\left(\frac{k_{0}w}{2}\cos\theta\right)}{\cos\theta} \right]^{2} \sin^{3}\theta \, d\theta \tag{11}$$

$$Y_{in} = Y_1 + Y_2 = 2G_1$$
 (12)

$$Z_{in} = \frac{1}{Y_{in}} = R_{in} = \frac{1}{2G_1}$$
(13)

However the above equation for input impedance does not take into consideration the mutual coupling between the radiating slots, so we can redefine the input resistance:

$$R_{in} = \frac{1}{2(G_1 \pm G_{12})} \tag{14}$$

Where:

- $G_{12}$ : Mutual conductance
- $G_1$ : Self conductance.
- (+) : Odd resonant modes
- (-): Even resonant modes

The mutual conductance is defined in term of field by the following expression:

$$G_{12} = \frac{1}{|V_0|^2} Re \iint_s E_1 \times H_2^* ds$$
(15)

The mutual conductance  $G_{12}$  is calculated using the following expression:

$$G_{12} = \frac{1}{120\pi^2} \times \int_{0}^{\pi} \left[ \frac{\sin\left(\frac{k_0 w}{2} \cos \theta\right)}{\cos \theta} \right]^2 J_0(k_0 L \sin \theta) \sin^3 \theta \, d\theta$$
(16)

Where:

 $J_0$  is the Bessel function of the first kind. The impedance characteristic is given by.

$$Z_{c} = \begin{cases} \frac{120 \pi}{\sqrt{\varepsilon_{reff}} \left[ \frac{W_{0}}{h} + 1.393 + 0.667 ln \left( \frac{W_{0}}{h} + 1.444 \right) \right]} & \frac{W_{0}}{h} \rangle 1 \\ \frac{60}{\sqrt{\varepsilon_{reff}}} ln \left[ \frac{8h}{W_{0}} + \frac{W_{0}}{4h} \right] & \frac{W_{0}}{h} \leq 1 \end{cases}$$
(17)

The input resistance is given by:

$$R_{in} = \frac{1}{2(G_1 + G_{12})} \times \left[ \cos^2(\beta_g L) + \frac{G_1^2 + B_1^2}{Y_c^2} \sin^2(\beta_g L) \right]^{-1}$$
(18)  
$$\left[ -\frac{B}{Y_c} \sin(2\beta_g L) \right]^{-1}$$

### IV. SERIES FED ARRAY

One is interested in the case of a linear array fed in series by a characteristic line microruban of impedance Zc as presented by Fig. 3. (a).

To calculate the input impedance of the printed antennas array, one supposes to exploit the electric model are equivalent of each aerial element established previously to lead to a complete electric modelling of the entire array. The equivalent diagram of this last is shown in the following Fig. 3. (b).



Fig.3. (a) The mask layout for the antennas array (b) Equivalent circuit of the antennas array



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$$G_1 = \frac{I_1}{120\pi^2} \tag{10}$$

Where  $I_1$  is the integral defined by:



According to the figure above, one notices that the transmission line model of the of is close to the axis 50 Ohm, while the moments method curve is one can far.

## B. 4.8 GHz antennas array

In this section, other geometry is analyzed by using the method proposed in this paper. The permittivity and the substrate thickness are 2.55 and 1.59 mm respectively and the operation frequency is 4.8 GHz. A probe of 50 Ohm is employ to feed the antennas array.

Fig.8 presents the mask layout for the antennas array presenting resonance at 4.8 GHz.



Fig.8. Antennas array architecture

The input computed return losses  $S_{11}$  of the antennas array functioning at 4.8 GHz has been reported in Fig.9.

According to the figure above, even though there is a shift in the resonant frequency, the transmission line model tracks the return loss profile predicted by the moment method very closely. The small shift in the resonant frequency can be attributed to a failure to consider the discintinuty between the antennas and the feed lines.



Fig. 9. Computed return losses

According to the figure above, even though there is a shift in the resonant frequency, the transmission line model tracks the return loss profile predicted by the moment method very closely. The small shift in the resonant frequency can be attributed to a failure to consider the discintinuty between the antennas and the feed lines.

The input phase of return loss of the antennas array is show in figure 11.



Fig.10. Computed phase of return

According to the figure above, one notices that the models have the same pace in spite of the undulations presented by the moment's method.



Fig. 12. Smith's chart of the input impedance return losses. Frequency points given by start = 4.0 GHz, stop = 6.0 GHz.



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The input impedance or the antenna has been calculated over a frequency range of 4.0- 6.0 GHz. It can be seen from Figure 12 that the comparison for the input impedance between transmission line model and the moment method results are in good agreement. One notices that the resonant frequency is very close to the axis of 50 Ohm.

## VI. CONCLUSION

A flexible and computation-efficient transmission line model is developed to analyse the antennas array. The results so far show that the transmission line model can be successfully used to predict the input characteristic of the antennas array over wide band frequencies. Even though the model is conceptually simple, it still produces accurate results in a relatively short period of computing time.

The results obtained highlighted an excellent agreement between the transmission line model and the moment's method.

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